

WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली  
सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)  
परीक्षार्थी प्रवेश-पत्र के अनुसार भरें

विषय Subject Mathematics (041)

परीक्षा का दिन एवं तिथि  
Day & Date of the Examination 20 March '13, Wednesday

उत्तर देने का माध्यम  
Medium of answering the paper English

प्रश्न पत्र के ऊपर लिखे कोड को दर्शाए  
Write Code No. as written on the  
top of Question Paper 65/1

अतिरिक्त उत्तर पुस्तिका (ओं) की संख्या  
No. of Supplementary answer-book(s) used Nil

किसी शारीरिक वशमता के प्रभावित हो तो संबंधित वर्ग में ✓ का निशान लगाए।  
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**B D H S C**

B = बूढ़िहान, D = दूक एवं बधिर, H = शारीरिक रूप से विकलंग, S = स्फस्टिक, C = डिस्लेक्सिक  
B = Blind, D = Deaf & Dumb, H = Physically Handicapped, S = Spastic, C = Dyslexic

क्या लेखन लिपिक उपलब्ध करवाया गया है / नहीं  
Whether writer provided Yes / No **NO**

\*एक खाने में एक अक्षर लिखें। नाम के प्रत्येक भाग के बीच एक खाना रिक्त छोड़ दें। यदि परीक्षार्थी का नाम 24 अक्षरों से अधिक है, तो केवल नाम के प्रथम 24 अक्षर ही लिखें।  
Each letter be written in one box and one box be left blank between each part of the name. In case Candidate's Name exceeds 24 letters, write first 24 letters.

कार्यालय उपयोग के लिए  
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केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली  
Central Board of Secondary Education, Delhi

सीनियर स्कूल सर्टिफिकेट परीक्षा (कक्षा बारहवीं)  
SENIOR SCHOOL CERTIFICATE EXAMINATION (CLASS XII)



प्रमाणित किया जाता है मैंने/हमने इस उत्तर पुस्तिका का मूल्यांकन प्रश्न पत्र के  
समुचित सेट के अनुसार और पूर्ण रूप से मूल्यांकन पद्धति के अनुसार किया है।

Certified that I/We have evaluated this answer-book according to the  
correct set of question paper and strictly as per the marking scheme.

$$1. \tan^{-1}(\sqrt{3}) - \cot^{-1}(\sqrt{3}) = \frac{\pi}{3} - (\pi - \cot^{-1}(\sqrt{3})) = \frac{\pi}{3} - (\pi - \frac{\pi}{6}) = \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{3\pi}{6} = -\frac{\pi}{2}$$

$$2. \tan^{-1}\left[2 \sin\left(2 \cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \tan^{-1}\left[2 \sin\left(2 \cdot \frac{\pi}{6}\right)\right] = \tan^{-1}\left[2 \sin \frac{\pi}{3}\right] = \tan^{-1}\left(2 \cdot \frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

$$3. 'A' \text{ is skew symmetric when } a_{ij} + a_{ji} = 0 \text{ so } a_{13} + a_{31} = 0 \Rightarrow x - 2 = 0 \Rightarrow x = 2$$

$$4. A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 2A \quad \therefore A^2 = 2A \quad \therefore k = 2$$

$$5. y = mx \text{ Differentiating wrt } x, \frac{dy}{dx} = m \quad \therefore y = x \frac{dy}{dx}$$

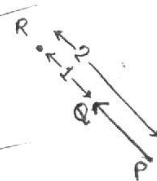
$$6. a_{32} = 5, A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix} = -(8 - 30) = 22 \quad \therefore a_{32} A_{32} = 5 \times 22 = 110$$

$$7. \vec{OR}(\vec{r}) = \frac{m \vec{OP} - n \vec{OQ}}{m - n} = \frac{2(3\vec{a} - 2\vec{b}) - 2(\vec{a} + \vec{b})}{2 - 2} = \frac{5\vec{a} - 3\vec{b}}{-1} = -\vec{a} + 4\vec{b}$$

$$8. \text{Given } \vec{a} \text{ is a unit vector, } |\vec{a}| = 1$$

$$\text{Now } (\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = \vec{x} \cdot \vec{x} - \vec{a} \cdot \vec{x} + \vec{x} \cdot \vec{a} - \vec{a} \cdot \vec{a} = 15$$

$$\text{Using } \vec{a} \cdot \vec{a} = |\vec{a}|^2 \text{ and } \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}, \text{ we get } |\vec{x}|^2 - |\vec{a}|^2 = 15 \Rightarrow |\vec{x}|^2 = 15 + 1 = 16$$



$$\therefore |\vec{x}| = \pm 4 \quad \text{but } |\vec{x}| > 0 \quad \therefore |\vec{x}| = 4 \quad \leftarrow \text{Ans}$$

9. Using distance formula:  $d = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{|(0)(2) + (0)(-3) + (0)(6) + 21|}{\sqrt{2^2 + (-3)^2 + 6^2}} = \frac{21}{7} = 3 \text{ units}$

10.  $R(x) = 3x^2 + 36x + 5$

$$MR(x) = R'(x) = 6x + 36 \quad R'(5) = 30 + 36 = 66$$

Money for welfare of employees is a nice thing, step.

There should be a growth in raising funds for the welfare of the employees.

### SECTION-B

11. Let  $f(x) = x^2 + 4$

$$f: \mathbb{R}_+ \rightarrow [4, \infty)$$

$$\text{Let } f(x_1) = f(x_2) \quad \forall x_1, x_2 \in \mathbb{R}_+$$

$$\Rightarrow x_1^2 + 4 = x_2^2 + 4 \Rightarrow (x_1 - x_2)(x_1 + x_2) = 0$$

$$\therefore x_1 > 0, x_2 > 0 \quad \therefore x_1 + x_2 > 0 \quad \text{Hence } x_1 = x_2$$

Thus  $f$  is one-one (injective)

$$\text{let } y = x^2 + 4 \Rightarrow x = \sqrt{y-4} \quad \{x = -\sqrt{y-4} \text{ rejected as } x > 0\}$$

Clearly, all values of  $y$  in the co-domain satisfy this relation to give  $x$  such that  $f(x) = x^2 + 4$ , hence  $f$  is an onto (surjective) function.

$f$  is one-one as well as onto, thus  $f$  is invertible

$$\text{Now let } y = f(y) = y^2 + 4. \text{ Once again, } y = \sqrt{y-4} \quad \{ \text{as } y > 0 \text{ and } f(y) \in [4, \infty) \}$$

Replacing  $y$  with  $f^{-1}(y)$  and  $f(y)$  with  $y$ , we get

$$f^{-1}(y) = \sqrt{y-4} \quad \text{Hence shown.}$$

$$12. \text{ let } \frac{3}{4} = \sin 2\theta \Rightarrow 2\theta = \sin^{-1}\left(\frac{3}{4}\right)$$

$$\text{so } \tan\left(\frac{1}{2} \sin^{-1}\frac{3}{4}\right) = \tan\left(\frac{1}{2} \cdot 2\theta\right) = \tan\theta$$

$$\text{let } \tan\theta = t \quad \text{then,}$$



$$\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta} \Leftrightarrow \Rightarrow \frac{3}{4} = \frac{2t}{1+t^2} \Rightarrow 3+3t^2 = 8t$$

$$\Rightarrow 3t^2 - 8t + 3 = 0$$

$$\frac{\pi}{2} < \theta < \pi$$

$$\Rightarrow t = \frac{8 \pm \sqrt{64 - 4(9)}}{6} = \frac{4 \pm \sqrt{7}}{3}$$

But  $\sin 2\theta = \frac{3}{4}$  which implies that  $0 < 2\theta < \frac{\pi}{2} \Rightarrow 0 < \theta < \frac{\pi}{4}$ .

Accordingly,  $0 < \tan \theta < \tan(\pi/4) \therefore 0 < \tan \theta < 1 \Rightarrow 0 < t < 1$

Thus  $t = \frac{4+\sqrt{7}}{3}$  is to be rejected, giving  $t = \tan^{-1}\left(\frac{1}{2} \sin^{-1} \frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$  H.P.

$$13. \text{ Let } \Delta = \begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

Applying  $R_1 \rightarrow R_1 + R_2 + R_3$ ,  $\Delta =$

$$\begin{vmatrix} 3x+3y & 3x+3y & 3x+3y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

$$\Delta = 3(x+y) \begin{vmatrix} 1 & 1 & 1 \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix}$$

Applying  $C_1 \rightarrow C_1 - C_3$  and  $C_2 \rightarrow C_2 - C_3$

$$\Delta = 3(x+y) \begin{vmatrix} 0 & 0 & 1 \\ y & -y & x+y \\ y & 2y & x \end{vmatrix}$$

Taking out common  $y$  from  $C_1$  and  $y$  from  $C_2$ ,

$$\Delta = 3(x+y) y^2 \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & x+y \\ 1 & 2 & x \end{vmatrix}$$

Expanding about  $R_1$ ,

$$\Delta = 3y^2(x+y) [1(1 \times 2 - (-1)(1))] = 3 \times 3y^2(x+y)$$

$$\therefore \Delta = 9y^2(x+y) \quad \text{Hence Proved.}$$

14. Given  $y^x = e^{y-x}$

Taking natural logarithm both sides,  $x \log y = y - x$  — ①

Differentiating with respect to  $x$ ,  $\log y + \frac{x}{y} \frac{dy}{dx} = \frac{dy}{dx} - 1$

$$\Rightarrow \frac{dy}{dx} \left(1 - \frac{x}{y}\right) = 1 + \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{1+\log y}{1-x/y} = \frac{y}{y-x} (1+\log y)$$

From eqn ①,  $y-x = x \log y \Rightarrow \frac{dy}{dx} = \frac{y}{x} \frac{(1+\log y)}{\log y}$

Also from eq. ①,  $y = x(1+\log y)$

$$\Rightarrow \frac{dy}{dx} = \frac{x(1+\log y)(1+\log y)}{x \log y}$$

$$\frac{dy}{dx} = \frac{(1+\log y)^2}{\log y} \quad \text{Proved}$$

15. Given  $f(x) = \sin^{-1} \left( \frac{2^{x+1} 3^x}{1+(36)^x} \right) = \sin^{-1} \left( \frac{2(6)^x}{1+(6^x)^2} \right)$  — ①

Let  $6^x = \tan \theta$

Differentiating w.r.t  $x$ ,  $6^x \log 6 = \sec^2 \theta \frac{d\theta}{dx}$  — ②

Putting  $6^x = \tan \theta$  in eqn ①,  $f(\theta) = \sin^{-1} \left( \frac{2 \tan \theta}{1+\tan^2 \theta} \right) = \sin^{-1} (\sin 2\theta) = 2\theta$



Thus  $\frac{d(f(\theta))}{d\theta} = 2$

Also Multiplying and dividing LHS by  $dx$ ,

$$\frac{df(\theta)}{d\theta} \cdot \frac{dx}{dx} = \frac{df(\theta)}{dx} \cdot \frac{dx}{d\theta} = 2 \Rightarrow \frac{d(f(\theta))}{dx} = 2 \frac{d\theta}{dx}$$

$$\Rightarrow \frac{d f(x)}{dx} = \frac{d}{dx} \left( \sin^{-1} \left( \frac{2^{x+1} 3^x}{1+36^x} \right) \right) = 2 \cdot \frac{6^x \log 6}{\sec^2 \theta}$$

$$= \frac{2 \log 6}{1 + \tan^2 \theta} 6^x = \frac{2 \cdot 6^x \log 6}{1 + 36^x}$$

$$= \left( \frac{2^{x+1} 3^x}{1 + (36)^x} \right) \log 6 \quad \leftarrow \text{Ans}$$

16. 
$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & 0 \leq x < 1 \end{cases}$$

$$f(0^+) = \lim_{h \rightarrow 0} f(0+h) = \frac{2(0+h)+1}{0+h-1} = \frac{2h+1}{h-1} = \frac{0+1}{0-1} = -1$$

$$f(0) = \frac{0+1}{0-1} = -1$$

$$f(0) = \lim_{h \rightarrow 0} f(0-h) = \lim_{h \rightarrow 0} \frac{\sqrt{1+kh} - \sqrt{1-kh}}{h}$$

Multiplying and dividing by  $\sqrt{1+kh} + \sqrt{1-kh}$  on both sides, we get

$$f(0) = \frac{(\sqrt{1+kh} - \sqrt{1-kh})(\sqrt{1+kh} + \sqrt{1-kh})}{h(\sqrt{1+kh} + \sqrt{1-kh})} = \lim_{h \rightarrow 0} \frac{1+kh - (1-kh)}{h(\sqrt{1+kh} + \sqrt{1-kh})}$$

$$= \lim_{h \rightarrow 0} \frac{2kh}{h(\sqrt{1+kh} + \sqrt{1-kh})} = \frac{2k}{\sqrt{1+0} + \sqrt{1-0}} = \frac{2k}{2} = k$$

For continuity at  $x=0$ ,  $f(0^+) = f(0) = f(0^-)$

$$\Rightarrow -1 = k \quad \leftarrow \text{Ans}$$

17. ~~18~~ Let  $I = \int \frac{x+2}{\sqrt{x^2+2x+3}} dx$

$$I = \frac{1}{2} \int \frac{(2x+2) + 2}{\sqrt{x^2+2x+3}} dx = \frac{1}{2} \int \frac{2x+2}{\sqrt{x^2+2x+3}} dx + \int \frac{1}{\sqrt{x^2+2x+3}} dx$$

$$\text{Let } x^2 + 2x + 3 = t$$

$$\Rightarrow 2x + 2 = \frac{dt}{dx} \quad \Rightarrow I = \frac{1}{2} \int \frac{dt}{\sqrt{t}} + \int \frac{dx}{\sqrt{(x+1)^2 + 2}}$$

$$= \frac{1}{2} \cdot 2\sqrt{t} + \log(x+1 + \sqrt{(x+1)^2 + 2}) + C$$

$C = \text{arbitrary constant}$

$$I = \sqrt{x^2 + 2x + 3} + \log(x+1 + \sqrt{x^2 + 2x + 3}) + C \quad \leftarrow \text{Ans}$$

$$18. \quad \text{Let } I = \int \frac{dx}{x^6(x^5+3)}$$

$$= \int \frac{dx}{x(x^5(1+3x^{-5}))} = \int \frac{dx}{x^6(1+3x^{-5})} = \int \frac{x^{-6} dx}{3x^{-5} + 1}$$

$$\text{Let } 3x^{-5} + 1 = t$$

$$\Rightarrow -15x^{-6} dx = dt \quad \therefore I = \int \frac{-1}{15} \frac{dt}{t} = \frac{-1}{15} \int \frac{dt}{t} = \frac{-1}{15} \log t + C$$

$$= \frac{-1}{15} \log(3x^{-5} + 1) + C$$

$$= \frac{-1}{15} [\log(3+x^5) - \log x^5] + C$$

$$= \frac{-1}{15} \log(x^5+3) + \frac{x}{3} + C$$

Ans

19. Let  $I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx$

using property of definite integrals that  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$ ,

$$\begin{aligned} I &= \int_0^{2\pi} \frac{1}{1+e^{\sin(2\pi-x)}} dx = \int_0^{2\pi} \frac{1}{1+e^{-\sin x}} dx \quad \{\sin(2\pi-x) = -\sin x\} \\ &= \int_0^{2\pi} \frac{1}{1+\frac{1}{e^{\sin x}}} dx = \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} dx \end{aligned}$$

$$I+I = 2I = \int_0^{2\pi} \frac{1}{1+e^{\sin x}} dx + \int_0^{2\pi} \frac{e^{\sin x}}{1+e^{\sin x}} dx = \int_0^{2\pi} \left( \frac{1+e^{\sin x}}{1+e^{\sin x}} \right) dx = \int_0^{2\pi} 1 \cdot dx = [x]_0^{2\pi}$$

$$\left\{ \text{Using } \int f(x) dx + \int g(x) dx = \int (f(x)+g(x)) dx \right\} = 2\pi - 0 = 2\pi$$

$$\therefore 2I = 2\pi \Rightarrow I = \pi \quad \text{Ans}$$

20. given  $\vec{a} = \hat{i} - \hat{j} + 7\hat{k}$   
 $\vec{b} = 5\hat{i} - \hat{j} + \lambda\hat{k}$

using additive properties of vectors,  $\vec{a} + \vec{b} = (1+5)\hat{i} + (-1-1)\hat{j} + (7+\lambda)\hat{k}$   
 $= 6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}$

Also  $\vec{a} - \vec{b} = (1-5)\hat{i} + (-1+1)\hat{j} + (7-\lambda)\hat{k}$   
 $= -4\hat{i} + (7-\lambda)\hat{k}$

For  $\vec{a} + \vec{b}$  to be perpendicular to  $\vec{a} - \vec{b}$ ,  $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$

$$\Rightarrow (6\hat{i} - 2\hat{j} + (7+\lambda)\hat{k}) \cdot (-4\hat{i} + (7-\lambda)\hat{k}) = 0$$

$$\Rightarrow -24 + (49 - \lambda^2) + 0 = 0$$

$$\Rightarrow \lambda^2 = 25$$

$$\Rightarrow \lambda = \pm 5$$



21. Let the given lines be  $L_1$  and  $L_2$

$$L_1 \equiv \frac{x-3}{1} = \frac{y-2}{2} = \frac{z+4}{2} \quad \text{and} \quad L_2 \equiv \frac{x-5}{3} = \frac{y+2}{2} = \frac{z-0}{6}$$

Any point  $P(k)$  on line  $L_1$  can be expressed as

$$P \equiv (k+3, 2k+2, 2k-4) \quad \text{--- ① } k \in \mathbb{R}$$

If  $L_1$  and  $L_2$  are intersecting (and are of course non-parallel), then there should be exactly one point of intersection, and in that case, only one real value of  $k$  should permit the existence of that point on  $L_2$ , so

substituting  $x, y, z$  of  $L_2$  by  $P(k)$ , we get

$$\frac{k+3-5}{3} = \frac{2k+2+2}{2}, \frac{2k-4-0}{6} \Rightarrow \frac{k-2}{3} = \frac{k+2}{2} = \frac{k-2}{3}$$

which is true for  $k = -4$  Hence the lines are intersecting

Putting this value in expression ①,  $P \equiv (2, -6, -12)$  ← Ans

22. Let  $E_1, E_2$  and be events defined as  
 $E_1 \rightarrow A$  comes to school in time  
 $E_2 \rightarrow B$  comes in times

Given  $p(E_1) = \frac{3}{7}$ ,  $p(E_2) = \frac{5}{7}$ . Also since  $E_1$  and  $E_2$  are independent events,  
we get  $p(E_1) \cdot p(E_2) = p(E_1 \cap E_2)$   
 $\Rightarrow p(E_1 \cap E_2) = \frac{3}{7} \cdot \frac{5}{7} = \frac{15}{49}$

$$\begin{aligned} p(\text{only A coming in time}) &= p(E_1) - p(E_1 \cap E_2) \\ &= p(\alpha) = \frac{3}{7} - \frac{15}{49} = \frac{6}{49} = p(\alpha) \end{aligned}$$

$$\text{Similarly, } p(\text{only B coming in time}) = p(E_2) - p(E_1 \cap E_2) = \frac{5}{7} - \frac{15}{49} = \frac{20}{49} = p(\beta)$$

$$p(\text{only either A or B coming in time}) = p(\alpha) + p(\beta) = \frac{6}{49} + \frac{20}{49} = \frac{26}{49} \quad \leftarrow \text{Ans}$$

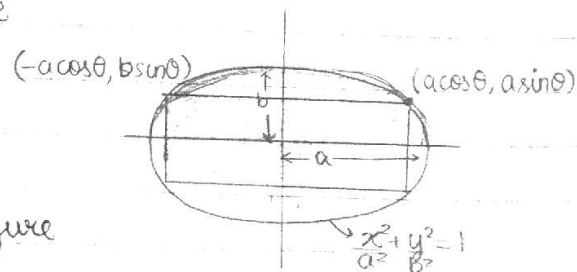
- ✓ One's sense of discipline improves if he/she comes to school in time.

## SECTION-C

23. Let any point on the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  be

parametrically represented as

$$P(\theta) = (a \cos \theta, b \sin \theta)$$



Then, area of rectangle shaded in the figure

$$A = l \cdot b$$

$$= (2a \cos \theta)(2b \sin \theta)$$

$$A = f(\theta) = 2ab \sin 2\theta$$

Clearly,  $f(\theta)$  is maximum when  $\sin 2\theta$  is max. i.e.  $2\theta = \pi/2$  or  $\theta = \pi/4$ .

Using calculus:  $A = f(\theta) = 2ab \sin 2\theta$

$$f'(\theta) = 4ab \cos 2\theta$$

$$f'(\theta) = 0 \text{ at } 2\theta = \frac{\pi}{2} \text{ or } \theta = \pi/4$$

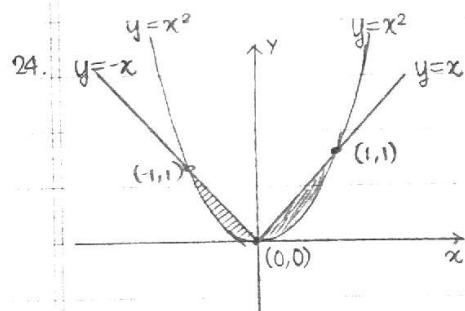
$$f''(\theta) = -8ab \sin 2\theta$$

$$f''(\pi/4) = -8ab < 0$$

thus  $f(\theta)$  is maximum at  $\theta = \pi/4$ .

Putting  $\theta = \pi/4$  in eqn ①,  $\& A_{\max} = 2ab$  ~~Ans~~ sq. units

Ans



The shaded area is the required area  
since both  $|x|$  and  $x^2$  are symmetrical about  
the  $y$  axis, the total area is twice of any one shaded part  
Hence

$$A = \text{required area} = 2 \left[ \int_0^1 x \, dx - \int_0^1 x^2 \, dx \right]$$

$$= 2 \int_0^1 (x - x^2) \, dx = 2 \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1$$

$$= 2 \left[ \frac{1}{2} - \frac{1}{3} - (0 - 0) \right]$$

$$= 2 \left( \frac{3-2}{6} \right) = \frac{1}{3} \text{ sq. units} \quad \text{Ans}$$

25. Given  $(\tan^{-1}y - x) dy = (1 + y^2) dx$

Let  $y = \tan \theta \quad \therefore dy = \sec^2 \theta d\theta \quad \Rightarrow (\theta - x) \sec^2 \theta d\theta = (1 + \tan^2 \theta) dx$   
 $= \sec^2 \theta dx$

$$\Rightarrow \frac{dx}{d\theta} + x = \theta$$

This is a linear differential equation of the form  $\frac{dy}{dx} + Py = Q$

I.F. =  $e^{\int 1 \cdot d\theta} = e^\theta$

Multiplying by I.F. on both sides,  $dx e^\theta + e^\theta d\theta x = \theta e^\theta d\theta$

$$\Rightarrow \int d(xe^\theta) = \int \theta e^\theta d\theta$$

$$\Rightarrow xe^\theta = e^\theta \theta - \int 1 \cdot e^\theta d\theta$$

$$= e^\theta \theta - e^\theta + c$$

$$\Rightarrow x = \theta - 1 + ce^{-\theta}$$

Putting back  $\theta = \tan^{-1}y$

$$x = \tan^{-1}y - 1 + ce^{-\tan^{-1}y}$$

At  $x=0, y=0$ , so

$$0 = 0 - 1 + ce^0 \Rightarrow c = 1$$

&

$$\Rightarrow x = \tan^{-1}y - 1 + e^{\tan^{-1}y} \quad \leftarrow \text{Ans}$$



26. Let the required line be  $L$

$L$  passes through  $(1, 2, 3)$

$$L \equiv \frac{x-1}{a} = \frac{y-2}{b} = \frac{z-3}{c} \quad \text{--- (1) } a, b, c \text{ being the D.R.s of the line } L$$

$L$  is parallel to planes  $P_1$  and  $P_2$ , and hence perpendicular to their normals

whose D.R.s are respectively  $a_1, b_1, c_1$  and  $a_2, b_2, c_2$

$$a_1 = 1, b_1 = -1, c_1 = 2$$

$$a_2 = 3, b_2 = 1, c_2 = 1$$

$$\text{Thus, } aa_i + bb_i + cc_i = 0 \quad i=1, 2$$

$$\Rightarrow a - b + 2c = 0$$

$$\text{and } 3a + b + c = 0$$

using cross multiplication to solve  $a, b, c$ ,

$$\frac{a}{-3} = \frac{+b}{+5} = \frac{c}{4}$$

substituting these ratios in the equation of line  $L$ ,

$$\frac{x-1}{-3} = \frac{y-2}{5} = \frac{z-3}{4}$$

its vector equation is  $\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$  Ans

27. Let  $N$  and  $S$  be two events defined as  
 $S \rightarrow$  the captain gets the number '6' on the die  
 $\bar{S} = N \rightarrow$  the captain does not get the number '6' on the die

Obviously  $p(S) = \frac{1}{6}$  and  $p(N) = p(\bar{S}) = 1 - p(S) = \frac{5}{6}$ .

Case 1: A wins

'A' can win if it gets a 6 before B. The following subcases are possible

$S, NNS, NNNNS, NNNNNNS, \dots, \infty$

This suggests that either  $S$  occurs, or both A & B lose and the next time A wins, and so on.  $\neq$

Total probability for A winning

$$\begin{aligned}
 p(A) &= p(S) + p(N)p(N)p(S) + (p(N))^4 p(S) + \dots \infty \\
 &= \frac{1}{6} + \frac{1}{6} \left(\frac{5}{6}\right)^2 + \frac{1}{6} \left(\frac{5}{6}\right)^4 + \dots \infty = \frac{1}{6} \left[ 1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \dots \infty \right] = \frac{1}{6} \cdot \frac{1}{1 - \frac{25}{36}}
 \end{aligned}$$

$$= \frac{1}{6} \cdot \frac{36}{11} = \frac{6}{11}$$

Case 2: B wins

Either A wins or B wins, so  $p(A) + p(B) = 1 \Rightarrow p(B) = 1 - p(A) = 1 - \frac{6}{11} = \frac{5}{11}$

As it can be seen, the decision of referee was not correct because probability of winning of A team is  $\frac{6}{11}$  is more than that of B  $\frac{5}{11}$

28.

Let

$x$  = number of units of A

$y$  = number of units of B

workers required for making 1 unit of A = 2

" " " of  $x$  units of A =  $2x$

workers required for making  $y$  units of B =  $3y$

so total no. of workers required =  $2x + 3y$

This number should be less than or equal to no. of available workers i.e. 30

$$\Rightarrow 2x + 3y \leq 30 \Leftrightarrow \frac{x}{15} + \frac{y}{10} \leq 1$$

similarly, no. of required units of capital =  $3x + y$

This should be less than or equal to available capital

$$\Rightarrow 3x + y \leq 17 \Leftrightarrow \frac{x}{17/3} + \frac{y}{17} \leq 1$$

Also, <sup>revenue</sup> ~~profit~~  $Z = 100x + 200y$

Also,  $x \geq 0$  and  $y \geq 0$  {no. of items cannot be negative}

Plotting the graphs of  $\frac{x}{15} + \frac{y}{10} = 1$ ,  $\frac{x}{17/3} + \frac{y}{17} = 1$ ,  $x \geq 0$  and  $y \geq 0$ , and

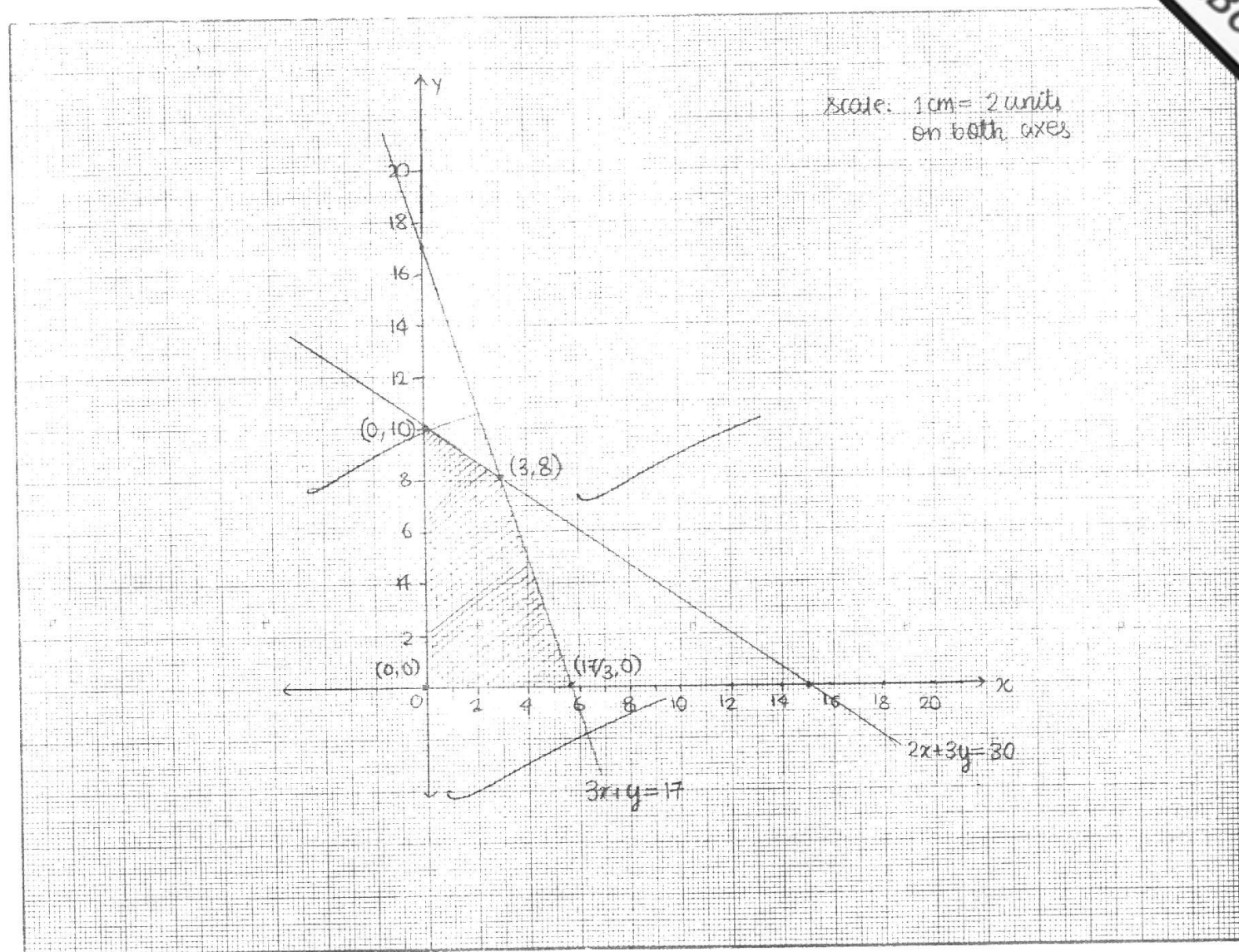
satisfying an arbitrary point (say  $(0,0)$ ) in these equations, we get the shaded region. Since an extremum lies on sharp points only, we plot a table for  $Z$  for 4 points only.

|     |   |                 |                |                 |
|-----|---|-----------------|----------------|-----------------|
| $x$ | 0 | 0               | $17/3$         | 3               |
| $y$ | 0 | 10              | 0              | 8               |
| $Z$ | 0 | <del>2000</del> | <del>567</del> | <del>1900</del> |
|     |   | 1200            | 567            | 1260            |

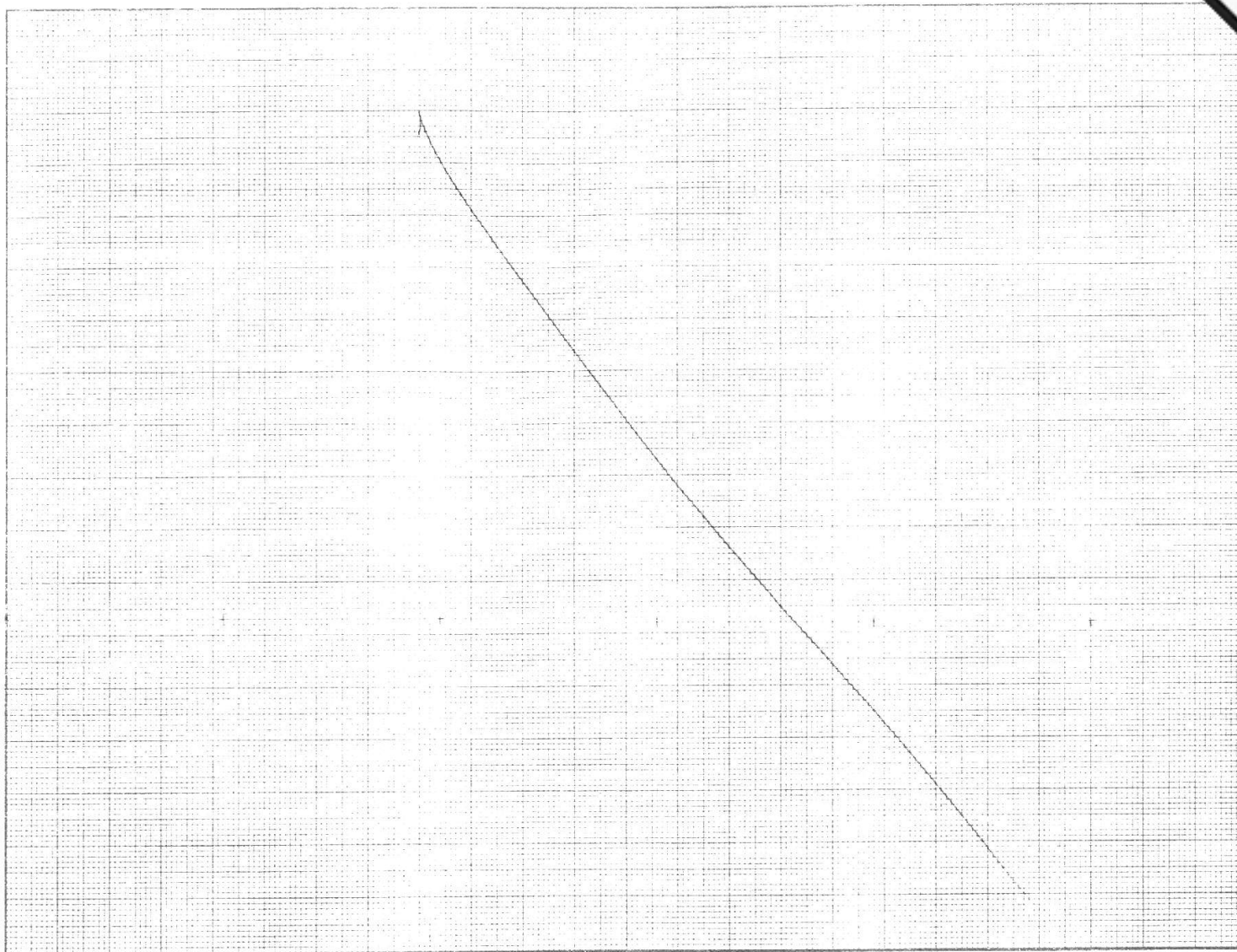
maximize( $z$ ):  $x=0, y=10$

or  $x=3, y=8$









Thus, ~~per~~ revenue will be maximized if ~~10~~<sup>3</sup> units of B and ~~10~~<sup>8</sup> units of A are manufactured. The corresponding revenue is ₹ ~~2000~~ 1260.

- ✓ Yes. The manufacturer is a noble person in thinking that men and women are equally capable of doing work and as such should be paid equally.

29. Let the number of awardees for honesty, helping others and supervisors be  $x, y$  and  $z$  respectively. We have

$$x + y + z = 12$$

$$3(y + z) + 2x = 33$$

$$x + z = 2y$$

$$\Rightarrow x + y + z = 12$$

$$2x + 3y + 3z = 33$$

$$x - 2y + z = 0$$

$$\text{Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}, \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

then  $AX = B$  yields the aforementioned equations

$X$  can be found as  $X = A^{-1}B$

$$|A| = 1(3+6) - (2-3) + (-4-3)$$

$$= 9 + 1 - 7 = 3 \neq 0 \quad \therefore |A| \neq 0 \therefore A^{-1} \text{ exists.}$$

$$\text{Now } A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$A_{11} = + \begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 9 \quad A_{12} = - \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 1 \quad A_{13} = + \begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -7$$

$$A_{21} = - \begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -3 \quad A_{22} = + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \quad A_{23} = - \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 3$$

$$A_{31} = + \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = 0 \quad A_{32} = - \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = -1 \quad A_{33} = + \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = 1$$

$$\text{adj } A = (\text{cof } A)^T = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A \quad \therefore X = A^{-1}B = \frac{1}{|A|} \text{adj } A \cdot B = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 \\ 12 \\ 15 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\Rightarrow x=3, y=4, z=5$$

$\therefore$  ~~3~~ 3 awardes for honesty, 4 for cooperation and 5 for supervision.

Leadership

✓ ~~Unity~~ is ~~an~~ an important quality that should be included in award categories, to ensure that there are some people that can take important decisions for the benefit of the colony.