FEDERAL PUBLIC SERVICE COMMISSI

**COMPETITIVE EXAMINATION FOR
RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT, 2011**

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PURE MATHEMATICS, PAPER-II

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS: 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **THREE** questions from **SECTION – A** and **TWO** questions from **SECTION – B.** All questions carry equal marks.
(ii) **Use of Scientific Calculator is allowed.**
(iii) **Extra attempt of any question or any part of the attempted question will not be considered.**

SECTION - A

Q.1. (a) Prove that every non-empty set of real numbers that has an upper bound also has an supremum in R. **(10)**

(b) If $x \in R$, set of real numbers, then there exists **n** $\in$ **N** such that **x < n**. **(10)**

Q.2. (a) Define continuity of a function at a point and also prove that if **f** and **g** be functions on **A** to R, where **A** $\subseteq$ R then **f + g** and f g are continuous at **C**. **(10)**

(b) If **f : I** $\to$ **R** is differentiable at **C** $\in$ **I**, then f is continuous at **C**. **(10)**

Q.3. (a) Evaluate $\int_1^5 \frac{dx}{\sqrt[3]{x-2}}$. **(08)**

(b) (i) Define Complete metric space. **(04)**

(ii) Prove that a sequence of real numbers is convergent iff it is a Cauchy sequence. This theorem is not in metric space, for justification give one example. **(08)**

Q.4. (a) Let **(x, d)** be a matric space and **A** a subset of **X**. Then prove that

(i) Interior A° of A is an open subset of **X**. **(05)**

(ii) A° is the largest subset of **X** contained in **A**. **(05)**

(b) State and prove Mean value theorem. **(10)**

Q.5. (a) If $\sum a_n$ converges absolutely then $\sum a_n$ converges. **(10)**

(b) Find the area enclosed by the parabola $y^2 + 16x - 71 = 0$ and the line **4x + y + 7 = 0** **(10)**

SECTION – B

Q.6. (a) Let $Z = (\cos\theta + i\,Sin\theta)$. Then prove that $Z^n = Cos\,n\theta \;\; i\,Sin\,n\theta$ for all **n**. **(10)**

(b) Using De Moivre's Theorem evaluate $\left(\frac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$. **(10)**

Q.7. (a) Expand **f(x) = x^2 , 0 < x < 2**π in a Fourier series if period is 2π. **(10)**

(b) If f(z) is analytic inside a circle C with centre at a, then for all Z inside C **(10)**

$$f(z) = f(a) + f'(a)(z-a) + \frac{f''(a)}{2!}(z-a)^2 + \dots$$

Q.8. (a) Evaluate the integral by using Cauchy integral Formula **(10)**

$$\int_c \frac{(4-3z)dz}{z(z-1)(z-2)} \quad \text{where C is a circle } |z| = \;^3/_2.$$

(b) Prove that $$\int_0^{2\pi} \frac{d\theta}{1-2pCos\theta - p^2} = \frac{2\pi}{1-p^2}.$$ **(10)**

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