FEDERAL PUBLIC SERVICE COMMISSION

COMETITIVE EXAMINATION FOR THE RECRUITMENT TO POST IN PBS-17, UNDER THE FEDERAL GOVERNMENT, 2004

APPLIED MATHEMATICS (PAPER-I)

TIME ALLOWED: THREE HOURS **MAX. MARKS: 100**

NOTE: Attempt **FIVE** questions in ALL according to the following instructions. **ALL** questions carry **EQUAL** marks.

(i) Question **NUMBER-8** is **COMPULSORY**.

(ii) Select at least **TWO** questions each from **SECTION – I and SECTION – II.**

SECTION – I

1.(a) With the usual notation, prove that $[\vec{a}\ \vec{b}\ \vec{c}]^2 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{a}.\vec{b} & \vec{a}.\vec{c} \\ \vec{b}.\vec{a} & \vec{b}.\vec{b} & \vec{b}.\vec{c} \\ \vec{c}.\vec{a} & \vec{c}.\vec{b} & \vec{c}.\vec{c} \end{vmatrix}$ (12)

(b) Prove that $\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$ (8)

2.(a) State Stoke's theorem and verify it for $\vec{A} = (2x - y)i - yz^2 j - y^2 z k$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (12)

(b) If $\vec{r} = xi + y j + zk$, then show that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r = |\vec{r}|$. (8)

3. (a) The least force which will move a weight up an inclined plane is of magnitude P. Show that the least force, acting parallel to the plane, which will move the weight upwards is $P\sqrt{1+\mu^2}$, where μ is the coefficient of friction. (12)

(b) Define the term centre of mass. Find the centre of mass of uniform solid hemisphere. (8)

SECTION – II

4. (a) Find the tangential and normal components of velocity and acceleration of a particle describing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (10)

(b) Find the velocity acquired by a block of wood of mass M lb., which is free to recoil when it is struck by a bullet of mass of m lb. moving with velocity v, in a direction passing through the centre of gravity.

If the bullet is embedded d ft., show that the resistance of the wood to the bullet, supposed uniform, is $\frac{Mmv^2}{2(M+m)gd}$ lb. wt. and that the time of penetration is $\frac{2d}{v}$ sec., during which time the block will move $\frac{ma}{M+m}$ ft. (10)

5.(a) A particle of mass m moves under a central force $mM3au^4 - 2(a^2 - b^2)u^5$ (a>b) and is projected from an apse at a distance $(a+b)$, with velocity $\frac{\sqrt{M}}{(a+b)}$. Show that the orbit is $r = a + b\cos\theta$. (8)

(b) Prove that the speed at any point of a central orbit is given by $v = ph$, where h is the areal speed and p is the perpendicular distance, from the centre of force, of the tangent at that point. Hence find the expression for v when a particle, subject to the inverse square law of force, describes (i) an elliptic (ii) a parabolic and (iii) a hyperbolic orbit. (12)

6.(a) Define the terms moment of inertia and product of inertia. State and prove parallel axes Theorem. (12)

(b) Find the moment of inertia of a uniform circular disc about an axis through the centre of the disc and perpendicular to the plane of the disc. (8)

7.(a) Define conservative force. Show that the total energy of a particle under the action of a conservative force is always conserved. (10)

(b) A battleship is steaming ahead with speed V, and a gun is mounted on the battleship so as to point straight backwards, and is set at an angle of elevation α. If v_0 is the speed of projectile (relative to the gun), show that the range is $\frac{2v_0}{g}\sin\alpha(v_0\cos\alpha - V)$ (10)

COMPULSORY SECTION

8. Write only your correct choice in the Answer Book. Do not write the question. Each part carries one mark.

(1) If M, L, T denote the dimensions of mass, length, time, respectively, then the dimensions of potential energy are

(a) MLT^{-1} (b) ML^2T^{-1} (c) ML^2T^2 (d) ML^2T^{-2} (e) none of these

(2) A force is described by its

(a) point of application only (b) direction only (c) point of application and direction only (d) direction and magnitude only (e) none of these

(3) The coefficient of static friction μ depends upon

(a) force of friction, $\vec{F}$ (b) normal reaction, $\vec{N}$ (c) nature of the surface

(d) direction of $\vec{F}\times\vec{N}$ (e) none of these

(4) If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\pi/4$, then $\frac{\vec{a}\times\vec{b}}{\vec{a}\bullet\vec{b}}$ is equal to

(a) 1 (b) $1/\sqrt{2}$ (c) $\sqrt{3}/2$ (d) $1/2$ (e) none of these

(5) If a particle executing harmonic motion describes a periodic path $x(t) = c\cos(2at/3 + \varepsilon)$, then its period of oscillation is

(a) $\frac{2\pi}{\varepsilon}$ (b) $2\pi\omega/c$ (c) $2a\pi/3$ (d) $3\pi/a$ (e) none of these

(6) The general motion of a rigid body is
(a) a rotation (b) a translation (c)vibration
(d) a translation followed by a rotation (e) none of these

(7) The power of a force is
(a) the time rate of change of Potential energy (b) the time rate of change of kinetic energy
(c) the time rate of change linear momentum (d) the time rate of change of angular momentum (e) None of these

(8) The impulse is equal to the
(a) increment in linear momentum (b) increment in angular momentum
(c) increment in linear velocity (d) increment in angular velocity (e) none of these

(9) Tension in a string is
(a) an external force (b) a non-conservative force (c) a constraint force
(d) a sliding force (e) None of these

(10) If φ is any scalar function, the $\vec{\nabla} \bullet (\vec{\nabla} \times \phi)$ is
(a) a vector (b) a scalar (c) zero (d) undefined (e) None of these

(11) The moment of inertia of uniform rod of mass m and length 2a about a perpendicular bisector is equal to
(a) $\frac{ma^3}{3}$ (b) $\frac{2ma^2}{3}$ (c) $\frac{ma^2}{3}$ (d) $\frac{4ma^2}{3}$ (e) none of these

(12) If i, j, k are the unit vectors along x, y, z - axes, respectively, then $2i \times (5k \times 3j)$ is
(a) 0 (b) $\vec{1}$ (c) $30k$ (d) $-30i$ (e) none of these

(13) For the simple harmonic motion of a particle, the force
(a) is minimum (b) is periodic (c) is constant and towards a fixed point
(d) is towards a fixed point and directly proportional to the acceleration
(e) none of these

(14) An apse is point on a central orbit which lies from the centre at
(a) a maximum distance (b) a minimum distance (c) both maximum and minimum distance (d) a maximum or a minimum distance (e) none of these

(15) A force is said to be conservative if, and only if,
(a) $(\vec{\nabla} \times \vec{\nabla}) \bullet \vec{F} = 0$ (b) $\vec{\nabla} \bullet \vec{F} = \vec{0}$ (c) $\vec{\nabla} \times \vec{F} = 0$ (d) $(\vec{\nabla} \times \vec{F}) \bullet \vec{\nabla} = \vec{0}$
(e) none of these

(16) If three forces are represented in magnitude, direction and position by the sides of a triangle taken in order, then they
(a) are equivalent to a single force (b) are in equilibrium (c) are equivalent to a couple
(d) are equivalent to a force and a couple (e) none of these

(17) With Sun at one focus, each planet describes
(a) circle (b) ellipse (c) parabola (d) conics (e) none of these

(18) Any three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar if, and only if,

(a) $\vec{A} \bullet (\vec{B} \times \vec{C}) = \vec{0}$ (b) $\vec{A} \times (\vec{B} \bullet \vec{C}) = \vec{0}$ (c) $\vec{A} \bullet (\vec{B} \bullet \vec{C})$

(d) $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{0}$ (e) none of these

(19) If the angular momentum $\vec{h} = (at-1)i + 5j - 3k$ of a particle is conserved, then a is equal to

(a) 1 (b) $\vec{1}$ (c) 0 (d) $\vec{2}$ (e) none of these

(20) When two rough bodies are in contact with each other, then in the case of limiting friction

(a) no friction force is acting between them
(b) friction force is acting but neither body is on the point of sliding along the other
(c) one of the bodies is on the point of sliding along the other
(d) one body slides along the other.
(e) none of these

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NOTE: Attempt **FIVE** questions in ALL according to the following instructions. ALL questions carry **EQUAL** marks.

(i) Question **NUMBER-8** is **COMPULSORY**.

(ii) Select at least **TWO** questions each from **SECTION – I** and **SECTION – II.**

SECTION – I

1.(a) Solve the following differential equation:

$$(2xy^2 + 2xye^{2x} + ye^{2x})dx + (2x^2y + xe^{2x})dy = 0 \quad (12)$$

(b) Find the orthogonal trajectories of the family $x^2 - cx + 4y = 0$ (8)

2.(a) Using power series method, find the solution of the differential equation $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (12)

(b) Verify whether or not $u(x, y, z) = \frac{y(z+x)}{y+z}$ is a solution of the differential equation $(y^2 + yz)dx + (z^2 + zx)dy + (y - xy)dz = 0$. (8)

3.(a) Find the general solution of the linear partial differential equation:

$$x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x + y)z, \quad (12)$$

(b) Using method of variation of parameters, solve the following differential equation

$$x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = x^4 \ln x \qquad (x > 0) \quad (8)$$

SECTION – II

4. (a) Define
(i) Symmetric tensor (ii) Skew-symmetric tensor (iii) Kronecker delta tensor
(iv) Levi-Civita tensor. Show that Kronecker delta is tensor of rank two (12)

(b) Using tensor notation, prove that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \bullet \vec{c})\vec{b} - (\vec{a} \bullet \vec{b})\vec{c}$ (8)

5.(a) Using Jacobi's iterative method, solve the following system of equations: (10)

$$4x_1 - x_2 - x_3 = 0.5$$
$$-x_1 + 4x_2 - x_4 = 1.3$$
$$-x_1 + 4x_3 - x_4 = 1.0$$
$$-x_2 - x_3 + 4x_4 = 1.8$$

(b) Using regula falsi method, find the root correct to four decimal places between 0.4 and 0.6 of the equation $\sin x = 5x - 2$. (10)

6.(a) Using Newton-Raphson method, derive the iterative procedure for function $f(x) = 1 - \frac{5}{x^2}$. If the initial approximation is 2, calculate the value of $\sqrt{5}$ correct to two decimal places. (10)

(b) Evaluate $\int_0^1 e^{-x^2} dx$, by using Simpson's $\frac{1}{3}$rd rule correct to six decimal places with $h = 0.1$. Also find the error bounds. (10)

7. (a) Find the value of $f(x)$ corresponding to $x = 5$ from the following table, by using Lagrange interpolation formula: (8)

x	1	3	4	6
F(x)	4	7	8	11

(b) Use Gauss-Seidel method to solve the following system of linear equations: (12)

$$x_1 - 0.25x_2 - 0.25x_3 = 50$$
$$-0.25x_1 + x_2 - 0.25x_4 = 50$$
$$-0.25x_1 + x_3 - 0.25x_4 = 25$$
$$-0.25x_2 - 0.25x_3 + x_4 = 25$$

COMPULSORY SECTION

8. Write down only your correct choice in the Answer Book. Do not write the question. Each part carries one mark.

(1) Any vector is a tensor of rank
(a) -1 (b) zero (c) *one* (d) *three* (e) none of these

(2) A homogeneous differential equation always:
(a) possesses a non-trivial solution (b) possesses a trivial solution (c) no solution at all
(d) possesses a singular solution (e) none of these

(3) If the numbers 0.3062, 0.25026 and 2.51392 are rounded, then absolute error in the number $z = 0.3062 - 0.25026 + 2.51392$ is
(a) 0.6×10^{-4} (b) 0.5×10^{-4} (c) 0.4×10^{-6} (d) 0.4×10^{-5} (e) none of these

(4) The general solution of the equation $x^2 p + y^2 q = (x^4 - y^4)$ is
(a) $(x^2 - y^2)/z$ (b) $(x^3 - y^3)/3$ (c) $(x^4 - y^4)/4$ (d) $(x^3 + y^3)/3$
(e) none of these

(5) The process of contraction always reduces the rank of a mixed tensor by
(a) 1 (b) 2 (c) 3 (d) 4 (e) none of these

(6) A fourth order partial differential equation in $u(x, y)$ is linear if it is of
(a) degree one in $u(x, y)$ and its 4^{th} order partial derivatives w.r.t. x
(b) degree one in $u(x, y)$ and its 4th order partial derivatives w.r.t y
(c) degree one in $u(x, y)$ and its all partial derivatives upto order four
(d) degree one in $u(x, y)$ and its all 4^{th} order partial derivatives
(e) none of these

(7) Gauss-Seidel is a
(a) a direct method for system of linear equations (b) an iterative method for system of nonlinear equations (c) a direct method for system of nonlinear equations
(d) an iterative method for system of linear equations (e) none of these

(8) The second order partial differential equation $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=0$ is
(a) a hyperbolic homogeneous (b) a hyperbolic nonhomogeneous
(c) a parabolic homogeneous (d) an elliptic homogeneous (e) None of these

(9) If A_{jk} is skew symmetric tensor of rank 2 and η_j is tensor of rank one, then $A_{jk}\eta_j\eta_k$ is equal to
(a) 1 (b) δ_{jk} (c) zero (d) 4 (e) none of these

(10) The equations $y'''+4y''+y=\sin x;\ y(0)=1,\ y'(0)=1,\ y'(1)=1$, define
(a) a third order initial value problem (b) an initial value problem
(c) a boundary value problem (d) a mixed boundary value problem
(e) none of these

(11) The numbers 4.0643 and .37487 are rounded, then the relative error in product of these numbers $z = 4.0643 \times .37487$ is
(a) $.2564\times10^{-4}$ (b) $.2564\times10^{-6}$ (c) $.2564\times10^{-5}$ (d) $.2564\times10^{-3}$ (e) none of these

(12) The differential equation $(3x^3y-y^3)dx+(x^3+3xy^2)dy=0$ has a singular point
(a) (0,0) (b) $(1,\sqrt{3})$ (c) $(1,-\sqrt{3})$ (d) no singular point (e) none of these

(13) If $A_{j_1j_2\ldots j_m}$ and $B^{k_1k_2\ldots k_n}$ are covariant and contravariant tensors of rank m and n, then $A_{j_1j_2\ldots j_m}B^{k_1k_2\ldots k_n}$ is
(a) a covariant tensor of rank (m+n) (b) a contravariant tensor of rank (m+n)
(c) a contravariant tensor of rank mn (d) a mixed tensor of rank mn
(e) None of these

(14) The differential equation $y'^2+y^2-4=0$ has a singular solution
(a) 1 (b) -2 (c) 4 (d) -4 (e) none of these

(15) The quantity $\delta^i_m\delta^j_n\delta^k_p\varepsilon_{ijk}A^p_{qr}$ is a tensor of rank
(a) zero (b) one (c) three (d) four (e) none of these

(16) The differential equation $(1-x^2)y''-2xy'+n(n+1)y=0$ is
(a) Bessel equation (b) Legendre equation (c) hypergeometric equation
(d) Poisson equation (e) none of these

(17) The value of the tensorial expression $\varepsilon_{132}\varepsilon_{123}\varepsilon_{131}+\varepsilon_{321}\varepsilon_{232}\varepsilon_{231}$ is equal to
(a) 1 (b) -1 (c) zero (d) (e) none of these

(18) A primitive of an ordinary differential equation is
(a) its general solution (b) its particular solution (c) its complementary solution
(d) its complete solution (e) none of these

(19) The error in Simpson's $\frac{1}{3}$rd rule is of the order
(a) h^2 (b) h^3 (c) $h^{1/2}$ (d) h^5 (e) none of these

(20) The partial differential equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ represents the
(a) transverse vibrations of a string
(b) longitudinal vibrations of a bar
(c) sound wave in space
(d) longitudinal sound waves
(e) none of these