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MATHEMATICS

(Three hours and quarter)

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*Answer **Question 1** from Section **A** and **14** questions from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.
The intended marks for questions or parts of questions are given in brackets [].

Mathematical formulae are given at the end of this question paper.
The use of calculator (fx-82/ fx-100) is allowed.

Section A

Answer **ALL** questions.

Directions: Read the following questions carefully. For each question there are four alternatives A, B, C and D. Choose the correct alternative and write it in your answer sheet.

Question 1 (2x15=30 Marks)

(i) The expanded form of $\sum_{i=1}^{7}(3-i)^2$ is

A 4 -1 + 0 + 4 -9 + 16.
B 1+4+9+16+ 25+ 36+49.
C 4 + 1 + 0 + 1 + 4 + 9 + 16.
D 2 + 1- 0 - 3+ 4 – 5 + 6 – 7.

(ii) The adjoint of the matrix $\begin{pmatrix} -3 & 2 \\ -1 & 5 \end{pmatrix}$ is

A $\begin{pmatrix} 3 & 2 \\ -1 & 5 \end{pmatrix}$.

B $\begin{pmatrix} 5 & 3 \\ -1 & -2 \end{pmatrix}$.

C $\begin{pmatrix} 5 & -2 \\ 1 & -3 \end{pmatrix}$.

D $\begin{pmatrix} -5 & -1 \\ -2 & 3 \end{pmatrix}$.

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(iii) The restrictions on the expression $\frac{x+1}{x^2-6x-7}$ are

A $x \neq 7, -1$.
B $x \neq -7, -1$.
C $x \neq -1$.
D $x \neq -7$.

(iv) If $g(a) = \frac{1}{a^2}$ has a vertical asymptote at $a = 0$, what is the vertical asymptote of $f(a) = \frac{1}{(a-2)^2}$?

A -2
B 0
C 2
D 4

(v) A body moves such that its distance D metres, after 't' seconds is given by $D = 3t^3 - 4t^2 + 5t - 6$. Its acceleration after 4 seconds would be

A 14 m/sec^2.
B 64 m/sec^2.
C 72 m/sec^2.
D 117 m/sec^2.

(vi) The value of $\lim_{x\to\infty} \frac{7\sin\left(\frac{x}{7}\right)}{x}$ is

A 0.
B $\frac{1}{7}$.
C 1.
D 7.

(vii) Which one of the following is the derivative of e^{2x}?

A $2x^3$
B $2x^3e^{2x}$
C $6x^2e^x$
D $6x^2e^{2x^3}$

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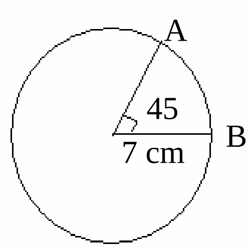
(viii) The integral value of $\int_{\pi}^{2\pi} \sin x dx$ is

A -2.
B 0.
C 1.
D 2.

(ix) The length of the arc AB in the diagram alongside is

A 2.75 cm.
B 5.5 cm.
C 11 cm.
D 44 cm.



(x) Which one of the following is the modulus of $\dfrac{-3+2i}{3-4i}$?

A $\dfrac{\sqrt{13}}{25}$
B $\sqrt{\dfrac{13}{25}}$
C $\sqrt{\dfrac{25}{13}}$
D $\dfrac{25}{\sqrt{13}}$

(xi) Which one of the following is the greatest surd?

A $\sqrt{2}$
B $\sqrt{3}$
C $\sqrt[3]{6}$
D $\sqrt[3]{7}$

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(xii) The regression co-efficient of y on x of the regression line $2x - 3y = 17$ is

A $\frac{-3}{2}$.

B $\frac{-2}{3}$.

C $\frac{2}{3}$.

D $\frac{3}{2}$.

(xiii) If $f(3) = -7, f'(3) = -5, \quad g(3) = -12$ and $g'(3) = 7$, the value of $(f \circ g)'(3)$ is

A -109.
B -11.
C 11.
D 49.

(xiv) The polar form of 3 + 4*i* is

A 4c is 36.9.
B 5c is 36.9.
C 4c is 53.1.
D 5c is 53.1.

(xv) Which one of the following is the conic represented by the equation $16x^2+9y^2 = 144$?

A hyperbola
B parabola
C ellipse
D circle

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Section B (70 Marks)

Answer any **14** questions. All questions in this section have equal marks.
Unless otherwise stated, you may round answers to 2 decimal places.

Question 2

a) Evaluate $\int \frac{\sqrt{1+\log x}}{x} dx$. [2]

b) If $y = v^5 - v^3$, where $v = 3x - x^2$. Determine $\left.\frac{dy}{dx}\right|_{x=-1}$ [3]

Question 3

a) A car depreciates by 20% per year. If a car is purchased for NU. 2, 50,000. When will its value be half of the original price? [2]

b) Find the derivative of $y = e^x$ by using the first principle method. [3]

Question 4

a) Determine $\int 3x \cos 5x dx$. [2]

b) Express $\frac{\sqrt{-4}+3}{\sqrt{-9}-5}$ in the form $a+bi$. [3]

Question 5

a) Determine the centre and radius of the equation of the circle $x^2 + y^2 + 6x - 4y + 9 = 0$. [2]

b) Determine the equation of the ellipse whose focus is (-1, 1), eccentricity $\frac{1}{2}$ and directrix is $x - y + 3 = 0$. [3]

Question 6

a) Determine the equation of the plane which passes through the points P(2, 1, 3), Q(3, -3, 4) and R(-1, 1, -4). [3]

b) Determine the equation of the tangent line to $y = \cos\theta$ where $\theta = \frac{\pi}{2}$. [2]

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Question 7

Prove that $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}, n \in N$ by mathematical induction method. [5]

Question 8

The sides of an equilateral triangle decrease at the rate of 10cm/sec. Determine the rate of decrease of the area of the triangle, when the area is 200 cm^2. [5]

Question 9

Find all the values of $\left(1+i\sqrt{3}\right)^{\frac{3}{4}}$ by using De Moivre's Theorem. [5]

Question 10

For the hyperbola, with equation $4x^2 - y^2 + 16x + 2y - 1 = 0$, determine:

(i) the centre;
(ii) the vertices;
(iii) the eccentricity;
(iv) the lengths of conjugate and transversal axis. [5]

Question 11

Solve the following system of linear equations using matrix method.

$-4x + 2y - 9z = 2$,
$3x + 4y + z = 5$,
$x - 3y + 2z = 8$. [5]

Question 12

A rectangular building is to be built on a 40m by 70 m rectangular plot in such a way that there is a path '*x*' metres wide surrounding the building. The building can occupy up to 70% of the plots area. What is the range of possible integral values for the width of the path? [5]

Question 13

a) If -2 is the root of the function $y = -3x^2 - 8x^2 - ax + 2$, find the value of *a*. [2]

b) Determine the square root of $27 + 18\sqrt{7}$. [3]

Question 14

a) A car battery loses 3% of its charge every day. Write an exponential equation in basic form and in base *e* form to model this situation, taking C as the remaining charge of the battery. [2]

b) A revolving watch tower torch is situated 1850m from a plane surface. It turns one revolution per minute. How fast does it sweep along the surface at a point 2550m from the nearest point? [3]

Question 15

Find the area bounded by $y = x^2 + 1$, $x = 0$, and $x = 2$ using the limit of sums. [5]

Question 16

a) Determine the volume of the solid generated by revolving the curve $y = x^2$ about the x- axis from $x = 1$ to $x = 2$. [2]

a) Determine the changes in the cost of living index of the living figures in Paro as given below. [3]

Items	Food	Rent	Clothing	Fuel	Others
Percentage expenditure	35	20	15	10	20
Price in 2000	250	60	80	50	200
Price in 2003	270	80	100	50	250

Question 17

For the function $y = x^3 - 3x^2 - 9x + 22$, find the:

(i) intercepts;
(ii) critical values;
(iii) inflection points;
(iv) maximum and minimum points.
(v) sketch the graph. [5]

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Question 18

Determine the co-efficient of correlation of the data and interpret the result. [5]

X	1	2	3	4	5	6	7	8	9	10	11
Y	4	7	10	13	16	19	22	25	28	21	34

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MATHEMATICS FORMULAE FOR BHSEC

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(6) $v(t) = h'(t)$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $S_n = \dfrac{a(1-r^n)}{1-r} = \dfrac{a(r^n - 1)}{r-1}$, where $r>1$

(6) $t_n = a + (n-1)d$

(7) $S_n = \dfrac{n}{2}[a + (n-1)d]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = x^n,\ y' = nx^{n-1}$

(3) $y = cf(x),\ y' = cf'(x)$

(4) $y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x)$,
$F'(x) = f(x)g'(x) + f'(x)g(x)$.

(6) $F(x) = \dfrac{f(x)}{g(x)}$,

$F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(7) $(f \circ g)'(x) = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

Coordinate Geometry

(1) $(y - y_1) = m(x - x_1)$

(2) $d = \sqrt{(x-a)^2 + (y-b)^2}$

Trigonometry

(1) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int gxdx - \int\left[\left(\dfrac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi \int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3} r^2 h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \dfrac{4\pi}{3} r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi r^2 + 2\pi rh$

(6) *Cylinder:* $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \dfrac{bh}{2}$, $A = \dfrac{\sqrt{3}}{4}x^2$,
A = $\sqrt{s(s-a)(s-b)(s-c)}$

(10) Rectangle: $A = lw$,

(11) Rectangle: $P = 2l + 2w$

(12) Square: $A = s^2$,

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(13) Square: $P = 4S$

(14) Rectangular Prism: $V = lwh$

Complex numbers

(1) $r = \sqrt{a^2 + b^2}$

(2) $\tan\theta = \frac{b}{a} \Rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

(3) If $z = r\text{cis}\theta$ then $z^n = r^n \text{cis} n\theta$

(4) $z^{\frac{1}{n}} = r^{\frac{1}{n}} cis(\frac{\theta}{n} + k\cdot\frac{360°}{n})\, f\ or\ k = 0,1,2,3,...n-1$

Second Degree Relations

(1) *Ellipse:* $\frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$

(2) *Hyperbola:* $\frac{X^2}{a^2} - \frac{y^2}{b^2} = 1$

(3) $e = \frac{c}{a}$

Geometry

(1) $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

(2) $(x, y, z) = \left(\frac{lx_2 + mx_1}{l+m}, \frac{ly_2 + my_1}{l+m}, \frac{lz_2 + mz_1}{l+m}\right)$

(3) For $a_1x + b_1y + c_1z = 0$ and $a_2x + b_2y + c_2z = 0,$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

(4) $l = \frac{\theta}{360^0} 2\pi r$

(5) $A = \frac{\theta}{360^0} \pi r^2$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $AA^{-1} = A^{-1}A = I$

(3) Inverse of $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\overline{x} = \frac{\sum fx}{n}$

(2) Median $= l_1 + \frac{l_2 - l_1}{f_1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum(x_i - \overline{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1{}^2 + n_2\sigma_2{}^2 + n_1d_1{}^2 + n_2d_2{}^2}{n_1 + n_2}}$

(5) $\overline{x}_{12} = \frac{m\overline{x}_1 + n\overline{x}_2}{m+n}$

(6) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(7) $I = \sum \frac{P_iW}{P_0W} \times 100$

(8) $\text{Cov}(X, Y) = \frac{1}{n}\sum(X - \overline{X})(Y - \overline{Y})$

(9) $r = \frac{\sum(x - \overline{x})(y - \overline{y})}{\sqrt{\sum(x - \overline{x})^2 \sum(y - \overline{y})^2}}$

(10) $r = \frac{\sum(x - \overline{x})(y - \overline{y})}{n\sigma_x\sigma_y}$

(11) $b_{YX} = \frac{\text{cov}(X, Y)}{\sigma_X{}^2} = r\frac{\sigma_y}{\sigma_x}$

(12) $Y - \overline{Y} = \frac{\text{cov}(X, Y)}{\sigma_x{}^2}(X - \overline{X})$

$= r\frac{\sigma_x}{\sigma_y}(X - \overline{X})$

(13) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(14) $\tau = \frac{2S}{n(n-1)}$

(15) $r = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$