UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9795 FURTHER MATHEMATICS

9795/02 Paper 2 (Further Application of Mathematics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2011 question papers for most IGCSE, Pre-U, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

UNIVERSITY of CAMBRIDGE International Examinations

1	**(i)**	$(X_1+X_2+X_3)-(Y_1+Y_2+Y_3+Y_4) \sim \text{N}((3\times 30-4\times 20),(3\times 9+4\times 4))$ i.e. N(10,43)	B1,B1	[2]
	(ii)	$z=\dfrac{0-10}{\sqrt{43}}=-1.52(5)$ (Accept 1.52 to 1.53)	M1A1	
		P($X_1+X_2+X_3>Y_1+Y_2+Y_3+Y_4$) = 0.936	A1	[3]
2	**(i)**	$\text{P(X} \le 5) = \text{e}^{-12.25}\left(1+12.25+\dfrac{12.25^2}{2!}+\dfrac{12.25^3}{3!}+\dfrac{12.25^4}{4!}+\dfrac{12.25^5}{5!}\right)$	M1A1	
		= 0.0174	A1	[3]
	(ii)	$z=\dfrac{5.5-12.25}{3.5}=-1.929$	M1A1	
		P(X ≤ 5) (from tables) = 1 – 0.9731 = 0.0269 (Allow 0.027(0))	A1	[3]
	(iii)	Result is not reliable (error is approximately 50%).	B1	
		The mean is not large enough.	B1	[2]
3	**(i) (a)**	$s^2=\dfrac{156.88+123.97}{15+10-2}=12.21$ (AG)	M1A1	[2]
	(b)	$\bar{x}_A=30.7$ $\bar{x}_B=33.4$	B1	
		$\nu=23$ $t_{23}(0.975)=2.069$	B1B1	
		95% confidence limits are: (33.4–30.7) $\pm 2.069\times 3.494\sqrt{\dfrac{1}{15}+\dfrac{1}{10}}$	M1A1√	
		95% confidence interval is: (–0.252, 5.65) (Accept –0.251 for LB)	A1	[6]
	(ii)	Since $0\in\text{CI}$ there is not enough evidence to suggest that the claim is valid.	M1A1√	[2]

4	(i)	$\mathrm{E}\left(X^2\right)= \int_0^\infty \frac{x^2}{\theta} e^{-\frac{x}{\theta}} \mathrm{d}x$	M1
		$= \left[-x^2 \mathrm{e}^{-\frac{x}{\theta}}\right]_0^\infty + \int_0^\infty 2x\mathrm{e}^{-\frac{x}{\theta}}\mathrm{d}x$	M1A1
		$= \left[-2x\theta \mathrm{e}^{-\frac{x}{\theta}}\right]_0^\infty + \int_0^\infty 2\theta\mathrm{e}^{-\frac{x}{\theta}}\mathrm{d}x$	M1
		$= \left[-2\theta^2 \mathrm{e}^{-\frac{x}{\theta}}\right]_0^\infty$	
		$= 2\theta^2$	A1 [5]
	(ii)	$\mathrm{E(T)} = \theta^2$	B1
		$\mathrm{E}(T) = \mathrm{E}\left[k\left(X_1{}^2 + X_2{}^2 + ... + X_n{}^2\right)\right]$	M1
		$= k\left[\mathrm{E}(X_1{}^2) + \mathrm{E}(X_2{}^2) + ... + \mathrm{E}(X_n{}^2)\right]$	A1
		$= kn\mathrm{E}(X^2)$	
		$\Rightarrow \theta^2 = 2kn\theta^2$	M1
		$\Rightarrow k = \frac{1}{2n}$	A1 [5]

5	**(i)** $\mathrm{P}(X=r)=q^{r-1}p \;\Rightarrow \mathrm{M}(t)=\sum_{1}^{\infty} q^{r-1}p\mathrm{e}^{tr}$	M1	
	$=\frac{p}{q}\sum_{1}^{\infty}(qe^{t})^{r}$	A1	
	$=\frac{p}{q}\times\frac{q\mathrm{e}^{t}}{1-q\mathrm{e}^{t}}=\frac{p\mathrm{e}^{t}}{1-q\mathrm{e}^{t}}$ (AG)	A1	[3]
	(ii) (a) $\mathrm{M}'(t)=\frac{(1-q\mathrm{e}^{t})p\mathrm{e}^{t}+p\mathrm{e}^{t}.q\mathrm{e}^{t}}{(1-q\mathrm{e}^{t})^{2}}=\frac{p\mathrm{e}^{t}}{(1-q\mathrm{e}^{t})^{2}}$	M1	
	$\mathrm{E}(X)=\mathrm{M}'(0)=\frac{p}{(1-q)^{2}}=\frac{p}{p^{2}}=\frac{1}{p}$	M1A1	
	(b) $\mathrm{M}''(t)=\frac{(1-q\mathrm{e}^{t})^{2}.p\mathrm{e}^{t}+2p\mathrm{e}^{t}(1-q\mathrm{e}^{t})q\mathrm{e}^{t}}{(1-q\mathrm{e}^{t})^{4}}$	M1	
	$\mathrm{E}(X^{2})=\mathrm{M}''(0)=\frac{p^{3}+2p^{2}q}{p^{4}}=\frac{p+2q}{p^{2}}=\frac{1+q}{p^{2}}$ (acf)	A1	
	$\mathrm{Var}(X)=\frac{1+q}{p^{2}}-\frac{1}{p^{2}}=\frac{q}{p^{2}}$ or $\frac{1-p}{p^{2}}$ (OE)	M1A1	[7]
	(iii) $\mu=6$ and $\sigma^{2}=30$ (or $\sigma=\sqrt{30}$)	B1	
	$\mathrm{P}\left(\lvert Y-6\rvert<\sqrt{30}\right)=\mathrm{P}(Y\le 11)$	M1	
	$=1-\left(\frac{5}{6}\right)^{11}=0.865$	A1	[3]

6	(i)	$F(x) = \int_0^x \frac{2}{\pi} dx = \left[\frac{2x}{\pi}\right]_0^x = \frac{2x}{\pi} \quad \left(0 < x < \frac{\pi}{2}\right)$	B1B1
		$\Rightarrow G(y) = \frac{2\sin^{-1} y}{\pi} \quad 0 < y < 1$	B1B1
		$\Rightarrow g(y) = \frac{dG}{dy} = \begin{cases} \frac{2}{\pi\sqrt{1-y^2}} & 0 < y < 1 \\ 0 & \text{Otherwise} \end{cases}$ (AG)	M1A1 [6]
		OR	
		For $X \sim U\left(0, \frac{\pi}{2}\right)$ and $y = \sin x$	B1
		$\int_0^{\frac{\pi}{2}} \frac{2}{\pi} dx = 1$	B1
		$x = 0 \Rightarrow y = 0$ and $x = \frac{\pi}{2} \Rightarrow y = 1$	
		$\frac{dy}{dx} = \cos x \Rightarrow \frac{dx}{dy} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-y^2}}$	M1A1
		$\int_0^1 \frac{2}{\pi} \cdot \frac{1}{\sqrt{1-y^2}} dy = 1$	A1
		$f(y) = \begin{cases} \frac{2}{\pi\sqrt{1-y^2}} & 0 < y < 1 \\ 0 & \text{Otherwise} \end{cases}$ (AG)	A1 [6]
	(ii) (a)	$\int_0^M \frac{2}{\pi\sqrt{1-y^2}} dy = \frac{1}{2}$	M1
		$\frac{2}{\pi} \arcsin M = \frac{1}{2}$	M1A1
		$M = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ (CAO)	M1A1 [5]
	(b)	$E(Y) = \int_0^1 \frac{2}{\pi} \cdot \frac{y}{\sqrt{1-y^2}} dy$	M1
		$= -\frac{2}{\pi}\left[\sqrt{1-y^2}\right]_0^1 = \frac{2}{\pi}$ (CAO)	M1A1 [3]

7	(i)	Uses conservation of energy: $0.3\times10\times1.5(\cos40^0-\cos75^0)=\frac{1}{2}\times0.3v^2$ where speed is v ms^{-1}. $\Rightarrow v=3.90$ (AG) (g = 9.8 gets M1A1A0).	M1A1 A1	 [3]
	(ii)	Transverse component = $-g\sin40^0=-6.428=-6.43$ ms^{-2} (Ignore sign). Radial component = $\frac{v^2}{r}=\frac{15.216...}{1.5}=10.1(4)$ ms^{-2} Magnitude of acceleration = $\sqrt{(6.428^2+10.14^2)}=12.0$ ms^{-2}.	B1 B1 B1√	 [3]
8	(i) (a)	Component of velocity perpendicular to wall = $0.4\times7\sin60^0$ (=2.425)	M1A1	[2]
	(b)	Uses impulse = change in momentum. $\lvert\text{Im}\rvert=0.3(7\sin60^0+0.4\times7\sin60)=2.55$ Ns	M1 A1	 [2]
	(ii)	Magnitude of velocity = $\sqrt{(7\cos60^0)^2+(0.4\times7\sin60^0)^2}=4.26$ ms^{-1}. Direction = $\tan^{-1}\left(\frac{0.4\times7\sin60^0}{7\cos60^0}\right)=34.7^0$	M1A1 M1A1	 [4]
9	(i)	$\mathbf{v}_A=(10\sin42^\circ)\mathbf{i}+(10\cos42^\circ)\mathbf{j}$ and $\mathbf{v}_B=15\mathbf{i}$ ${}_A\mathbf{v}_B=(10\sin42^\circ-15)\mathbf{i}+(10\cos42^\circ)\mathbf{j}$ (Accept –8.31**i** + 7.43**j**).	M1A1	[2]
	(ii)	$\mathbf{r}_A=(10\sin42^\circ)t\mathbf{i}+(10\cos42^\circ)t\mathbf{j}$ and $\mathbf{r}_B=15t\mathbf{i}+13\mathbf{j}$ ${}_A\mathbf{r}_B=(10\sin42^\circ-15)t\mathbf{i}+(10\cos42^\circ\ t-13)\mathbf{j}$ (Accept –8.31t**i** + [7.43t – 13]**j**).	M1A1	[2]
	(iii)	By considering relative motion, one object is taken to be at rest and the other moving relative to it. Hence, when closest together, the relative position vector is perpendicular to the relative velocity vector; (i.e. dot product = 0).	B1	[1]
	(iv)	${}_A\mathbf{r}_B\cdot{}_A\mathbf{v}_B=(10\sin42^\circ-15)^2t+10\cos42^\circ(10\cos42^\circ\ t-13)=0$ $\Rightarrow(100\sin^242^\circ-300\sin42^\circ+225)t+100\cos^242^\circ\ t-130\cos42^\circ=0$ $\Rightarrow t=\frac{130\cos42^\circ}{(325-300\sin42^\circ)}=0.777$ or 47 minutes (i.e. 1247)	M1A1 M1A1	 [4]

10	**(i)**	$T_1 = \dfrac{20x}{2}$, where T_1 is the tension in *AP* (or *BP*)	M1A1	
		$T_2 = \dfrac{10(3-x)}{1}$, where T_2 is the tension in the other string.	A1	
		$10x = 30 - 10x \Rightarrow x = 1.5$ So *AP* is 3.5 metres.	A1	[4]
	(ii)	Apply Newton II: $\dfrac{10(1.5-y)}{1} - \dfrac{20(1.5+y)}{2} = 0.5\ddot{y}$	M1A1	
		$\Rightarrow \ddot{y} = -40y$ ($\Rightarrow \varpi = \sqrt{40}$) (AG)	A1	[3]
	(iii)	Use $y = a\cos\varpi t$	M1A1	
		$-0.5 = \cos(\sqrt{40}t)$ (a= 1)		
		$\dfrac{2\pi}{3} = \sqrt{40}t \Rightarrow t = \dfrac{\pi}{3\sqrt{10}}$ (= 0.331)	M1A1	[4]
11	**(i)**	Take x and y axes along and perpendicular to plane. $y = V\sin\theta\, t - 0.5g\cos\alpha\, t^2$	M1	
		$y = 0$ on landing $\Rightarrow t = \dfrac{2V\sin\theta}{g\cos\alpha}$	M1A1	
		$x = v\cos\theta\left(\dfrac{2V\sin\theta}{g\cos\alpha}\right) - \dfrac{1}{2}g\sin\alpha\left(\dfrac{2V\sin\theta}{g\cos\alpha}\right)^2$ (up plane).	M1A1	
		$\Rightarrow x = \dfrac{2V^2\sin\theta}{g\cos^2\alpha}(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ (AG)	A1	[6]
	(ii)	$\dfrac{2V^2\sin\theta}{g\cos^2\alpha}(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ (down plane).	B1	[1]
	(iii)	$4(\cos\theta\cos\alpha - \sin\theta\sin\alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$ $\Rightarrow 3\cos\theta\cos\alpha = 5\sin\theta\sin\alpha$	M1A1√	
		$\Rightarrow \dfrac{3}{\tan\alpha} = 5\tan\theta$	M1	
		$\Rightarrow \tan\theta = 1.2 \Rightarrow \theta = 50.2^0$ (Allow $\tan^{-1}1.2$ or 0.876 rad.)	A1A1	[5]

12	**(i)**	Driving force at speed v is $\dfrac{400000}{v}$ N	B1	
		Resistive forces total $20000+\dfrac{250000\times 10}{500}=25000$ (No penalty for g=9.8 here.)	M1A1	
		At maximum speed these are equal $\dfrac{400000}{v}=25000 \Rightarrow v=16\text{ ms}^{-1}$. (Allow 16.1 from g= 9.8)	M1A1	[5]
	(ii)	Apply Newton II $\dfrac{400000}{v}-25000=250000v\dfrac{dv}{dx} \Rightarrow \dfrac{16}{v}-1=10v\dfrac{dv}{dx}$	M1A1	
		$\Rightarrow \dfrac{1}{10}\int dx=\int\dfrac{v^2}{16-v}dv \Rightarrow \dfrac{1}{10}\int dx=\int\left(-v-16+\dfrac{256}{16-v}\right)dv$	M1A1√	
		$\Rightarrow \dfrac{x}{10}=-\dfrac{v^2}{2}-16v-256\ln\lvert 16-v\rvert+c$	M1	
		$v=6$ when $x=0 \Rightarrow c=114+256\ln 10$ (703.5) (acf) (Or use of limits)	M1A1	
		$\Rightarrow \dfrac{x}{10}=114+256\ln 10-72-192-256\ln 4=256\ln\left(\dfrac{5}{2}\right)-150$ when v = 12.	M1	
		$\Rightarrow x=846$ (AWRT)	A1	[9]