

**Modified Enlarged 18 pt**

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS**

**Tuesday 14 June 2022 – Morning**

**Level 3 Cambridge Technical in Engineering**

**05823/05824/05825/05873**

**Unit 23: Applied mathematics for engineering**

**Time allowed: 2 hours plus your additional time allowance**

**You must have:**

**the Formula Booklet for Level 3  
Cambridge Technical in Engineering  
(with this document)**

**a ruler (cm/mm)**

**a scientific calculator**

**Please write clearly in black ink.**

**Centre  
number**

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**Candidate  
number**

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**First name(s)** \_\_\_\_\_

**Last name** \_\_\_\_\_

**Date of  
birth**

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**READ INSTRUCTIONS OVERLEAF**

## INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

Write your answer to each question in the space provided. If you need extra space use the lined pages at the end of this booklet. The question numbers must be clearly shown.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.

## INFORMATION

The total mark for this paper is 80.

The marks for each question are shown in brackets [ ].

## ADVICE

Read each question carefully before you start your answer.

**BLANK PAGE**

Answer ALL the questions.

- 1 FIG. 1 opposite shows a side view of an ironing board supported in a horizontal position on two sloping legs AB and CD which each have a length of 1.1 m.

Leg AB is fixed on a hinge located on the underside of the board at point A while leg CD is locked into an anchor point at C, which is also located on the underside of the board.

The legs are connected together by a pivot at point P which is 0.4 m from both A and C.

- (i) The flat surface of the ironing board is 0.8 m above the floor.

Calculate the angle between the two legs,  $\theta$ , shown in FIG. 1.

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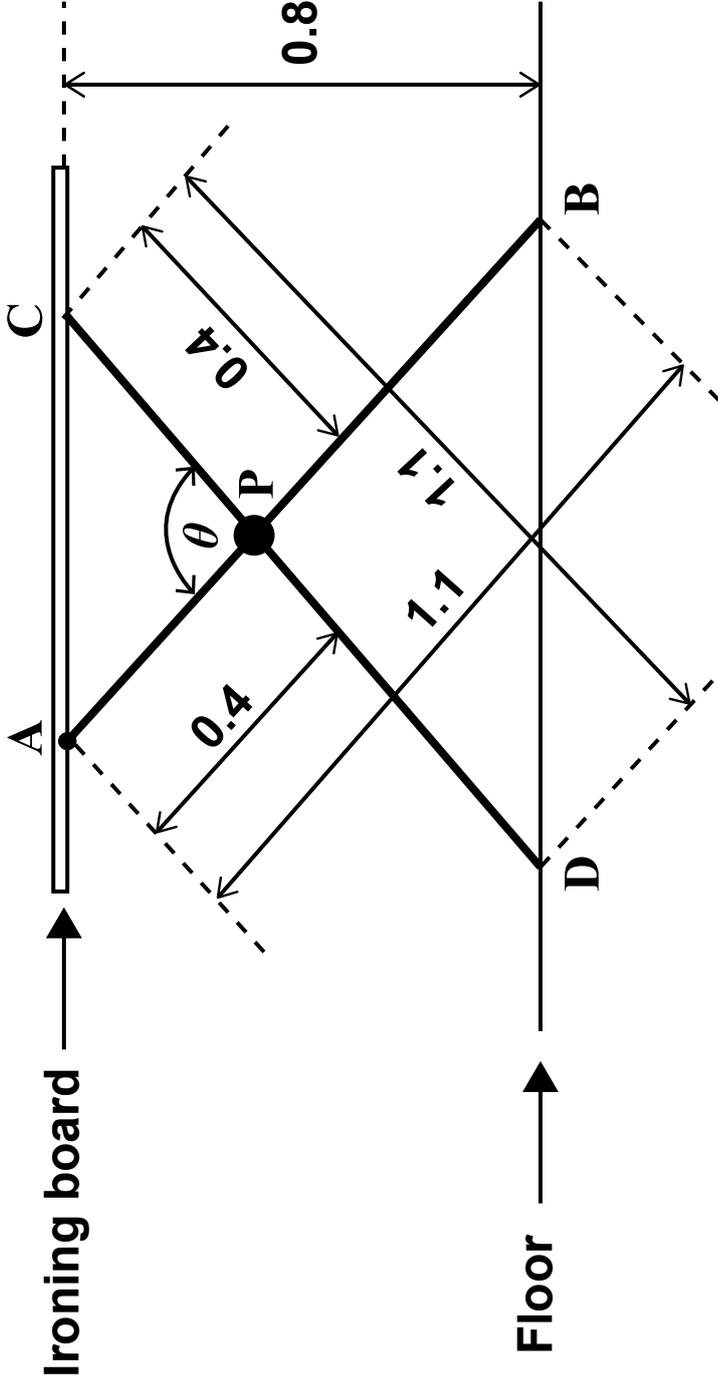
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[3]

**FIG. 1**



**Not to scale**

(ii) Calculate the distance AC.

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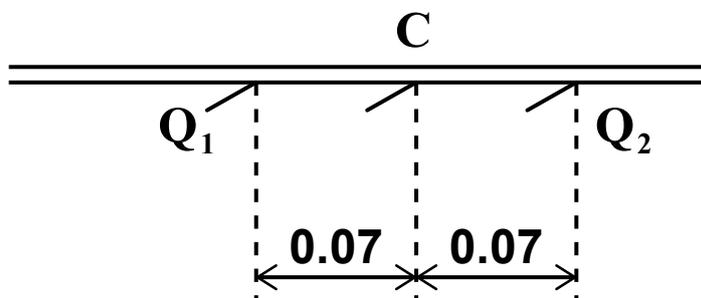
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[3]

In order to allow the ironing board to be used at a height greater than or less than 0.8 m above the floor, the designer introduces two further anchor points at  $Q_1$  and  $Q_2$ , each 0.07 m from the original anchor point C, as shown in FIG. 2.

FIG. 2



(iii) Calculate the minimum height at which the board can be used, AND state which anchor point is used. [6]

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- 2 A small helicopter has mass 400 kg. The lift force,  $F_N$ , provided by its rotor is modelled by  $F = 0.02 N^2 \rho$ , where  $N$  rpm is the rotational speed of the rotor and  $\rho \text{ kg m}^{-3}$  is the density of air.

In this question you should assume that the density of air at ground level is  $1.22 \text{ kg m}^{-3}$  and that this diminishes linearly by  $0.032 \text{ kg m}^{-3}$  for every 250 m of height above the ground.

- (i) Calculate the maximum rotational speed of the rotor at which the free-standing helicopter will remain on the ground.

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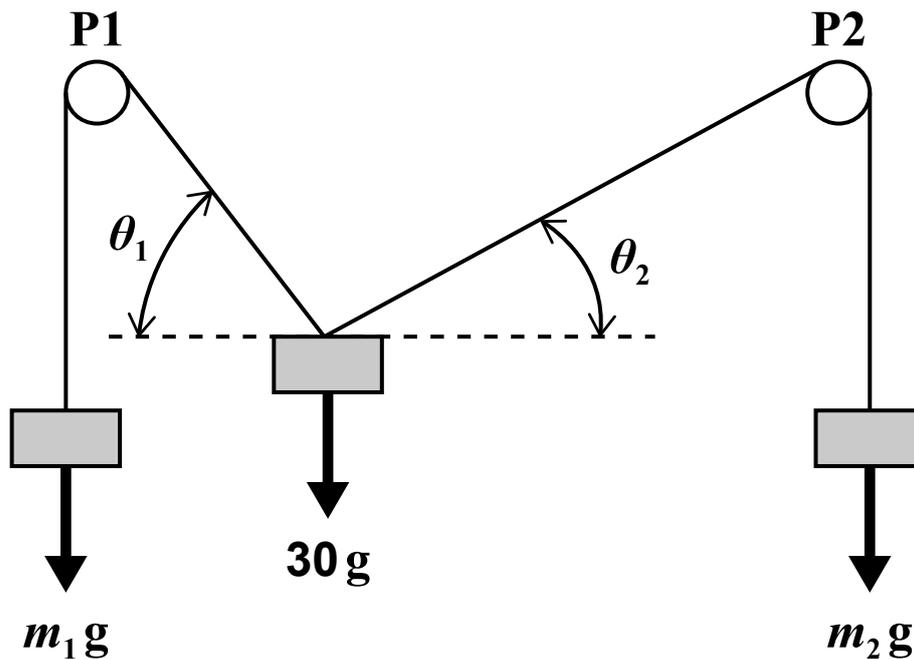
[2]







## 3 FIG. 3



A mass of 30 kg is supported in equilibrium by two light cables. The cables each pass over one of two fixed frictionless pulleys, P1 and P2, located above and on either side of the mass. Masses  $m_1$  kg and  $m_2$  kg hang vertically from the other ends of the two cables.

The end of the cable that passes over P1 is attached to the mass of  $m_1$  kg; this cable makes an angle of  $\theta_1$  with the horizontal. The other end of the cable that passes over P2 is attached to the mass of  $m_2$  kg; this cable makes an angle of  $\theta_2$  with the horizontal as shown in FIG. 3.

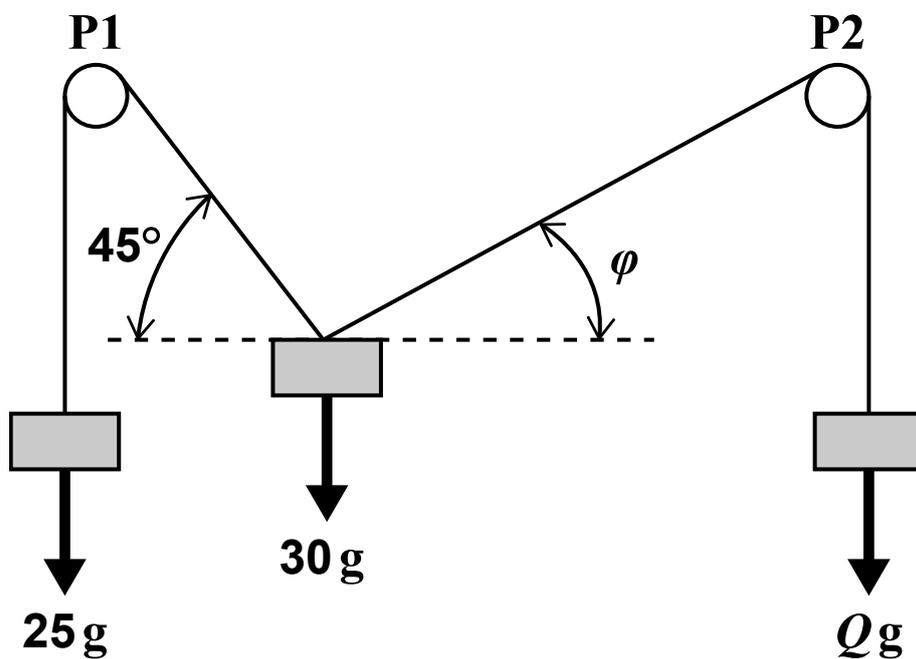


The mass of  $m_1$  kg is replaced by a mass of 25 kg, and the mass of  $m_2$  kg is replaced by a mass of  $Q$  kg.

The cable passing over P1 still makes an angle of  $45^\circ$  with the horizontal. The cable passing over P2 now makes an angle of  $\phi$  with the horizontal, as shown in FIG. 4.

The system is again in equilibrium.

FIG. 4









- (b) The capacity of lead-acid batteries used in road vehicles is rated in terms of Amp-hours (Ah). A fully charged battery rated at 80 Ah should deliver a constant current of 5 A for a discharge time of 16 hours. However, if the same battery is discharged with a higher current, for example 8 A, then the total discharge time will be less than 16 hours.

Peukert's formula predicts the time, in hours, to completely discharge a fully charged battery with a constant current.

Peukert's formula is often stated as follows.

$$t = H \left( \frac{C}{IH} \right)^k,$$

where  $t$  is the predicted discharge time in hours,

$C$  is the rated capacity in Amp-hours,

$H$  is the rated discharge time in hours,

$I$  is the constant discharge current in Amps,

$k$  is Peukert's constant. This is a dimensionless parameter and typically has a value between 1.1 and 1.3.

- (i) Given that  $H = 16$ ,  $C = 80$  and  $k = 1.2$ , use Peukert's formula to complete the following table.

Discharge current $I$ (Amps)	3	7
Predicted discharge time $t$ (hours)		

Use the space below for working if needed. [2]

- (ii) Under experimental conditions a new battery is tested with the following parameters.

$$H = 16, C = 80, I = 8, t = 9.$$

Calculate the value of  $k$  in Peukert's formula giving your answer correct to 2 decimal places. [3]

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- 5 A new domestic storage heater is being tested. The heater is powered until the heat energy output flow,  $q$ , is 2500 W. The heater is then disconnected from the power and moved into a small room. The temperature in the room is  $T^\circ\text{C}$ ; when the heater is first put in the room  $T = 16$ . The heater's energy output flow is then used to heat the air in the room for a period of 12 hours. As time passes, the heater's energy output flow decreases according to the following formula.**

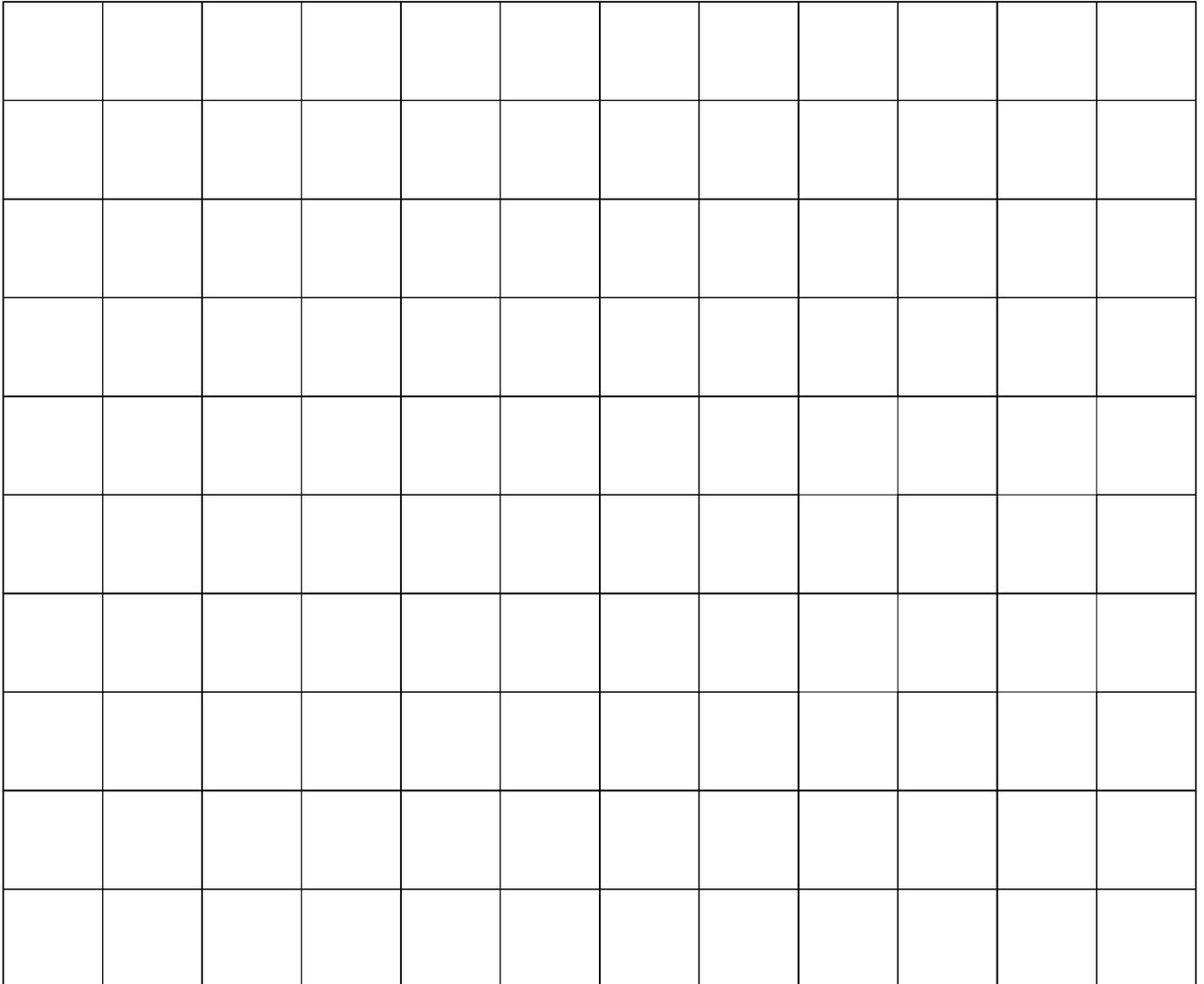
**$q = 2500 e^{-t/5}$ , where  $t$  hours is the time after the heater was moved into the room.**

**During the 12-hour period the temperature in the room is modelled by the following equation.**

$$T = 16 + \frac{q\sqrt{t}}{500}$$



(iii) Sketch a graph of  $T$  against  $t$  on the grid below for  $t$  in the range 0 to 12. [3]





- (ii) The electrical mains domestic supply in the UK is an alternating voltage given by  $v = V \sin(\omega t)$ , where  $V$  is peak voltage,  $\omega$  is frequency in radians per second and  $t$  is time in seconds.

The root mean square (RMS) value of this voltage is given by

$$\sqrt{\frac{1}{T} \int_0^T V^2 \sin^2(\omega t) dt},$$

where  $T$  seconds is the period of one complete voltage cycle.

- (A) Show that  $T = \frac{1}{50}$  when  $\omega = 100\pi$ .

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[1]

- (B) Calculate the RMS value when  $V = 330$  and  $\omega = 100\pi$ . [6]

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