# MATHEMATICS

***(Three hours)***

*Answer* **Question 1** *from Part I* and **six** questions from *Part II, choosing either (i) or (ii) from each question.*

*All working ,including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.*

*The intended marks for questions or parts of questio0ns are given in brackets [ ]*

***Mathematical tables and square paper are provided.***
***Slide rule, may be used.***

## PART I

Answer **all** questions

**Question 1**

(i) Find the equation of the hyperbola whose eccentricity is $\frac{5}{2}$, co-ordinates of the focus are (2, 0) and the equation of the directrix is $4x - 3y - 2 = 0$ [3]

(ii) Find the derivatives of the function $e^x.\log(1+x^2)$. [3]

(iii) Determine real values of $x$ and $y$ for which the following statement is true :

$$x - i\,y = \frac{2 + I}{1 + i}$$ [3]

(iv) Find the equation of the plane the normal to which the origin is of length 5 units and which makes angles with the axes of $x$, $y$ and $z$ equal to $135^0$, $45^0$, $45^0$ respectively [3]

(v) Using properties of determinants prove that:

$$\begin{vmatrix} a^2 & bc & c^2+ca \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$$ [4]

(vi) Verify Rolle's theorem for the function $f(x) = e^x.\cos x,\ x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ [4]

(vii) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$ [4]


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(viii) Evaluate $\int \frac{x+\sin x}{1+\cos x}dx$ [4]

Find the equation of the sphere concentric with $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$ but of double the radius.

(ix) Two dice are thrown. Find the probability of getting an odd number on one dice and a multiple of 3 on the other. [4]

(x) Calculate the standard deviation for the following distribution using assumed mean of 20: [4]

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 8 | 11 | 17 | 20 | 25 | 30 | 35 |
| $f$ | 2 | 3 | 4 | 1 | 5 | 7 | 3 |

## PART – II (60 Marks)

Answer either (i) or (ii) from each question.

**Question 2** (i)

(a) Solve the following linear equations by matrix method.

$12y = 9 + 5x$; $7x = 6y - 8$ [3]

b) Determine the value of $\lambda$ for which the $6x^2-5xy-6y^2+14x+5y+y+\lambda=0$ represents a pair of straight lines. [3]

**OR**

**Question 2** (ii)

(a) Using Cramer's rule, solve the following system of equations :

$$2y - 3z = 0$$
$$x + 3y = 4$$
$$3x + 4y = 3$$
[3]

(b) Find the equation of the bisector of the angles between the pairs of lines given by $10x^2 - 11xy - 6y^2 = 0$ [3]


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**Question 3** (i)

Prove that:

$$\text{Cos}^{-1}\frac{1}{\sqrt{1+x^2}}+\sin^{-1}\frac{1}{\sqrt{x^2+2x+2}}=\tan^{-1}(x^2+x+1)$$

[6]

**OR**

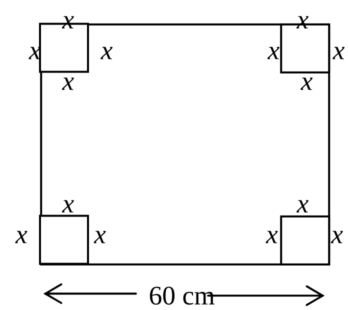
**Question 3** (ii)

Solve the equation: $\sin^{-1} + \sin^{-1}2x = \frac{\pi}{3}$

[6]

**Question 4.** (i)

a) An open top box of maximum possible volume is to be made from a square piece of tin of side 60cm by cutting equal squares out of its corners and then folding up the tin to form the sides of the box as shown in the diagram alongside. Show that the length of a side of the square cut out is 10cm.




[5]

b) Solve:

$$\text{Sin } x.\frac{dy}{dx} + y\text{ Cos } x = x, \sin x, \text{ given } y = 1, \text{ when } x = 0$$ [5]

c) Evaluate: $\int_0^2 (x^2+1)dx$ as the limit of a sum. [4]

d) Evaluate by L-Hospital's theorem [3]

$$Lt_{x\to 0}\ \frac{\tan x - x}{x - \sin x}$$

**OR**

**Question 4** (ii)

(a) Prove that the curves $x^2 = 4y$ and $y^2 = 4x$ meet at the origin and at the point (4, 4) and find the volume obtained by rotating the area between them about the $x$-axis. [5]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b) Solve the differential equation: $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$. [5]

c) Evaluate: $\int \frac{x^3 dx}{x^2+5x+4} dx$ [4]

d) If $y = (\log x)^{\log x}$, find $\frac{dy}{dx}$ [3]

**Question 5** (i)

a) Find the area of the parallelogram for which the vectors represented by the diagonals are $2\hat{i} - 3\hat{j} + 5\hat{k}$ and $^-2\hat{i} + 2\hat{j} + 2\hat{k}$. [3]

b) Prove that: $\vec{a}\times(\vec{b}\times\vec{c})+\vec{b}\times(\vec{c}\times\vec{a})+\vec{c}\times(\vec{a}\times\vec{b}) = 0$ [3]

c) Find the equation of the plane passing through the line of intersection of the planes $2x - y = 0$ and $3z - y = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$. [4]

d) Find the vertex, focus and length of latus of the parabola $x^2 + 4y + 3x = 2$ [4]

**OR**

**Question 5** (ii)

a) Find the equation of the ellipse whose vertices are $(2, ^-2)$ and $(2, 4)$ and whose eccentricity is $\frac{1}{3}$. [3]

b) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. [4]

c) Prove that if the diagonals of a parallelogram intersect at right angles then all the sides of the parallelogram are equal. [3]

d) Find the value of $\lambda$ for which the four points with position vectors $3\hat{i} - 2\hat{j} - \hat{k}$; $2\hat{i} + 3\hat{j} - 4\hat{k}$; $-\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} + 5\hat{j} + \lambda\hat{k}$ are coplanar. [4]

**Question 6** (i)

Illustrate and explain the region of the Argand's plane represented by the inequality

$|z + i| \le |z + 3|$ [5]

**OR**


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**Question 6** (ii)

Show that the continued product of the four values of

$\left(\frac{1}{2} + i\frac{V3}{2}\right)^{3/4} = 1.$ [5]

**Question 7** (i)

(a) Using weighted Arithmetic Mean method, find the index number from the following data and interpret your result. [4]

| Commodity | Base year (1995) Price in Nu. | Current year (2000) Price in Nu. | Weight |
|---|---|---|---|
| Rice | 10 | 15 | 20 |
| Wheat | 27 | 32 | 15 |
| Oil | 53 | 58 | 10 |
| Fish | 74 | 82 | 12 |
| Sugar | 23 | 28 | 13 |

(b) In a contest, two judges awarded points to eight candidates in the following manner :

| **Candidate** | **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** |
|---|---|---|---|---|---|---|---|---|
| Judge X | 52 | 67 | 40 | 72 | 55 | 48 | 60 | 43 |
| Judge Y | 52 | 51 | 43 | 54 | 56 | 40 | 60 | 49 |

Find the rank correlation coefficient by using spearman's method.

[3]

(c) One bag contains 5 white and 4 black balls. Another bag contains 7 white and 9 black balls. A ball is transferred from the first bag to the second and then a ball is drawn from the second. Find the probability that the ball drawn is white [5]

**OR**

**Question 7** (ii)

(a) From the following data, obtain the regression equation of $y$ on $x$ : [4]

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 9 | 8 | 10 | 12 | 11 |


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(b) Obtain the 4- year centered moving average for the following series of observations :

| **Year** | **1990** | **1991** | **1992** | **1993** | **1994** | **1995** | **1996** | **1997** |
|---|---|---|---|---|---|---|---|---|
| Annual Sale (Nu. 000) | 3.6 | 4.3 | 4.3 | 3.4 | 4.4 | 5.4 | 3.4 | 2.4 |

Find the values up to one decimal place. [3]

(c) On an average of 12 games of chess played by A and B, A wins 6, B wins 4 and 2 games end in a tie. A and B played in a tournament of 3 games. Calculate the probability that

i) at least 2 games end in a tie

ii) A and B win alternate games, no games gets tied up. [5]


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