Eton College King's Scholarship Examination 2012

MATHEMATICS B
(One and a half hours)

- *Answer as many questions as you can.*
- *Each of the ten questions carries ten marks.*
- *Show all your working.*
- *Calculators are not allowed.*

1. a) What is 15% of 25% of 0.4?

   b) Find $\dfrac{5}{1001} + \dfrac{3}{2002}$ as a single, reduced fraction.

   c) Find $\dfrac{4}{142857} - \dfrac{5}{285714} + \dfrac{7}{571428}$ as a single, reduced fraction.

2. a) James is three times as old as Nick but eight years younger than Mark. The sum of their three ages is 127 years.

   i) Let j represent James's age. Show that
   $$\tfrac{7}{3}\,j + 8 = 127$$

   ii) Hence find how old James is.

   b) Gavin is four times as old as Tom, who is six years younger than Xavier, who is a third of the age of John, who is a year younger than Gavin. How old is Xavier?

3. a) Find the Highest Common Factor (HCF) of:

   i) 21 and 27

   ii) 22 and 28

   iii) 23 and 29

   iv) 24 and 30

   v) 25 and 31

   vi) 26 and 32

   b) What do you notice about the HCF of two numbers differing by 6?

   c) For two whole numbers n and $n + 12$, what are the possible values of the HCF?

   d) Find the HCF of 227472 and 227490

4. a) Put the following groups of numbers in ascending order, showing your reasoning clearly:

   i) $\tfrac{7}{9}$, $\tfrac{10}{13}$, 0.8

   ii) $(-0.873)^3$, $(-0.8)^2$, $\tfrac{2}{3}$

   b) Find and simplify the square of each of the following:

   i) $2\sqrt{5}$

   ii) $3\sqrt{7}$

   c) Put the following group of numbers in ascending order, showing your reasoning clearly:

   $$4\sqrt{30}, \; 5\sqrt{5}, \; 3\sqrt{13}$$

5. A circle of radius r cm with centre at point O is inscribed in a triangle ABC, so that touches the sides of the triangle at points L, M, and N as shown in the diagram. Triangle ABC has lengths $AC = 12$cm, $AB = 5$cm and $BC = 13$cm. Note that $\angle OLB$, $\angle OMC$ and $\angle ONA$ are all right angles.


Diagram not to scale
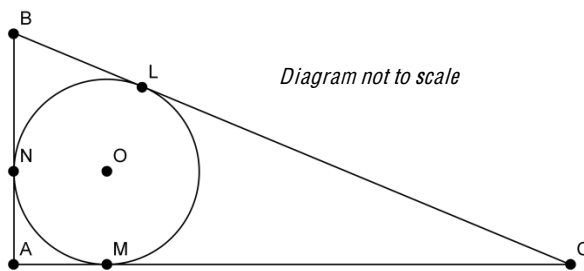
a) Find the area of triangle ABC.

b) Prove that $AMON$ is a square.

c) Show that triangle AOC has area $6r$ cm^2

d) Find the areas of triangles AOB and BOC in terms of r, showing your working clearly.

e) Hence find the radius of the circle.

6. a) Travelling at an average speed of 196000 km/hr, *The Explorer* takes 147 days to travel from a certain star to another star a certain fixed distance away. Returning at an average speed of 28000 km/hr, how long does *The Explorer* take?

b) Sharing 10000 litres of fuel equally between 22 identical trucks allows each to travel 4200 miles. How far would each go if there were one fewer truck and 1000 litres less fuel to share around?

7. a) In the diagram on the right, $AC = AD = BD$. Let $\angle BCA = x$.

Diagram not to scale

   i) Find $\angle ABC$ in terms of x.

   ii) Show that $\angle BAC = 180° - \frac{3}{2}x$

   If, in addition, $AB = CB$

   iii) Find x.
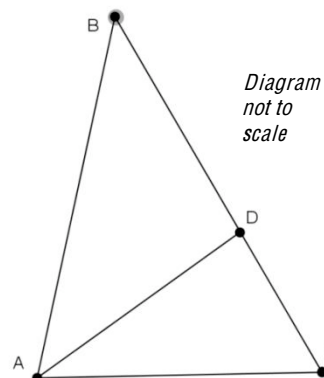
b) As shown in the diagram on the right, $ABDF$ is a rhombus, ACE is an equilateral triangle, and $AB = AC$. Find $\angle ABC$ through geometric reasoning (a scale diagram will gain no credit).

Diagram not to scale
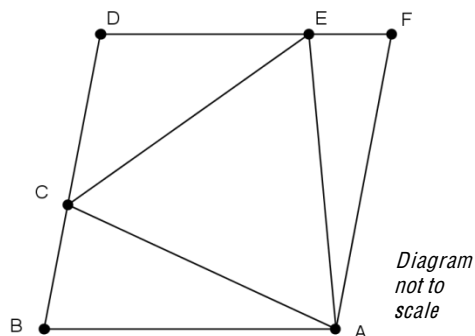
8. Write an algebraic expression for each of the following:

   a) I travel x miles at an average speed of 60 mph, then at an average of y mph I travel 30 miles. Find the number of hours I spend travelling in terms of x and y.

   b) I earned e pounds but gave c pounds to charity. I paid 20% of the remainder in tax and was then given p pounds as a present. How much did I have in the end, in terms of e, c, and p?

   c) In West Cornwall, I met w women each carrying s sacks, each containing c cats. However, a third of the women then lost half their sacks, and the remaining women lost half their cats from a third of their sacks. How many cats remained, it terms of w, s and c?

9. a) x is a two digit number. When its digits are reversed, it becomes the two digit number y, which is smaller than x. The difference between x and y is a one digit number. If we write x as $10a + b$ then:

      i) write y in terms of a and b;

      ii) find $x - y$ in terms of a and b;

      iii) deduce the numerical value of $x - y$.

   b) x is a three digit number. When its digits are reversed, it becomes the three digit number y. The difference between x and y is z, where z is positive and its last digit is 4. Find the numerical value of z.

10. The *integers* are all the whole numbers, but including negative whole numbers and zero; therefore -7, 5, 0, -342 and 245 are all integers.

    a) Explain (very briefly) why, when considered as integers:

       i) -7 is a factor of 42;

       ii) 3 is a factor of -9;

       iii) zero is not a factor of 12.

    b) List the integer factors of 10.

    c) In each part, find all values of n such that both expressions are integers:

       i) n and $\dfrac{10}{n}$

       ii) n and $\dfrac{26 + n}{n}$

       iii) $2n$ and $\dfrac{7}{n}$

       iv) n and $\dfrac{n+16}{n+1}$

[End of Paper]