Serial SSR/1

Code No. **65/1/3**

Roll No. | | | | | | | | |

Candidates must write the Code on the title page of the answer- book.

- Please check that this question paper contains **11** printed pages.
- Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- Please check that this question paper contains **29** questions.
- **Please write down the Serial Number of the questions before attempting it.**

MATHEMATICS

Time allowed: 3 hours] [Maximum marks: 100

General Instructions:

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each. Section B comprises of 12 questions of four marks each and Section C. comprises of 7 questions of sex marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is nor permitted.

Section A

1. Find the co-factor of a_{12} in the following:

$$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$$

2. Evaluate: $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

3. If f(x) = x + 7 and g(x) = x – 7, $x \in$ R, find (fog) (7)

4. Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

5. Find the value of x and y if :$2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

6. Evaluate: $\int_0^1 \frac{dx}{1+x^2}$

7. Find a unit vector in the direction of $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

8. Evaluate: $\int \frac{x^2}{1+x^3} dx$

9. For what value of λ are the vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ perpendicular to each other?

10. Find the angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$

Section B

11. Let A = $\begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$. Express A as sum of two matrices such that one is symmetric and the other is skew symmetric.

OR

If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, verify that $\mathbf{A}^2 - 4A - 5I = 0$

12. For what value of k is the following function continuous at x = 2?

$$f(x) = \begin{cases} 2x+1 & ; \ x<2 \\ k & ; \ x=2 \\ 3x-1 & ; \ x>2 \end{cases}$$

13. (i) Is the binary operation *, defined on set N, given by a*b = $\frac{a+b}{2}$ for all a, b∈ N, commutative?

(ii)Is the above binary operation *associative?

14. Find the equation of tangent to the curve x = sin 3t, y = cos 2t, at t = $\pi/4$.

15. Solve the following differential equation:

$(x^2 - y^2)\, dx + 2\, xy\, dy = 0$

give that y = 1 when x = 1

OR

Solve the following differential equation:

$\frac{dy}{dy} = \frac{x(2y-x)}{x(2y+x)}$, if y = 1 when x = 1

16. Solve the following differential equation:

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

17. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes.

18. Find the shortest distance between the following lines:

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ **OR**

Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance from the point (1, 2, 3)

19. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b}=\hat{j}-\hat{k},$ find a vector $\vec{c}$ such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$

OR

If $\vec{a}+\vec{b}+\vec{c}=0$ and $\left|\vec{a}\right|=3,\left|\vec{b}\right|=5$ and $\left|\vec{c}\right|=7,$ show that the angle between $\vec{a}$ and $\vec{b}$ is 60^o.

20. Solve for x:

$$\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$$

21. If y = $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$, *find* $\frac{dy}{dx}$

22. Evaluate: $\int_0^1 \cot^{-1}\left[1-x+x^2\right]dx$

Section C

23. Using properties of determinants, prove the following:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2\,(a+b+c)^3$$

24. Using integration, find the area lying above x-axis and included between the circle $x^2+y^2 = 8x$ and the parabola $y^2 = 4x$.

25. Using properties of definite integrals, evaluate the following:

$$\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$$

26. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

OR

Show that the height of the cylinder of maximum, volume that can be inscribed in a one of height h is $\frac{1}{3}h$.

27. A factory owner purchases two types of machines, A and B for his factory. The requirements and the limitations for the machines area as follows:

Machine	Area occupied	Labour force	Daily output (in units)
A	$1000m^2$	12 men	60
B	$1200m^2$	8 men	40

He has maximum area of $9000m^2$ available, and 72 skilled labourers show can operate both the machines. How many machines of each type should he buy to maximise the daily output?

28. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident involving a scooter, a car and a truck are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver.

29. Find the equation of the plane passing through the point (-1, -1, 2) and perpendicular to each of the following planes:

$$2x + 3y - 3z = 2 \text{ and } 5x - 4y + z = 6$$

OR

Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel to the line $\frac{x+3}{2} = \frac{y-3}{7} = \frac{z-2}{5}$