AGA KHAN UNIVERSITY EXAMINATION BOARD

HIGHER SECONDARY SCHOOL CERTIFICATE

CLASS XI EXAMINATION

MAY 2012

Mathematics Paper II

Time allowed: 2 hours 15 minutes Marks 65

INSTRUCTIONS

Please read the following instructions carefully.

1. Check your name and school information. Sign that it is correct.

I agree that this is my name and school. **Candidate's signature**

2. RUBRIC. There are TWELVE questions. Answer ALL TWELVE questions. Choices are specified inside the paper.

3. When answering the questions:

   Read each question carefully.
   Use a black pencil for diagrams. DO NOT use coloured pencils.
   DO NOT use staples, paper clips, glue, correcting fluid or ink erasers.
   Complete your answer in the allocated space only. DO NOT write outside the answer box.

4. The marks for the questions are shown in brackets ( ).

5. You may use a scientific calculator if you wish.

Q.1. (Total 4 Marks)

i. If $z = a + ib$, then prove that $|z| = |-z|$. (2 Marks)

ii. If $z_1 = 2 + 3i$, $z_2 = a + (b - 1)i$ and $z_1 = z_2$, then find the value of b. (2 Marks)

Q.2. (Total 7 Marks)

i. Find the Adjoint of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 3 & 2 \end{bmatrix}$. (4 Marks)

ii. By using the properties of determinant show that $\begin{vmatrix} 6 & 7 & 2a \\ 8 & 3 & 3a \\ 14 & 10 & 5a \end{vmatrix} = 0$. (3 Marks)

Q.3. (Total 5 Marks)

If $7 , \frac{31}{4} , \frac{17}{2} , \frac{37}{4} , 10 , \frac{43}{4} , ...$ is an arithmetic progression, then find its 13^{th} term.
Also find the sum of the first 100 terms.

(ATTEMPT EITHER PART a OR PART b OF Q.4.)

Q.4. (Total 3 Marks)

a. Insert TWO geometric means between 4 and 32.

b. Find the sum to n terms of the series whose n^{th} term is $4n^3 - 2n$.

(ATTEMPT EITHER PART a OR PART b OF Q.5.)

Q.5. (Total 4 Marks)

a. How many FOUR letter passwords can be formed from the word "COMPUTER" when each letter is used once. Also find the number of passwords when all the letters used at a time and each letter is used once. (4 Marks)

b.

i. Write the sample space of an experiment when three coins are tossed simultaneously. (1 Mark)

Hence, find the probability of an event getting

ii. at most two heads. (2 Marks)

iii. exactly one head. (1 Mark)

Q.6. (Total 6 Marks)

i. Find the middle term in the expansion of $\left(a x^{2} + \frac{b}{x} \right)^{12}$. (4 Marks)

ii. State the principle of Mathematical Induction. (2 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.7.)

Q.7. (Total 7 Marks)

a.

i. Find the values of y for the equation $2^{1+y} + 2^{1-y} = 5$. (4 Marks)

ii. Find a quadratic equation whose roots are $-1+\sqrt{3}\,i$ and $-1-\sqrt{3}\,i$. (3 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.7.)

b.

i. Use synthetic division to find the value of k if $(x - 2)$ is a factor of $x^3 - 2x^2 - 2x + k$. (3 Marks)

ii. Find the values of y for the system of equations $x + 2 = 2y$ and $x^2 + y^2 = 4$. (4 Marks)

Q.8. (Total 2 Marks)

Decide whether $f(x) = x^3 - 3x$ is an even or an odd function ? Also justify the answer.

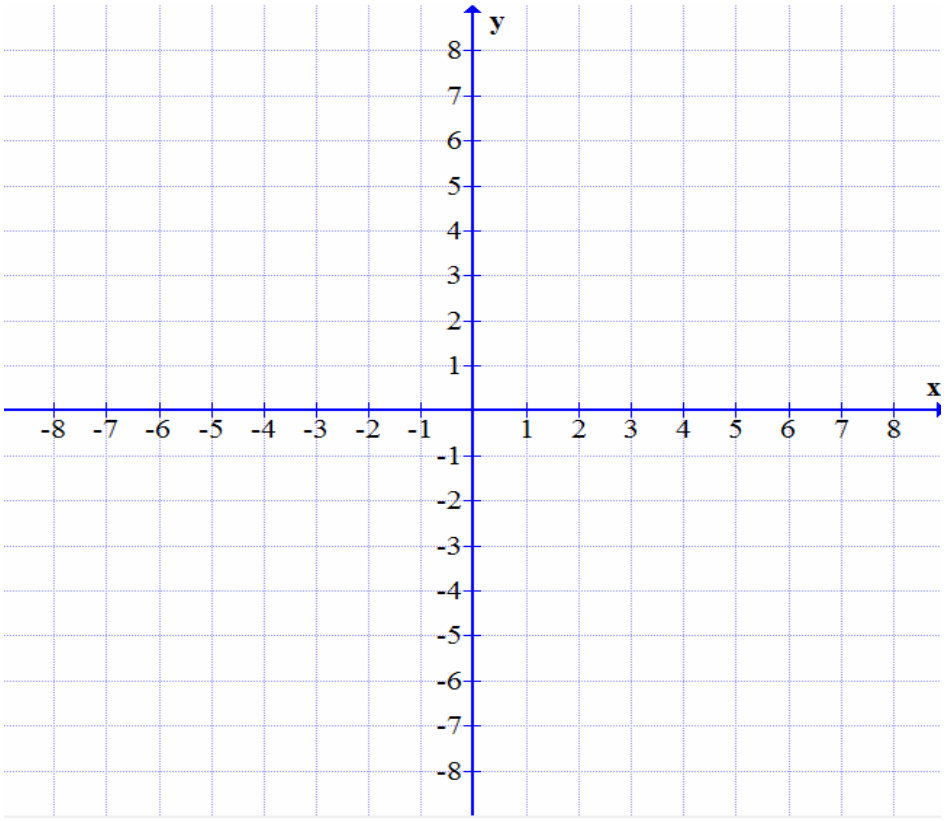
Q.9. (Total 6 Marks)

Graph the solution region of the following system of linear inequalities and find any one corner point.

$3x + 2y \leq 6$

$4x + y \geq -8$

$y \geq 0$



(ATTEMPT EITHER PART a OR PART b OF Q.10.)

Q.10. (Total 6 Marks)

a.

i. Prove that $\sin(\alpha + \beta)\sin(\alpha - \beta) = \cos^2\beta - \cos^2\alpha$. (4 Marks)

ii. Prove that $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$. (2 Marks)

b.

i. Prove that $\sin 7\theta + \sin 3\theta = 2\sin 5\theta \cos 2\theta$. (2 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.10.)

ii. Prove that $\dfrac{\tan\frac{\theta}{2} - \cot\frac{\theta}{2}}{\tan\frac{\theta}{2} + \cot\frac{\theta}{2}} = -\cos\theta$. (4 Marks)

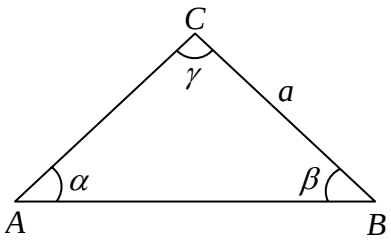
(ATTEMPT EITHER PART a OR PART b OF Q.11.)

Q.11. (Total 8 Marks)

a.

i. Two boys are on the opposite sides of a 50 m high tree and angles of elevation of the top of the tree are $26°$ and $32°$, draw a diagram and find the distance between the two boys. (5 Marks)

ii. Find the value of a for the ΔABC if $\beta = 45°$, $\gamma = 25°$ and $c = 2\sqrt{2}$. (3 Marks)



(ATTEMPT EITHER PART a OR PART b OF Q.11.)

b.

i. If the measures of the sides of a ΔABC are 6 cm, 6 cm and 8 cm , then find the circum-radius (R) and in-radius (r) of a circle associated to the ΔABC. (6 Marks)

ii. Write the formula of $\sin\frac{\alpha}{2}$ and $\cos\frac{\alpha}{2}$ in terms of $s = \frac{a+b+c}{2}$, where a, b and c are the measures of the sides of a triangle. (2 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.12.)

Q.12. (Total 7 Marks)

a.

i. Find the period, domain and range of $2 \sin 2x$. (4 Marks)

ii. Find the maximum and minimum value of $5 + 3 \cos 3\theta$. (3 Marks)

(ATTEMPT EITHER PART a OR PART b OF Q.12.)

b. Solve the equation $4\sin^2 x - 3 = 0$. (7 Marks)

Please use this page for rough work

Please use this page for rough work

Please use this page for rough work

Please use this page for rough work