UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9794 MATHEMATICS

9794/02 Paper 2 (Pure Mathematics and Mechanics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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UNIVERSITY of CAMBRIDGE
International Examinations

1	**(i)**	Substitute $x = 4$ into equation or attempt factorisation of $(x - 4)$	M1	
		Verify $y(4) = 0$ or that $(x - 4)$ is a factor	A1	[2]
	(ii)	*May be seen in part **(i)***		
		$x^3 - 12x - 16 = (x-4)(x^2 + 4x + 4)$	B1 B1	
		$= (x-4)(x+2)(x+2)$	B1	[3]
2	**(i)**	Attempt to multiply out brackets	M1	
		Obtain $61 - 28\sqrt{3}$.	A1	[2]
		SC For answer given without working – B1		
	(ii)	$\sqrt{125} = 5\sqrt{5}$ seen	B1	
		Multiply numerator and denominator by $2 - \sqrt{5}$, and expand.	M1	
		and use of $(2+\sqrt{5})(2-\sqrt{5}) = -1$.	A1	
		Obtain $25 - 10\sqrt{5}$. AG	A1	[4]
3		$\frac{dv}{dx} = \sin 3x, \quad u = x$	M1	
		$v = -\frac{1}{3}\cos 3x, \quad \frac{du}{dx} = 1$	A1	
		Obtain an expression of the form $f(x) \pm \int g(x)dx$	M1	
		Obtain $-\frac{x}{3}\cos 3x + \int \frac{1}{3}\cos 3x \, dx$	A1√	
		$= -\frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x + c$ CAO	A1	[5]
4	**(i)**	Shape of each graph (concavity).	B1 B1	
		Asymptote at $\frac{\pi}{2}$	B1	
		Max/Min points clearly indicated at $x = 0$ and π.	B1	[4]
	(ii)	Evidence that $\sec x = \frac{1}{\cos x}$	B1	
		Multiply by $\cos x$, obtaining a quadratic.	M1	
		Solve quadratic.	M1	
		Solutions $x = \pi$	A1	
		and $x = 0.841$	A1	[5]
		SC For either both in degrees or one in degress and one in radians – A1A0		

5	**(i)**	Attempt to solve $c = 1$ (or $c < 1$) for at least one drug, and obtain a solution.	M1	
		Obtain 54.9 (hours) for Antiflu;	A1	
		Obtain 23.0 (hours) for Coldcure.	A1	[3]
(ii)		Two ***decaying*** exponentials in the first quadrant showing	M1	
		correct intercepts on the c-axis and crossing for some $t > 0$.	A1	[2]
(iii)		Assume additive nature of the concentrations:	M1	
		$5\mathrm{e}^{-0.07\times 30} + 5\mathrm{e}^{-0.07\times 10} = 3.10.$	A1	[2]
6	**(i)**	$\mathrm{d}u = 2x\mathrm{d}x$ or equivalent used	M1	
		Substitute to obtain $\int \frac{1}{2}\mathrm{e}^{-\frac{1}{2}u}\,\mathrm{d}u$	A1	
		Obtain $\left[-\mathrm{e}^{-\frac{1}{2}u}\right]$	A1	
		Evaluate: 0.5 WWW	A1	[4]
		SC For 0.5 without working – B2		
	(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 1\times\mathrm{e}^{-\frac{1}{2}x^2} + x\times(-x)\times\mathrm{e}^{-\frac{1}{2}x^2}$	M1 A1	
		Equate to zero and find at least one point	M1	
		Stationary points $\left(1, \mathrm{e}^{-0.5}\right)$; $\left(-1, -\mathrm{e}^{-0.5}\right)$.	A1	[4]
7	**(i)**	**(a)** Not invertible	B1	
		Not 1–1 or equivalent	B1	[2]
		(b) (Minimum value of –1 at $x = 1$)	B1	
		$-1 \le \mathrm{f}(x)$	B1	[2]
		[B1 for correct interval; B1 for correct inequality]		
	(ii)	**(a)** $\mathrm{gh}(x) = \sin^2 x.$	B1	
		Obtain $\frac{1}{2}(1-\cos 2x)$ with some working AG	B1	[2]
		(b) Sine wave	M1	
		Period of π	A1	
		Completely correct	A1	[3]

8	**(i)**	**(a)**	$\dfrac{dx}{d\theta} = 1 - \cos\theta$	B1	
			$\dfrac{dy}{d\theta} = \sin\theta$	B1	
			$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \Big/ \dfrac{dx}{d\theta} = \dfrac{\sin\theta}{1-\cos\theta}$	M1	
			$= \dfrac{2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta}{2\sin^2\frac{1}{2}\theta} = \cot\frac{1}{2}\theta$	A1 AG	
			At least two of $\theta = ... - 2\pi, 0, 2\pi...$ without any incorrect values	B1	[5]
		(b)	Rearranging $y = x$ to give		
			$\theta = 1 + \sin\theta - \cos\theta$	M1	
			$= 1 + A\sin(\theta - \alpha)$	M1	
			where $A = \sqrt{2}$	A1	
			and $\alpha = \dfrac{\pi}{4}$	A1	[4]
		(c)	Consider sign of $\theta - 1 - \sqrt{2}\sin\left(\theta - \dfrac{\pi}{4}\right)$ at $\theta = \dfrac{\pi}{2}, \pi$	M1	
			Change of sign implies root:		
			$\left(\dfrac{\pi}{2} - 2(\text{negative}) \text{ and } \pi - 2\ (\text{positive})\right)$	A1	[2]
	(ii)		$\dfrac{dy}{dx} = \dfrac{\sin\theta}{2-\cos\theta}$	B1	
			$\dfrac{d^2y}{dx^2} = \dfrac{d}{d\theta}\left(\dfrac{dy}{dx}\right) \times \dfrac{d\theta}{dx}$ or equivalent	M1	
			$= \dfrac{2(2\cos\theta - 1)}{(2-\cos\theta)^3}$ AEF, unsimplified	A1	
			$\dfrac{d^2y}{dx^2} = 0 \Rightarrow y = \dfrac{3}{4}$	A1	[4]

9	(i)	P has x-coordinate k.	B1	
		Region R has area $\frac{1}{2}k\times k^2-\int_0^{(k)}x^2\mathrm{d}x$ or $\int_0^{(k)}kx-x^2dx$	M1	
		$=\frac{1}{2}k^3-\frac{1}{3}k^3$		
		$=\frac{1}{6}k^3$ AG	A1	[3]
	(ii)	$\int_0^a kx-x^2\mathrm{d}x=\frac{1}{12}k^3$ or equivalent.	M1	
		$=\left[\frac{1}{2}kx^2-\frac{1}{3}x^3\right]_0^a$	A1	
		$\Rightarrow k^3-6ka^2+4a^3=0$ AG	A1	[3]
	(iii)	Differentiate the implicit equation wrt t:	M1	
		$3k^2\frac{\mathrm{d}k}{\mathrm{d}t}-12a\frac{\mathrm{d}a}{\mathrm{d}t}k-6a^2\frac{\mathrm{d}k}{\mathrm{d}t}+12a^2\frac{\mathrm{d}a}{\mathrm{d}t}=0$	A1 (< 3 errors)	
			A1 CAO	
		Make substitutions and obtain $\frac{\mathrm{d}a}{\mathrm{d}t}=1$.	A1	[4]
		<u>*OR*</u>:		
		Differentiate the implicit equation wrt a or k	M1	
		$3k^2\frac{\mathrm{d}k}{\mathrm{d}a}-12ak-6a^2\frac{\mathrm{d}k}{\mathrm{d}a}+12a^2=0$ or $3k^2-12a\frac{\mathrm{d}a}{\mathrm{d}k}k-6a^2+12a^2\frac{\mathrm{d}a}{\mathrm{d}k}=0$	A1	
		Relate connected rates of change	M1	
		Make substitutions and obtain $\frac{\mathrm{d}a}{\mathrm{d}t}=1$.	A1	
	(iv)	$\left(\text{The formula } \frac{\mathrm{d}a}{\mathrm{dt}}=\frac{k^2-2}{2(k-1)} \text{ may appear}\right)$		
		Attempt to factorise k^3-6k+4 with linear factor $(k-2)$	M1	
		Obtain $(k-2)(k^2+2k-2)$	A1	
		Solve quadratic factor and obtain either or both of $k=\pm\sqrt{3}-1$	A1	
		Correctly substitute into derivative formula and attempt to simplify	M1	
		Obtain either or both of $\frac{\mathrm{d}a}{\mathrm{d}t}=1\pm\sqrt{3}$.	A1	[5]

10	**(i)**	Any valid method, for example		
		$AB.AC = (-4\mathbf{i}+3\mathbf{k}).(3\mathbf{i}+4\mathbf{k})$	M1	
		$= -12+12=0$ Hence result.	A1	[2]
	(ii)	Resolving along AB: $T_{AB} = 20\cos\left(\tan^{-1}\frac{4}{3}\right)$	M1	
		Obtain 12N.	A1	
		Resolving along AC: $T_{AC} = 20\sin\left(\tan^{-1}\frac{4}{3}\right) = 16\text{N}$	A1	[3]
		SC Both answers either unassigned or swapped – B1		
	(iii)	The vector tension is 12 x unit vector in *AB* direction	M1	
		$= -9.6\mathbf{i} + 7.2\mathbf{j}$	A1√ A1√	[3]
		Or $= -a\mathbf{i} + b\mathbf{j}$ where $\frac{a}{b} = \frac{4}{3}$ and $a^2 + b^2 = (their\ T_{AB})^2$		
11	**(i)**	Use of $x = Vt\cos\theta$ and $y = Vt\sin\theta - \frac{1}{2}gt^2$	M1 A1	
		Solving $y = 0$ for t and substitute in x formula	M1	
		$R = \frac{2V^2\sin\theta\cos\theta}{g}\left(=\frac{V^2\sin 2\theta}{g}\right)$	A1 AG	[4]
	(ii)	(symmetry of the trajectory) implies $R = 100\text{m}$	B1	[1]
	(iii)	$V = \sqrt{1000}\ \left(=10\sqrt{10}\right)\text{ms}^{-1}$ (= $\sqrt{g \times theirR}$)	B1√	
		Solving		
		$30 = 10\sqrt{10}t\ \sin\frac{\pi}{4}$	M1	
		Obtain $t = \frac{3}{\sqrt{5}}$ or $t = \frac{x}{V\cos\theta}$ and substitute later	A1 M1	
		Obtain $h = 21\text{m}$	A1	[5]

12	**(i)**	All forces shown: Applied, weight and reaction.	B1	[1]
	(ii)	Net force up the slope $20-20\sin 30=10(\text{N})$	B1	
		Use $'\text{Force}=\text{mass}\times\text{acceleration}' \Rightarrow a=5\text{ms}^{-2}$	B1	
		Applying 'suva' with $u=0$ and $a=5$	M1	
		$v=5t$.	A1	[4]
	(iii)	Let U and $V(V>U)$ be the speeds of the particles up the slope after the collision. An attempt at both of		
		COM: $2\times 15-1\times 5=2\times U+1\times V$	M1 A1√	
		NEL: $0.2\times(15-(-5))=V-U$	M1 A1√	
		Obtain $U=7\text{ms}^{-1}$	A1	
		'suva' gives $v=7-5T$, where T is time after impact.	A1√	[6]
13	**(i)**	As the system is in equilibrium, the tension in the string is $T=mg$	B1	
		Resolving at right angles to the plane:	M1	
		$R+T\sin\alpha=2mg\cos\alpha$		
		giving $R=mg(2\cos\alpha-\sin\alpha)$.	A1 AG	[3]
	(ii)	By implication $\alpha\le 45°$ (condone boundary case only)	M1	
		$\cos\alpha\ge\frac{1}{\sqrt{2}}$; $\sin\alpha\le\frac{1}{\sqrt{2}}$	A1	
		$R\ge mg\left(\frac{2}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$	A1 AG	[3]
	(iii)	Resolving up the slope		
		$F=2mg\sin\alpha-T\cos\alpha=mg(2\sin\alpha-\cos\alpha)$	M1	
		For this to be positive	A1	
		and combined with first line of solution of **(ii)**		
		$0.5<\tan\alpha\le 1$	A1 AG	[3]
	(iv)	Using $F=\mu R$	M1	
		$\mu=\frac{2\sin\alpha-\cos\alpha}{2\cos\alpha-\sin\alpha}=\frac{2\tan\alpha-1}{2-\tan\alpha}$	A1	
		Max value of μ is 1 when $\tan\alpha=1$.	A1	[3]