BUSINESS MATHEMATICS

(Three hours and a quarter)

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*Answer **Question 1** from Section **A** and **10 Question**s from Section **B**.*
All working, including rough work, should be done on the same sheet as, and adjacent to, the rest of the answer.

*The intended marks for the **Question**s or parts of **Question**s are given in brackets [ ]*
Mathematical formulae are given at the end of this Question paper.
The use of calculator (fx-82/fx-100) is allowed without memory.
Diagrams given in this question booklet are not in scale.

Section A(30 Marks)

Answer **ALL** questions

Directions: Read the following questions carefully. For each question there are four alternatives **A**, **B**, **C** and **D**. Choose the correct alternative and write it in your answer sheet.

Question 1. **(2×15=30 Marks)**

(i) Summation notation for 1+7+17+31 is

A $\sum_{1=1}^{4} 2i^2 - 1$

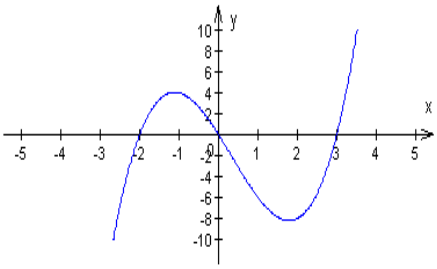
B $\sum_{1=1}^{4} 2i - 1$

C $\sum_{1=1}^{4} 4i^2 - 3$

D $\sum_{1=1}^{4} i$

This booklet contains 12 pages

(ii) The Factors of the polynomial of the given graph are



A $(x(x+2),(x+3)$

B $x,(x+2),(x-3)$

C $x(x-2),(x-3)$

D $x,(x-2),(x+3)$

(iii) If $\tan\theta = \frac{12}{5}, \quad 0 < \theta < \frac{\pi}{2}$, then the value of $\cos 2\theta$ is

A $\frac{169}{119}$

B $-\frac{169}{119}$

C $\frac{119}{169}$

D $-\frac{119}{169}$

(iv) Which of the following is a rational number?

A $\left(5-\sqrt{3}\right)^2$

B $\left(\sqrt{2}+1\right)^2$

C $\left(\sqrt{8}+\sqrt{2}\right)^2$

D $\left(\sqrt{3}-\sqrt{2}\right)^2$

(v) The restrictions of the expression $\frac{x^2-4}{x^2+x-6} \div \frac{x^2+3x}{x^2+7x+12}$ are

A $x \neq \{-3,0,2\}$

B $x \neq \{-3,0\}$

C $x \neq \{-3,2\}$

D $x \neq \{-2,0,3\}$

(vi) For $y = \frac{x-3}{2-x}$ Evaluate $\left.\frac{dy}{dx}\right|_{x=1}$

A -1

B 3

C 5

D -3

(vii) If $y = u^2 + 4u$ and $u = \sqrt{x}$ *then* $\frac{dy}{dx}$ is equal to

A $2\sqrt{x} + 4$

B $1 - \frac{2}{\sqrt{x}}$

C $1 + \frac{2}{\sqrt{x}}$

D $4x + 8\sqrt{x}$

(viii) The point on the curve $y = \sin x$ at which the tangent is horizontal is

A $\left(1, \frac{\pi}{2}\right)$

B $\left(-\frac{\pi}{2}, -1\right)$

C $\left(-\frac{\pi}{2}, 1\right)$

D $\left(\frac{\pi}{2}, 1\right)$

(ix) Given $y = x^x$ then $\frac{dy}{dx}$ is

A $\left(1 + \log_e x\right)$

B $x^x\left(1 + \log_e x\right)$

C $\frac{1}{x^x\left(1 + \log_e x\right)}$

D $\frac{x^x}{1 + \log_e x}$

(x) $\int \frac{x^3+x^2+x+1}{x}dx$ is equal to

A $x^2+x+1+\frac{1}{x}+c$

B $\log_e\left(x^3+x^2+x+1\right)+c$

C $\frac{x^3}{3}+\frac{x^2}{2}+x+\log_e x+c$

D $2x+1+\frac{1}{x^2}+c$

(xi) $\int \frac{\cos(\log_e x)}{x}dx$ is equal to

A $-\sin(\log_e x)+c$

B $\frac{\cos^2(\log_e x)}{2}+c$

C $-\cos(\log_e x)+c$

D $\sin(\log_e x)+c$

(xii) If $A=\begin{pmatrix}2 & 3\\ 1 & 4\end{pmatrix}$ then $A\cdot(AdjA)$ is

A $\begin{pmatrix}4 & -3\\ -1 & 2\end{pmatrix}$

B $\begin{pmatrix}5 & 0\\ 0 & 5\end{pmatrix}$

C $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

D $\begin{pmatrix}7 & -12\\ 4 & -7\end{pmatrix}$

(xiii) Find $\frac{dy}{dx}$ when $y = \log_5 x^3$

A $\frac{3}{x}$

B $\frac{3}{x} \log_{10} e$

C $\frac{3}{x} \log_e 5$

D $\frac{3}{x} \log_5 e$

(xiv) The mean marks in mathematics of class A, and class B are 50 and 60 respectively. The mean marks of the combined class is 54. If the strength of the class A is 60, then the strength of class B is

A 90
B 50
C 40
D 55

(xv) The vertical asymptote of $y = \frac{x-2}{x^2+2x-8}$ is

A $x = 4$
B $x = -4$
C $x = 2, x = -4$
D $x = 2, x = 4$

Section B (70 marks)

Answer any 10 questions. All questions in this section have equal marks. Unless otherwise stated, you may round answers to decimal places.

Question 2

a) Simplify $\frac{x-3}{x^2-9} - \frac{x+5}{x^2+8x+15}$ **[3]**

b) Using mathematical Induction , prove that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots\ldots\ldots\ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}, \forall n \in N$$ **[4]**

Question 3

a) Determine a quadratic equation f(x) such that $f(2) = 21$, $f'(2) = 14$ and $f''(2) = 6$ **[3]**

b) A rectangular shopping complex is to be built on a 80m by 60m rectangular plot in such a way that there is a path x meter wide surrounding the building. The building can occupy up to 70% of the plot's area. What is the range of possible integral values for the width of the path (x) **[4]**

Question 4

(a) Convert the exponential function $y = 20(x)^5$ into base –e form **[2]**

(b) The following table shows the relationship between the ages of husbands (x) and wives (y) in a town. Using this, find the regression line of Y on X. hence estimate the age of wife when the age of husband is 30. **[5]**

Ages of husband (x)	25	22	28	26	35	20
Age of wife (y)	18	15	20	19	22	14

Question 5

(a) Evaluate $\sum_{i=1}^{20} i(2-3i)^2$ **[4]**

(b) Evaluate $\int x \sin x dx$ **[3]**

Question 6

a) Find the square root of $8 + 2\sqrt{15}$ **[3]**

b) The population of a town in the year 2005 was 0.15 million. The population increases by 15% p.a.

i) Estimate the population of the town in the year 2009

ii) When will the population of the town become double the population of 2005? (Give both the answer to the nearest integer) **[4]**

Question 7

(a) Using $\sin(A+B) = \sin A\cos B + \cos A\sin B$, Prove that $sin3x = 3\sin x - 4\sin^3 x$ **[4]**

(b) Evaluate $\int_1^e \frac{\log_e x}{x} dx$ **[3]**

Question 8

(a) For what values of x is the graph of $y = x^3 + x$ below the graph of $y = 6 - 4x^2$ **[4]**

(b) Solve the equations by matrix method

$$8x + 3y - 2 = 0$$
$$5x - 4y - 13 = 0$$ **[3]**

Question 9

(a) Calculate the area bound by the $y = x(x-4)$ and the X – axis. This area is rotated about X-axis through four right angles to generate a solid. Calculate the volume of the solid generated. **[5]**

(b) Calculate the mean deviation about median of the following data
65, 35, 50, 42, 55 **[2]**

Question 10

(a) Evaluate $\int \left(\sqrt{x} - 2\right)^2 dx$ **[2]**

(b) Determine the dimensions of a rectangular playground with maximum area that can be enclosed with 676m of fencing
[5]

Question 11

(a) Given $X = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, does X^{-1} exist? Give a reason, Also find Adj X **[4]**

(b) At a certain instant, the area of a circular oil spill on the ocean increases at the rate of $2cm^2/\sec$. Determine the rate of increase of the radius when the area is 50cm^2 **[3]**

Question 12

(a) If $y = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$, show that $\frac{dy}{dx} = \sec^2 x$ **[4]**

(b) Given $y = \frac{1-x}{1+x}$, show that $(1+x)^2 \frac{dy}{dx} + 2 = 0$ **[3]**

Question 13

(a) Find the equation of the tangent to the curve $y = 5^x$ at x = 1 **[3]**

(b) Describe the transformation that should be applied to the graph of $y = e^x$ to obtain the graph of $y = 5e^x - 4$ **[2]**

(c) The following table is taken from the family budget of a group of people from a village. Calculate the cost of living index for the year 2007 **[2]**

Expenses on	Weight	Price in 2005	Price in 2007
Rice	40	200	220
Oil	15	120	150
Dal	10	80	90
Kharang	35	150	200

Question 14

For the curve $y = x^4 - 8x + 16$, find

(i) The x and y intercepts
(ii) The local maximum and minimum points
(iii) Point of inflexion
(iv) With the help of the above information, sketch the curve(Use the graph Paper) **[7]**

Functions and Equations

(1) $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

(2) $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

(3) $a^2 - b^2 = (a+b)(a-b)$

(4) $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

(5) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sequence and series

(1) $\sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}$

(2) $\sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6}$

(3) $\sum_{i=1}^{n} i^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

(4) $t_n = ar^{n-1}$

(5) $t_n = a + (n-1)d.$

(6) $S_n = \dfrac{a(1-r^n)}{1-r}$ where $r<1$

$= \dfrac{a(r^n - 1)}{r-1}$, Where $r > 1$

(7) $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Differentiation

(1) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(2) $y = x^n,\ y' = nx^{n-1},$

(3) $y = cf(x),\ y' = cf'(x)$

(4) $y = f(x) \pm g(x),\ y' = f'(x) \pm g'(x)$

(5) $F(x) = f(x)g(x),$

$F'(x) = f(x)g'(x) + f'(x)g(x)$

(6) $F(x) = \dfrac{f(x)}{g(x)}$,

$F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

(7) $f \circ g(x)' = f'g(x) \times (g'x)$

(8) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

(9) $v(t) = h'(t)$

Coordinate Geometry

(1) $(y - y_1) = m(x - x_1)$

(2) $\sqrt{(x-a)^2 + (y-b)^2}$

Trigonometry

(1) $Sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

(2) $Cos(A \pm B) = CosA \cos B \mp \sin A \sin B$

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

(4) $\sin^2\theta + \cos^2\theta = 1$

Logarithmic Exponentials

(1) $y = y_0(1+r)^x$

(2) $y = y_0 e^{kx}$

(3) $A = P(1+r)^n$

Integration

(1) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[\left(\dfrac{d}{dx}f(x)\right)\int g(x)dx\right]dx$

(2) $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$

(3) $V = \pi\int_a^b y^2 dx$

(4) $A = \int_a^b y dx$

Measurement

(1) Cone: $V = \dfrac{\pi}{3} r^2 h$

(2) Cone: $SA = \pi rl + \pi r^2$

(3) Sphere: $V = \dfrac{4\pi}{3} r^3$

(4) Sphere: $SA = 4\pi r^2$

(5) Cylinder: $SA = 2\pi rh + 2\pi r^2$

(6) Cylinder: $V = \pi r^2 h$

(7) Circle: $A = \pi r^2$

(8) Circle: $C = 2\pi r$

(9) Triangle: $A = \frac{bh}{2}$, $A = \frac{\sqrt{3}}{4}x^2$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

(10) Rectangle: $A = lw$,

(11) Rectangle $P = 2l + 2w$

(12) Square: $A = s^2$,

(13) Square $P = 4s$

(14) Rectangular Prism: $V = lwh$

Matrices

(1) $C_{ij} = (-1)^{i+j} M_{ij}$

(2) $A A^{-1} = A^{-1} A = I$

(3) *Inverse of* $A = A^{-1} = \frac{1}{\det A} \cdot adjA$

Data & Probability

(1) $\bar{x} = \frac{\sum fx}{n}$

(2) Median $= l_1 + \frac{l_2 - l_1}{f1}(m - c)$

(3) $\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}}$

(4) $\sigma_{12} = \sqrt{\frac{n_1\sigma_1^2 + n_1\sigma_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1 + n_2}}$

(5) $\sigma = \sqrt{\frac{\sum f(x_i - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{N} - \left(\frac{\sum fx}{N}\right)^2}$

(6) $\bar{x}_{12} = \frac{m\bar{x}_1 + n\bar{x}_2}{m+n}$

(7) $I = \frac{\sum \frac{P_1}{P_0} \times 100}{n}$

(8) $I = \frac{\sum p_1 w}{\sum p_0 w} \times 100$

(9) $\mathrm{Cov(X,Y)} = \frac{1}{n}\sum (X - \bar{X})(Y - \bar{Y})$

(10) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sqrt{\sum (x - \bar{x})^2 \sum (y - \bar{y})^2}}$

(11) $r = \frac{\sum (x - \bar{x})(y - \bar{y})}{n\sigma_x \sigma_y}$

(12) $b_{YX} = \frac{\mathrm{cov(X,Y)}}{\sigma_x^2} = r\frac{\sigma_y}{\sigma_x}$

(13) $Y - \bar{Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_x^2}(X - \bar{X})$

$$= r\frac{\sigma_x}{\sigma_y}(X - \bar{X})$$

(14) $b_{xy} \times b_{yx} = r\frac{\sigma_y}{\sigma_x} \times r\frac{\sigma_x}{\sigma_y}$

(15) $\tau = \frac{2S}{n(n-1)}$

(16) $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$