VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education
2007

FURTHER MATHEMATICS
Written examination 1

Monday 5 November 2007

Reading time: 11.45 am to 12.00 noon (15 minutes)
Writing time: 12.00 noon to 1.30 pm (1 hour 30 minutes)

MULTIPLE-CHOICE QUESTION BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of modules*	*Number of modules to be answered*	*Number of marks*
A	13	13			13
B	54	27	6	3	27
					Total 40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved graphics calculator or approved CAS calculator or CAS software and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question book of 42 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.
- Working space is provided throughout the book.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

At the end of the examination
- You may keep this question book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Working space

SECTION A

Instructions for Section A

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Core – Data analysis

The following information relates to Questions 1 and 2.

The dot plot below shows the distribution of the number of bedrooms in each of 21 apartments advertised for sale in a new high-rise apartment block.

Question 1

The mode of this distribution is

A. 1
B. 2
C. 3
D. 7
E. 8

Question 2

The median of this distribution is

A. 1
B. 2
C. 3
D. 4
E. 5

Question 3

A student obtains a mark of 56 on a test for which the mean mark is 67 and the standard deviation is 10.2.

The student's standardised mark (standard z-score) is closest to

A. -1.08

B. -1.01

C. 1.01

D. 1.08

E. 49.4

Question 4

The length of 3-month-old baby boys is approximately normally distributed with a mean of 61.1 cm and a standard deviation of 1.6 cm.

The percentage of 3-month-old baby boys with length greater than 59.5 cm is closest to

A. 5%

B. 16%

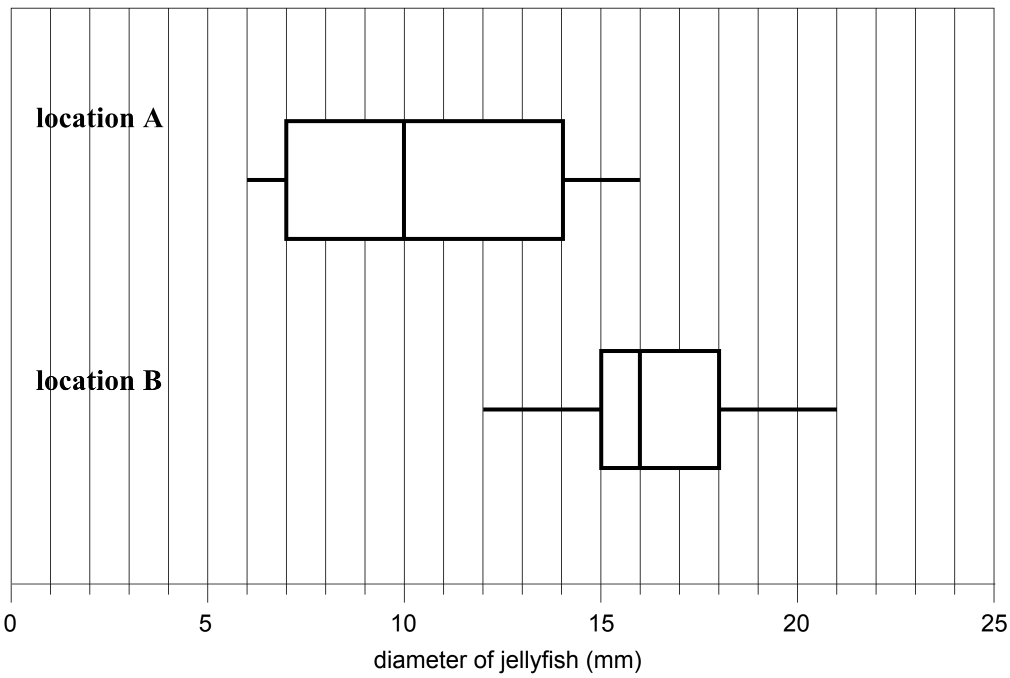
C. 68%

D. 84%

E. 95%

The following information relates to Questions 5 and 6.

Samples of jellyfish were selected from two different locations, A and B. The diameter (in mm) of each jellyfish was recorded and the resulting data is summarised in the boxplots shown below.



Question 5

The percentage of jellyfish taken from location A with a diameter greater than 14 mm is closest to

A. 2%

B. 5%

C. 25%

D. 50%

E. 75%

Question 6

From the boxplots, it can be concluded that the diameters of the jellyfish taken from location A are generally

A. similar to the diameters of the jellyfish taken from location B.

B. less than the diameters of the jellyfish taken from location B and less variable.

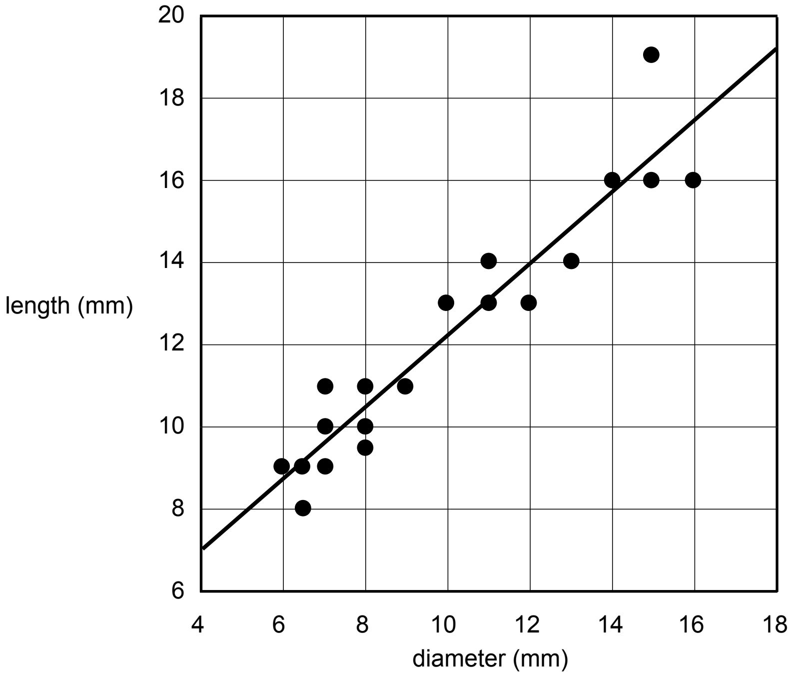
C. less than the diameters of the jellyfish taken from location B and more variable.

D. greater than the diameters of the jellyfish taken from location B and less variable.

E. greater than the diameters of the jellyfish taken from location B and more variable.

The following information relates to Questions 7 and 8.

The lengths and diameters (in mm) of a sample of jellyfish selected from another location were recorded and displayed in the scatterplot below. The least squares regression line for this data is shown.



The equation of the least squares regression line is

$$length = 3.5 + 0.87 \times diameter$$

The correlation coefficient is $r = 0.9034$

Question 7

Written as a percentage, the coefficient of determination is closest to

A. 0.816%

B. 0.903%

C. 81.6%

D. 90.3%

E. 95.0%

Question 8

From the equation of the least squares regression line, it can be concluded that for these jellyfish, on average

A. there is a 3.5 mm increase in *diameter* for each 1 mm increase in *length*.

B. there is a 3.5 mm increase in *length* for each 1 mm increase in *diameter*.

C. there is a 0.87 mm increase in *diameter* for each 1 mm increase in *length*.

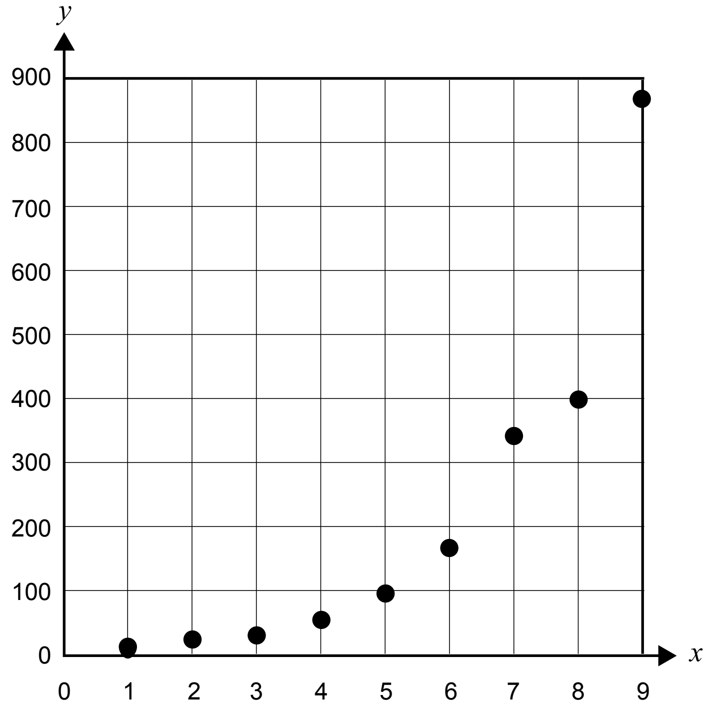
D. there is a 0.87 mm increase in *length* for each 1 mm increase in *diameter*.

E. there is a 4.37 mm increase in *diameter* for each 1 mm increase in *length*.

Question 9

A student uses the following data to construct the scatterplot shown below.

x	1	2	3	4	5	6	7	8	9
y	12	25	33	58	98	168	345	397	869



To linearise the scatterplot, she applies a **log *y*** transformation; that is, a log transformation is applied to the y-axis scale.

She then fits a least squares regression line to the transformed data.

With x as the independent variable, the equation of this least squares regression line is closest to

A. $\log y = -217 + 88.0\,x$

B. $\log y = -3.8 + 4.4\,x$

C. $\log y = 3.1 + 0.008\,x$

D. $\log y = 0.88 + 0.23\,x$

E. $\log y = 1.58 + 0.002\,x$

Question 10

The relationship between the variables

size of car (1 = small, 2 = medium, 3 = large)

and

salary level (1 = low, 2 = medium, 3 = high)

is best displayed using

A. a scatterplot.

B. a histogram.

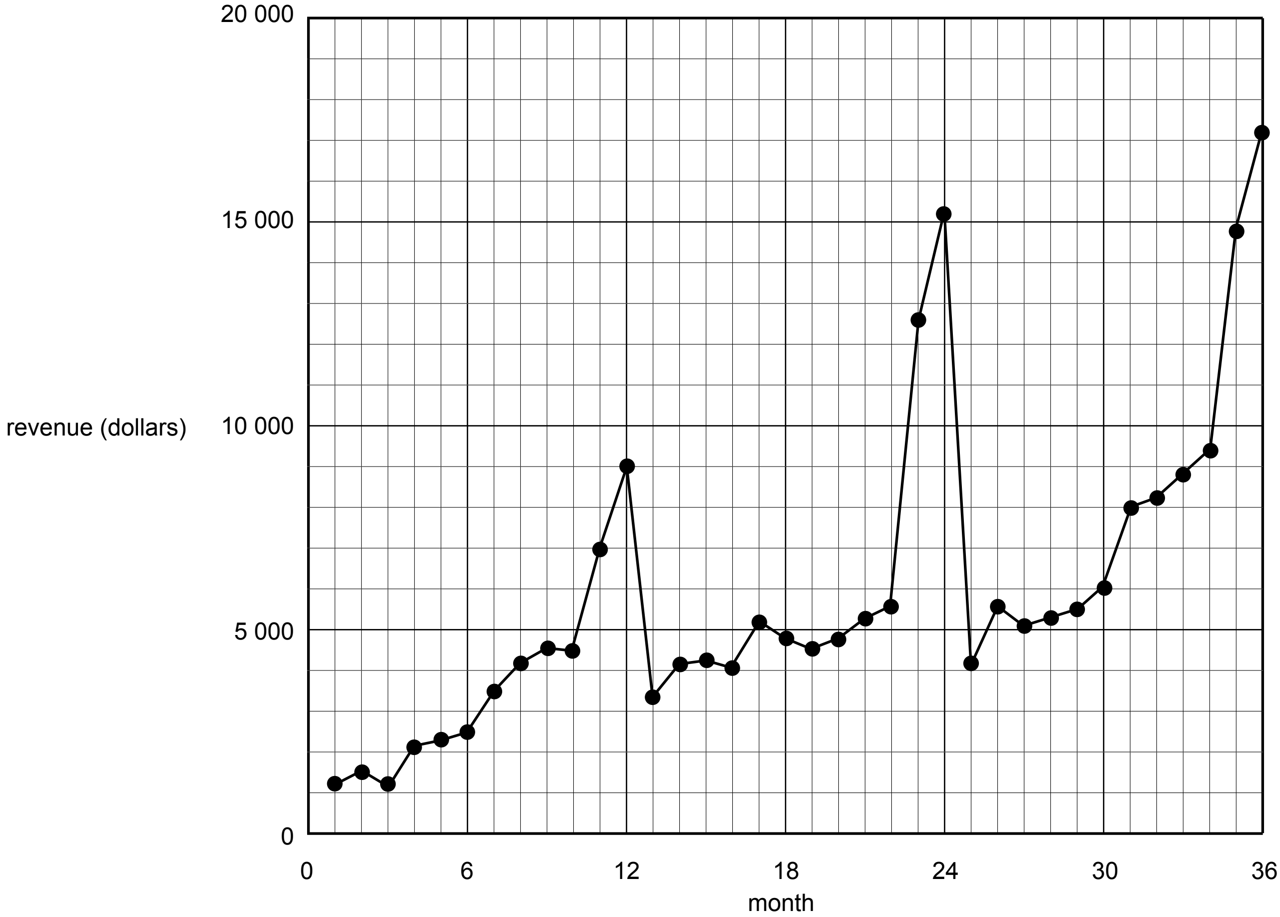
C. parallel boxplots.

D. a back-to-back stemplot.

E. a percentaged segmented bar chart.

The following information relates to Questions 11, 12 and 13.

The time series plot below shows the revenue from sales (in dollars) each month made by a Queensland souvenir shop over a three-year period.



Question 11

This time series plot indicates that, over the three-year period, revenue from sales each month showed

A. no overall trend.

B. no correlation.

C. positive skew.

D. an increasing trend only.

E. an increasing trend with seasonal variation.

Question 12

A three median trend line is fitted to this data.

Its slope (in dollars per month) is closest to

A. 125

B. 146

C. 167

D. 188

E. 255

Question 13

The revenue from sales (in dollars) each month for the first year of the three-year period is shown below.

Month	Revenue ($)
January	1236
February	1567
March	1240
April	2178
May	2308
June	2512
July	3510
August	4234
September	4597
October	4478
November	7034
December	8978

If this information is used to determine the seasonal index for each month, the seasonal index for **September** will be closest to

A. 0.80

B. 0.82

C. 1.16

D. 1.22

E. 1.26

END OF SECTION A

SECTION B

Instructions for Section B

Select **three** modules and answer **all** questions within the modules selected in pencil on the answer sheet provided for multiple-choice questions.

Show the modules you are answering by shading the matching boxes on your multiple-choice answer sheet.

Choose the response that is **correct** for the question.

A correct answer scores 1, an incorrect answer scores 0.

Marks will **not** be deducted for incorrect answers.

No marks will be given if more than one answer is completed for any question.

Module 1: Number patterns

Before answering these questions you must **shade** the Number patterns box on the answer sheet for multiple-choice questions.

Question 1

For the geometric sequence

$$24, 6, 1.5\ldots$$

the common ratio of the sequence is

A. -18
B. 0.25
C. 0.5
D. 4
E. 18

Question 2

The yearly membership of a club follows an arithmetic sequence.
In the club's first year it had 15 members.
In its third year it had 29 members.
How many members will the club have in the fourth year?

A. 8
B. 22
C. 36
D. 43
E. 57

Question 3

The difference equation

$$t_{n+1} = at_n + 6 \text{ where } t_1 = 5$$

generates the sequence

$$5, 21, 69, 213\ldots$$

The value of a is

A. -1
B. 3
C. 4
D. 15
E. 16

The following information relates to Questions 4 and 5.

The number of waterfowl living in a wetlands area has decreased by 4% each year since 2003.

At the start of 2003 the number of waterfowl was 680.

Question 4

If this percentage decrease continues at the same rate, the number of waterfowl in the wetlands area at the start of 2008 will be closest to

A. 532
B. 544
C. 554
D. 571
E. 578

Question 5

W_n is the number of waterfowl at the start of the nth year.

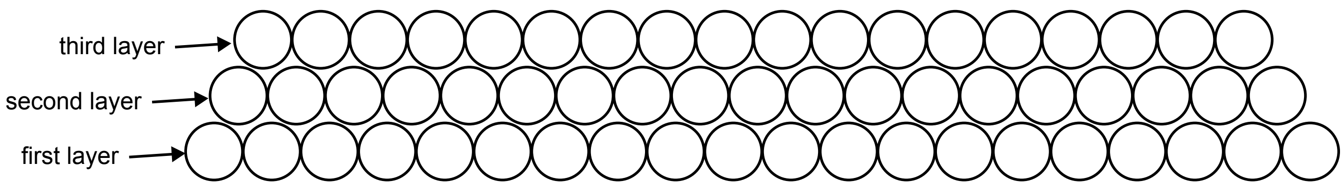
Let $W_1 = 680$.

The rule for a difference equation that can be used to model the number of waterfowl in the wetlands area over time is

A. $W_{n+1} = W_n - 0.04n$
B. $W_{n+1} = 1.04\ W_n$
C. $W_{n+1} = 0.04\ W_n$
D. $W_{n+1} = -0.04\ W_n$
E. $W_{n+1} = 0.96\ W_n$

Question 6

In the first three layers of a stack of soup cans there are 20 cans in the first layer, 19 cans in the second layer and 18 cans in the third layer.



This pattern of stacking cans in layers continues.

The maximum number of cans that can be stacked in this way is

A. 190
B. 210
C. 220
D. 380
E. 590

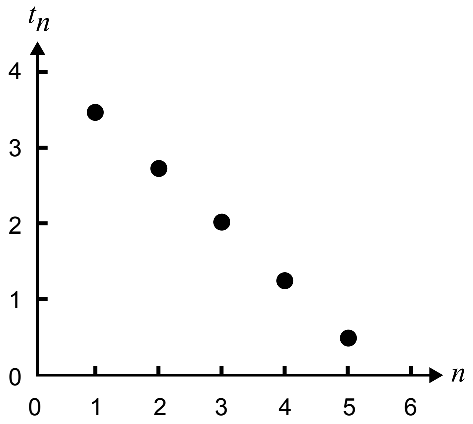
Question 7

The first term, t_1, of a geometric sequence is positive.

The common ratio of this sequence is negative.

A graph that could represent the first five terms of this sequence is

A.



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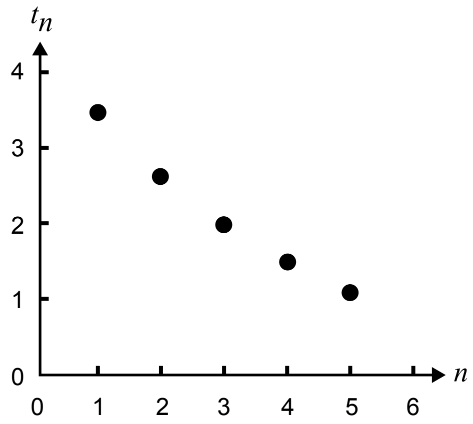
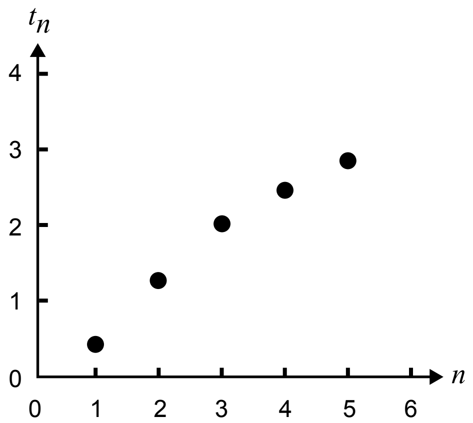
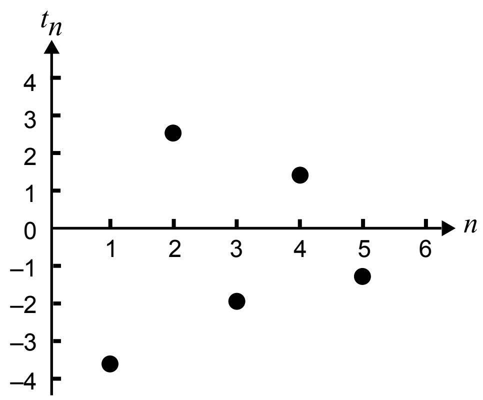


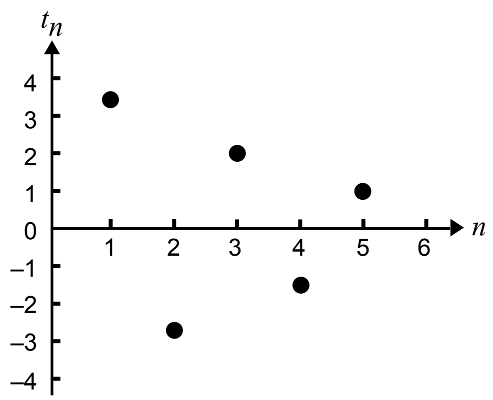
C.



D.



E.



Question 8

The first four terms of a sequence are

12, 18, 30, 54

A difference equation that generates this sequence is

A. $t_{n+1} = t_n + 6$ $\quad t_1 = 12$

B. $t_{n+1} = 1.5t_n$ $\quad t_1 = 12$

C. $t_{n+1} = 0.5t_n + 12$ $\quad t_1 = 12$

D. $t_{n+1} = 2t_n - 6$ $\quad t_1 = 12$

E. $t_{n+2} = t_{n+1} + t_n$ $\quad t_1 = 12, t_2 = 18$

Question 9

At the end of the first day of a volcanic eruption, 15 km^2 of forest was destroyed.

At the end of the second day, an additional 13.5 km^2 of forest was destroyed.

At the end of the third day, an additional 12.15 km^2 of forest was destroyed.

The total area of the forest destroyed by the volcanic eruption continues to increase in this way.

In square kilometres, the total amount of forest destroyed by the volcanic eruption at the end of the fourteenth day is closest to

A. 116

B. 119

C. 150

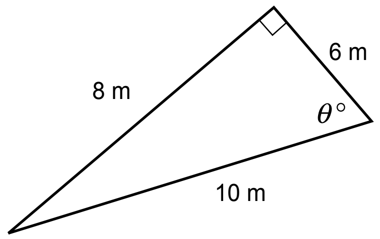
D. 179

E. 210

Module 2: Geometry and trigonometry

Before answering these questions you must **shade** the Geometry and trigonometry box on the answer sheet for multiple-choice questions.

Question 1



For the triangle shown, the value of $\cos \theta°$ is equal to

A. $\frac{6}{10}$

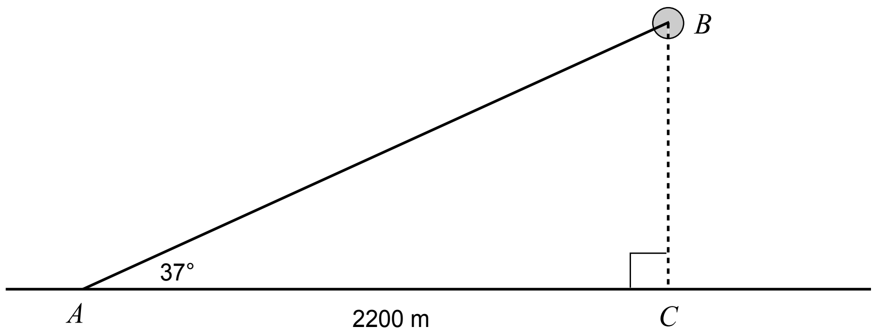
B. $\frac{6}{8}$

C. $\frac{8}{10}$

D. $\frac{10}{8}$

E. $\frac{8}{6}$

Question 2



For an observer on the ground at A, the angle of elevation of a weather balloon at B is 37°.
C is a point on the ground directly under the balloon. The distance AC is 2200 m.
To the nearest metre, the height of the weather balloon above the ground is

A. 1324 m

B. 1658 m

C. 1757 m

D. 2919 m

E. 3655 m

Question 3

A rectangle is 3.79 m wide and has a perimeter of 24.50 m.

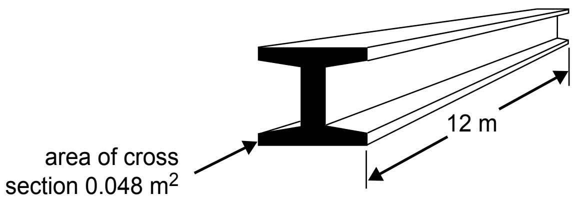
Correct to one decimal place, the length of the diagonal of this rectangle is

A. 9.2 m
B. 9.3 m
C. 12.2 m
D. 12.3 m
E. 12.5 m

Question 4

A steel beam used for constructing a building has a cross-sectional area of 0.048 m^2 as shown.

The beam is 12 m long.



In cubic metres, the volume of this steel beam is closest to

A. 0.576
B. 2.5
C. 2.63
D. 57.6
E. 2 500

Question 5

A block of land has an area of 4000 m^2.

When represented on a map, this block of land has an area of 10 cm^2.

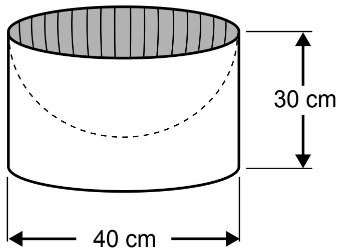
On the map 1 cm would represent an actual distance of

A. 10 m
B. 20 m
C. 40 m
D. 400 m
E. 4000 m

Question 6

A solid cylinder has a height of 30 cm and a diameter of 40 cm.

A hemisphere is cut out of the top of the cylinder as shown below.



In square centimetres, the total surface area of the remaining solid (including its base) is closest to

A. 1 260

B. 2 510

C. 6 280

D. 7 540

E. 10 050

Question 7

A closed cubic box of side length 36 cm is to contain a thin straight metal rod.

The maximum possible length of the rod is closest to

A. 36 cm

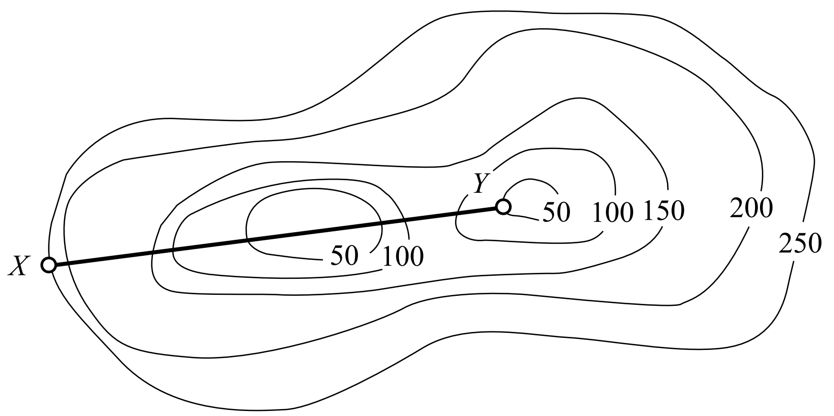
B. 51 cm

C. 62 cm

D. 108 cm

E. 216 cm

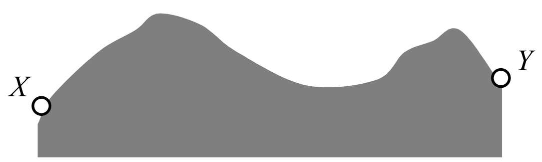
Question 8



contour interval = 50 m

For the contour map shown above, the cross section along the line segment *XY* could be

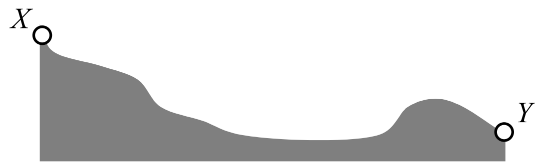
A.



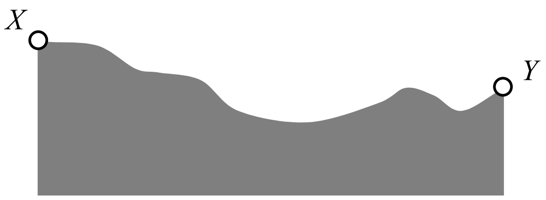
B.

C.

D.



E.



Question 9

The points M, N and P form the vertices of a triangular course for a yacht race.

$MN = MP = 4$ km.

The bearing of N from M is 070°

The bearing of P from M is 180°

Three people perform different calculations to determine the length of NP in kilometres.

Graeme $NP = \sqrt{16 + 16 - 2 \times 4 \times 4 \times \cos 110^\circ}$

Shelley $NP = 2 \times 4 \times \cos 35^\circ$

Tran $NP = \dfrac{4 \times \sin 110^\circ}{\sin 35^\circ}$

The correct length of NP would be found by

A. Graeme only.

B. Tran only.

C. Graeme and Shelley only.

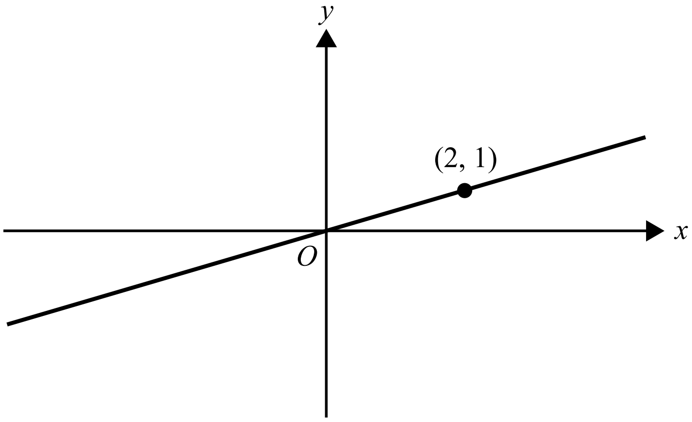
D. Graeme and Tran only.

E. Graeme, Shelley and Tran.

Module 3: Graphs and relations

Before answering these questions you must **shade** the Graphs and relations box on the answer sheet for multiple-choice questions.

Question 1



The line above passes through the origin and the point (2, 1).

The slope of this line is

A. –2

B. –1

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

E. 2

Question 2

A builder’s fee, C dollars, can be determined from the rule $C = 60 + 55n$, where n represents the number of hours worked.

According to this rule, the builder’s fee will be

A. \$60 for 1 hour of work.

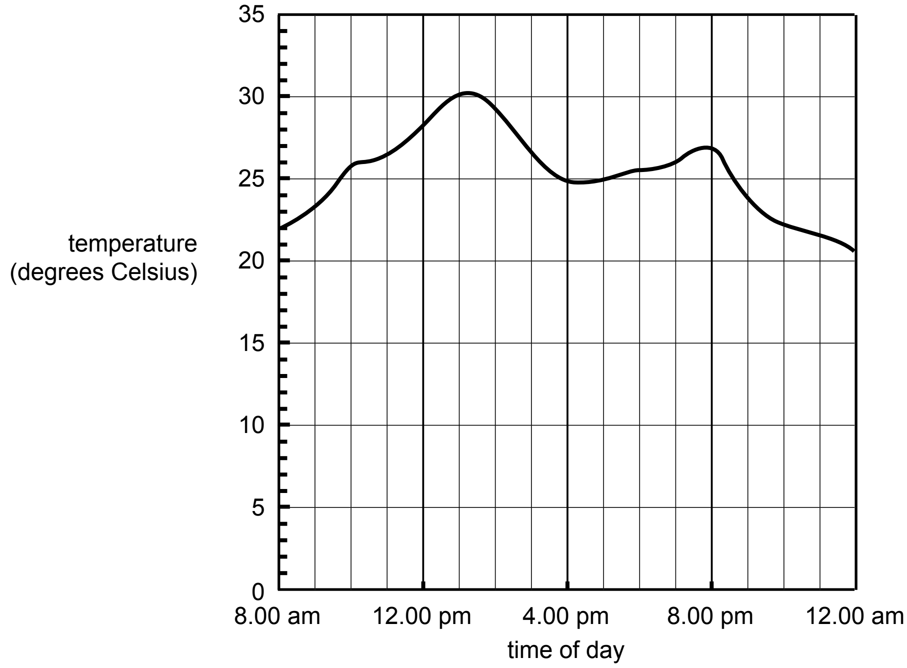
B. \$110 for 2 hours of work.

C. \$500 for 8 hours of work.

D. \$550 for 10 hours of work.

E. \$1150 for 10 hours of work.

Question 3



The graph above represents the temperature, in degrees Celsius, over a 16-hour period.

During this period, the minimum temperature occurred at

A. 8.00 am

B. 1.20 pm

C. 4.15 pm

D. 7.45 pm

E. 12.00 am

Question 4

Paul makes rulers. There is a fixed cost of \$60 plus a manufacturing cost of \$0.20 per ruler.

Last week Paul was able to break even by selling his rulers for \$1 each.

The number of rulers Paul sold last week was

A. 50

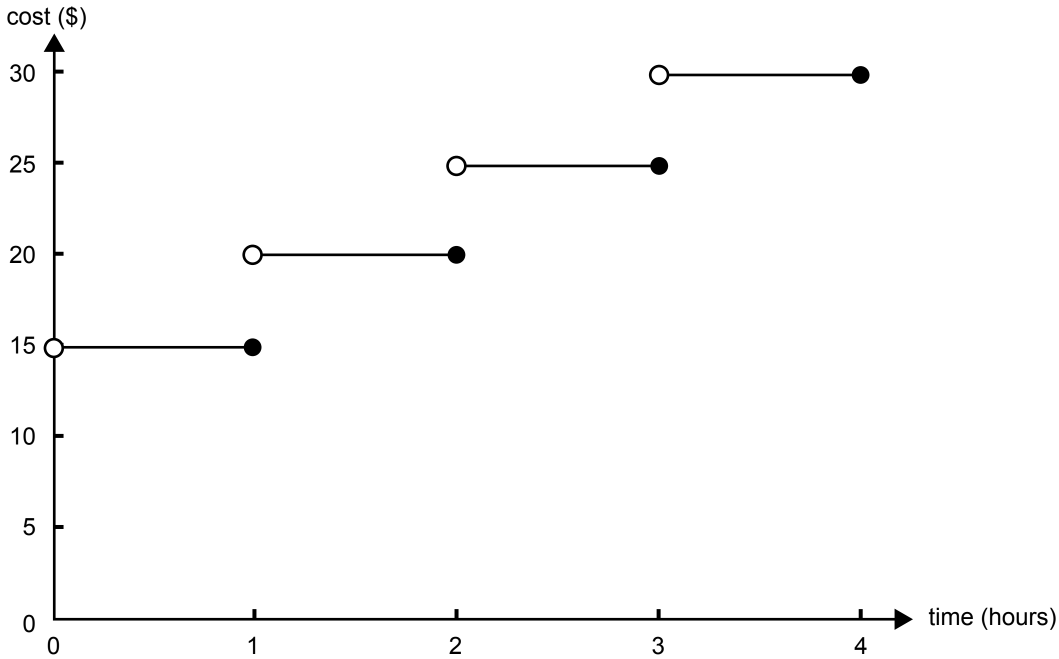
B. 75

C. 90

D. 120

E. 150

Question 5



The cost of hiring one motorbike for up to 4 hours is shown in the graph above.

Two motorbikes were hired.

The **total** charge for hiring the two motorbikes was $45.

The time for which each motorbike was hired could have been

A. 1 hour and 2 hours.

B. 1 hour and 3 hours.

C. 1.5 hours and 2 hours.

D. 1.5 hours and 3 hours.

E. 2 hours and 3.5 hours.

Question 6

Russell is a wine producer. He makes both red and white wine.

Let x represent the number of bottles of red wine he makes and y represent the number of bottles of white wine he makes.

This year he plans to make at least twice as many bottles of red wine as white wine.

An inequality representing this situation is

A. $y \leq x + 2$

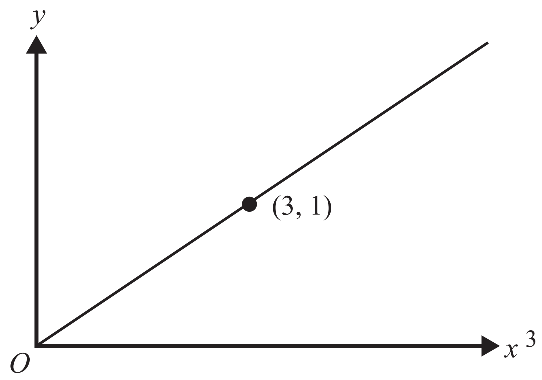
B. $y \leq 2x$

C. $y \geq 2x$

D. $x \leq 2y$

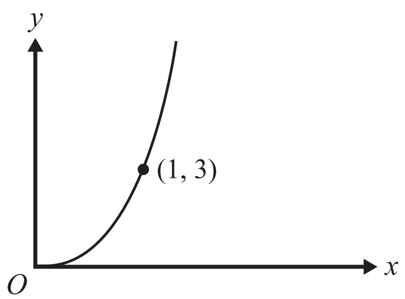
E. $x \geq 2y$

Question 7

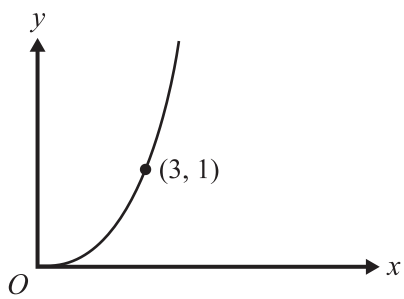


The graph above shows a relationship between y and x^3.
The graph that shows the same relationship between y and x is

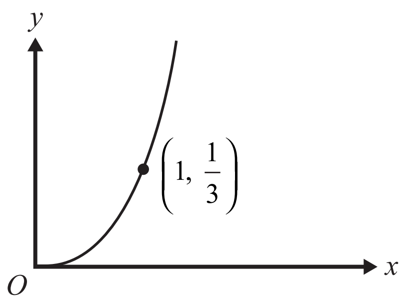
A.



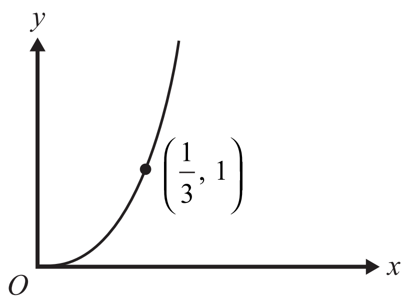
B.



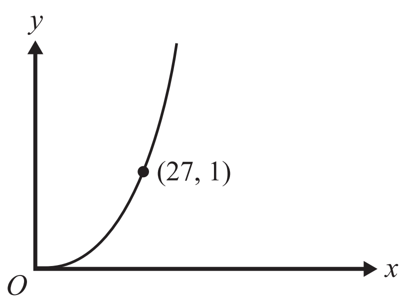
C.



D.



E.



Question 8

Which one of the following pairs of simultaneous linear equations has no solution?

A. $3x - y = 5$
$4x + y = 9$

B. $2x - y = 1$
$4x - 2y = 3$

C. $x + 3y = 0$
$2x - y = 7$

D. $x - 3y = 10$
$3x + 2y = 8$

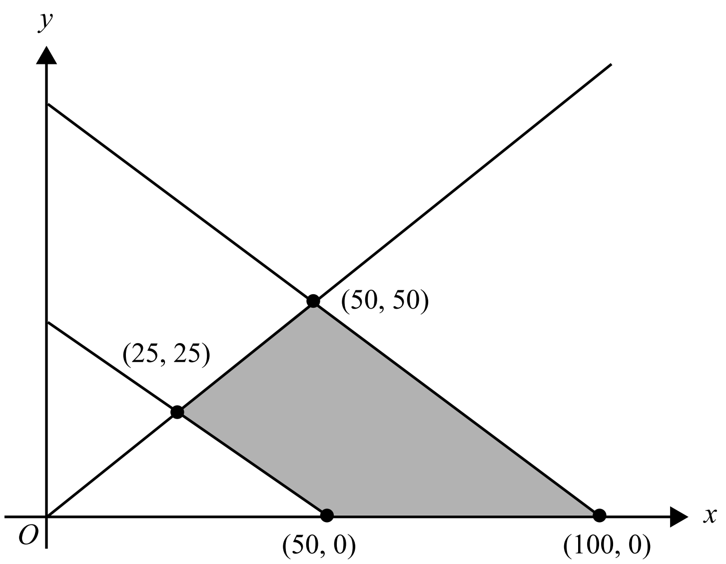
E. $4x + y = -6$
$2x - y = 0$

Question 9

The following five constraints apply to a linear programming problem.

$$x \geq 0, \quad y \geq 0, \quad x + y \geq 50, \quad x + y \leq 100, \quad y \leq x$$

In the diagram below, the shaded region (with boundaries included) represents the feasible region for this linear programming problem.



The aim is to maximise the objective function $Z = 2x + ky$.

If the maximum value of Z occurs **only** at the point (100, 0), then a possible value for k is

A. 1

B. 2

C. 3

D. 4

E. 5

Module 4: Business-related mathematics

Before answering these questions you must **shade** the Business-related mathematics box on the answer sheet for multiple-choice questions.

Question 1

An agent charged $20 commission for selling a rare book for $500.

What percentage of the selling price is the commission?

A. 4%

B. 5%

C. 20%

D. 25%

E. 40%

Question 2

A car is valued at $30 000 when new.

Its value is depreciated by 25 cents for each kilometre it travels.

The number of kilometres the car travels before its value depreciates to $8 000 is

A. 32 000

B. 55 000

C. 88 000

D. 120 000

E. 550 000

Question 3

A sum of money is invested in an account paying simple interest at a rate of 8% per annum.

The total interest earned on this investment over 6 years is $27 000.

The sum of money invested is

A. $12 960

B. $45 000

C. $56 250

D. $202 500

E. $337 500

Question 4

The price of a property purchased in 2006 was $200 000.

Stamp duty was paid on this purchase according to the schedule below.

Price range	**Rate**
$0 – $20 000	1.4 per cent of the price of the property
$20 001 – $115 000	$280 plus 2.4 per cent of the price in excess of $20 000
$115 001 – $868 000	$2 560 plus 6 per cent of the price in excess of $115 000
More than $868 000	5.5 per cent of the price

The amount of stamp duty paid was

A. $2560

B. $2800

C. $5100

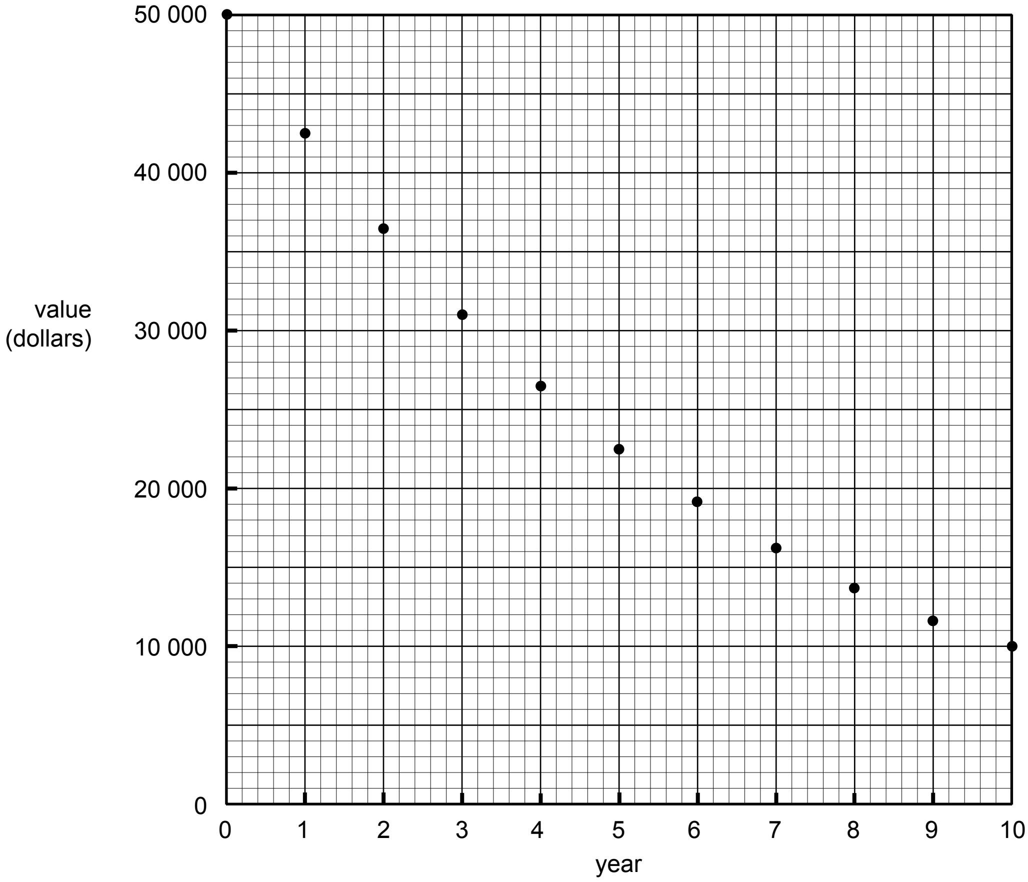
D. $7660

E. $9460

Question 5

A new kitchen in a restaurant cost \$50 000. Its value is depreciated over time using the reducing balance method.

The value of the kitchen in dollars at the end of each year for ten years is shown in the graph below.



Which one of the following statements is **true**?

A. The kitchen depreciates by \$4000 annually.

B. At the end of five years, the kitchen's value is less than \$20 000.

C. The reducing balance depreciation rate is less than 5% per annum.

D. The annual depreciation rate increases over time.

E. The amount of depreciation each year decreases over time.

Question 6

\$10 000 is invested at a rate of 10% per annum compounding half yearly.

The value, in dollars, of this investment after five years, is given by

A. $10\,000 \times 0.10 \times 5$

B. $10\,000 \times 0.05 \times 10$

C. $10\,000 \times 0.05^{10}$

D. $10\,000 \times 1.05^{10}$

E. $10\,000 \times 1.10^{5}$

Question 7

At the start of each year Joe's salary increases to take inflation into account.

Inflation averaged 2% per annum last year and 3% per annum the year before that.

Joe's salary this year is $42 000.

Joe's salary two years ago, correct to the nearest dollar, would have been

A. $39 900

B. $39 925

C. $39 926

D. $39 976

E. $39 977

Question 8

Brad investigated the cost of buying a $720 washing machine under a hire purchase agreement.

A deposit of $180 is required and the balance will be paid in 24 equal monthly repayments.

A flat interest rate of 12% per annum applies to the balance.

Brad correctly calculated the monthly repayment to be

A. $22.50

B. $25.20

C. $26.10

D. $27.90

E. $29.70

Question 9

Petra borrowed $250 000 to buy a home. The interest rate is 7% per annum, calculated monthly on the reducing balance over the life of the loan. She will fully repay the loan over 20 years with equal monthly instalments.

The total amount of interest she will pay on the loan is closest to

A. $215 000

B. $266 000

C. $281 000

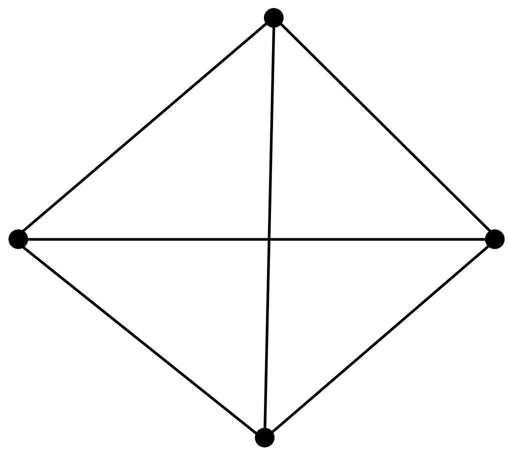
D. $350 000

E. $465 000

Module 5: Networks and decision mathematics

Before answering these questions you must **shade** the Networks and decision mathematics box on the answer sheet for multiple-choice questions.

Question 1



A mathematical term that could **not** be used to describe the graph shown above is

A. complete.

B. planar.

C. simple.

D. undirected.

E. tree.

Question 2

A connected planar graph has 12 edges.

This graph could have

A. 5 vertices and 6 faces.

B. 5 vertices and 8 faces.

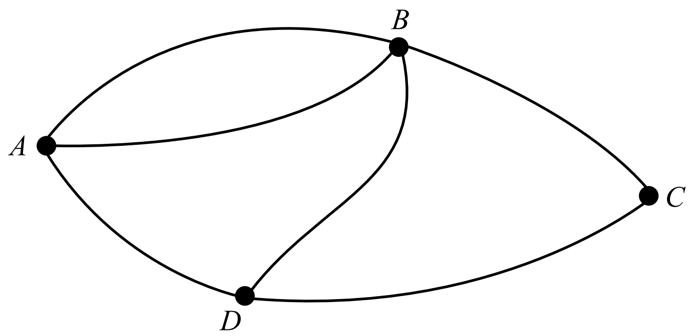
C. 6 vertices and 8 faces.

D. 6 vertices and 9 faces.

E. 7 vertices and 9 faces.

Question 3

Consider the following graph.



An adjacency matrix that could be used to represent this graph is

A. $\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

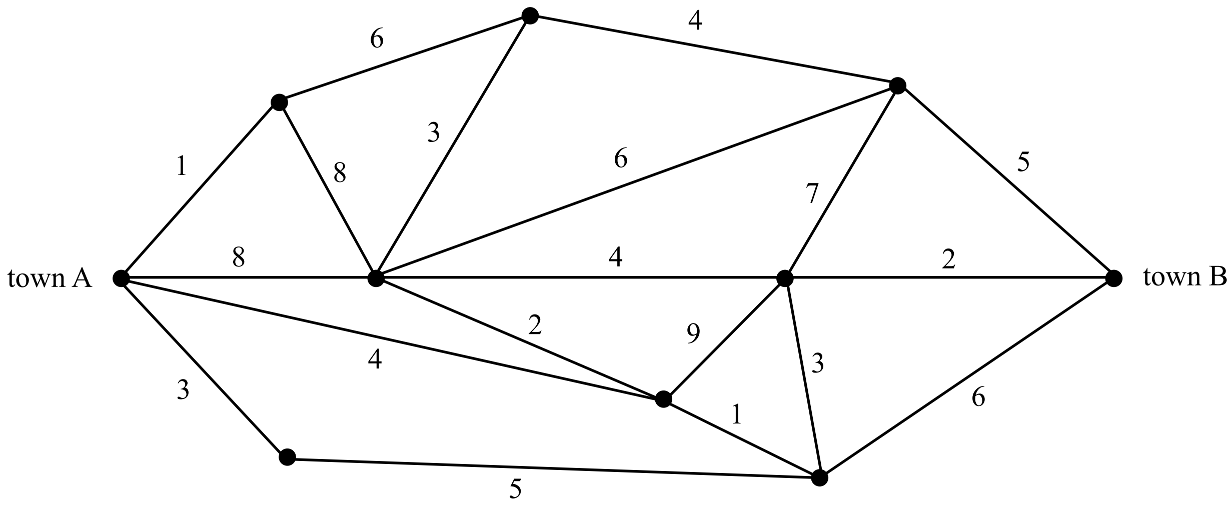
C. $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Question 4

The following network shows the distances, in kilometres, along a series of roads that connect town A to town B.



The shortest distance, in kilometres, to travel from town A to town B is

A. 9

B. 10

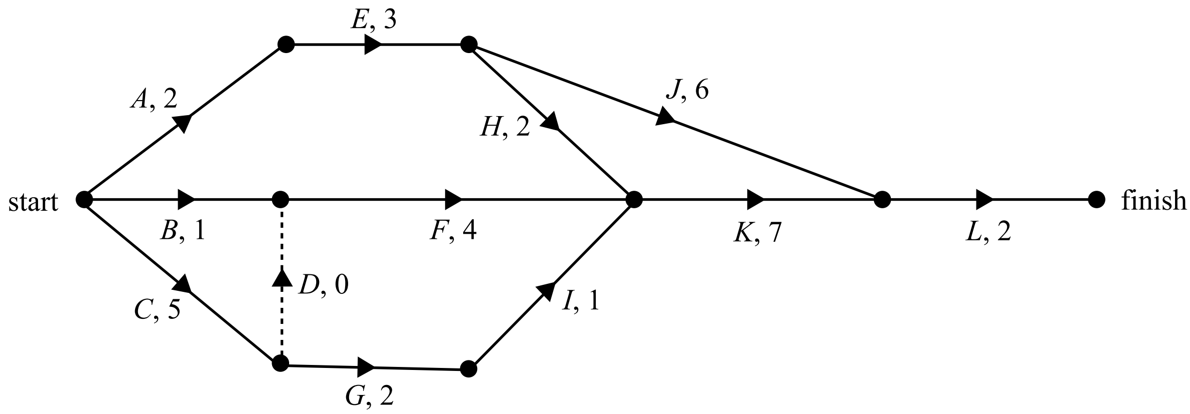
C. 11

D. 12

E. 13

Questions 5 and 6 relate to the following information.

The following network shows the activities that are needed to complete a project and their completion times (in hours).



Question 5

Which one of the following statements regarding this project is **false**?

A. Activities A, B and C all have the same earliest start time.

B. There is only one critical path for this project.

C. Activity J may start later than activity H.

D. The shortest path gives the minimum time for project completion.

E. Activity L must be on the critical path.

Question 6

The earliest start time for activity L, in hours, is

A. 11

B. 12

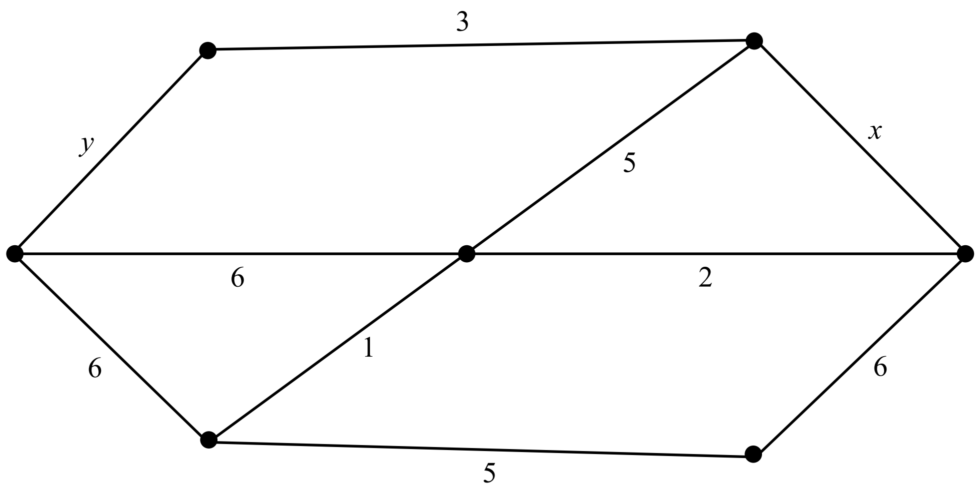
C. 14

D. 15

E. 16

Question 7

The minimal spanning tree for the network below includes two edges with weightings x and y.



The length of the minimal spanning tree is 19.

The values of x and y could be

A. $x = 1$ and $y = 7$

B. $x = 2$ and $y = 5$

C. $x = 3$ and $y = 5$

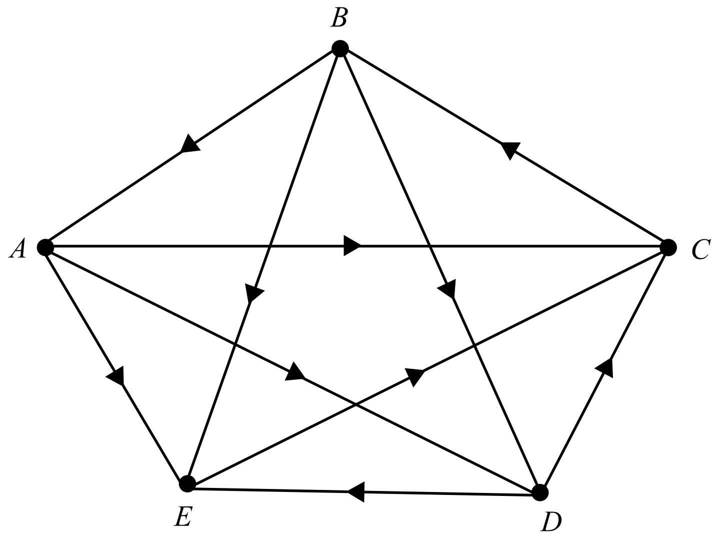
D. $x = 4$ and $y = 5$

E. $x = 5$ and $y = 6$

Questions 8 and 9 relate to the following information.

There are five teams, A, B, C, D and E, in a volleyball competition. Each team played each other team once in 2007.

The results are summarised in the directed graph below. An arrow from A to E signifies that A defeated E.



Question 8

In 2007, the team that had the highest number of two-step dominances was

A. team A

B. team B

C. team C

D. team D

E. team E

Question 9

In 2008, two new teams, F and G, will join the competition.

As in 2007, each team will play every other team once.

Compared to 2007, the number of **extra** games that will be played in 2008 will be

A. 10

B. 11

C. 12

D. 21

E. 42

Module 6: Matrices

Before answering these questions you must **shade** the Matrices box on the answer sheet for multiple-choice questions.

Question 1

The matrix sum $\begin{bmatrix} 0 & -4 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 5 & 4 \\ -2 & 2 \end{bmatrix}$ is equal to

A. $\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$

B. $\begin{bmatrix} 0 & 0 \\ 0 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 5 & -4 \\ 0 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 5 & -4 & 4 \\ 2 & -2 & 5 & 2 \end{bmatrix}$

E. $\begin{bmatrix} 0 & -4 & 5 & 4 \\ 2 & 5 & -2 & 2 \end{bmatrix}$

Question 2

The number of tourists visiting three towns, Oldtown, Newtown and Twixtown, was recorded for three years. The data is summarised in the table below.

	2004	**2005**	**2006**
Oldtown	975	1002	1390
Newtown	2105	1081	1228
Twixtown	610	1095	1380

The 3 × 1 matrix that could be used to show the number of tourists visiting the three towns in the year **2005** is

A. $\begin{bmatrix} 975 & 1002 & 1390 \end{bmatrix}$

B. $\begin{bmatrix} 1002 & 1081 & 1095 \end{bmatrix}$

C. $\begin{bmatrix} 975 \\ 1002 \\ 1390 \end{bmatrix}$

D. $\begin{bmatrix} 1002 \\ 1081 \\ 1095 \end{bmatrix}$

E. $\begin{bmatrix} 975 & 1002 & 1390 \\ 2105 & 1081 & 1228 \\ 610 & 1095 & 1380 \end{bmatrix}$

Question 3

If $A = \begin{bmatrix} 8 & 4 \\ 5 & 3 \end{bmatrix}$ and the product $AX = \begin{bmatrix} 5 & 6 \\ 8 & 10 \end{bmatrix}$ then X is

A. $\begin{bmatrix} 24 & -14 \\ 13 & -7.5 \end{bmatrix}$

B. $\begin{bmatrix} -4.25 & -5.5 \\ 9.75 & 12.5 \end{bmatrix}$

C. $\begin{bmatrix} -3.75 & 7 \\ -6.5 & 12 \end{bmatrix}$

D. $\begin{bmatrix} 25 & 11 \\ -19.5 & -8.5 \end{bmatrix}$

E. $\begin{bmatrix} 0.625 & 1.5 \\ 1.6 & 3.333 \end{bmatrix}$

Question 4

Consider the following system of three simultaneous linear equations.

$$2x + z = 5$$
$$x - 2y = 0$$
$$y - z = -1$$

This system of equations can be written in matrix form as

A. $\begin{bmatrix} 2 & 1 \\ 1 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 2 & 1 & 5 \\ 1 & -2 & 0 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

D. $\begin{bmatrix} 2 & 1 & 0 \\ 1 & -2 & 0 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 2 & 1 \\ 1 & -2 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Question 5

An international mathematics competition is conducted in three sections – Junior, Intermediate and Senior. There are money prizes for gold, silver and bronze levels of achievement in each of these sections.
Table 1 shows the number of students who were awarded prizes in each section.

Table 1 – Number of students awarded prizes

Achievement level	Junior	Intermediate	Senior
Gold	21	12	10
Silver	16	18	14
Bronze	21	26	24

Table 2 shows the value, in dollars, of each prize.

Table 2 – Value of prizes ($)

Achievement level	Junior	Intermediate	Senior
Gold	75	100	200
Silver	25	40	80
Bronze	10	15	20

A matrix product that gives the **total** value of all the **Silver** prizes that were awarded is

A. $\begin{bmatrix} 25 & 40 & 80 \end{bmatrix} \begin{bmatrix} 16 \\ 18 \\ 14 \end{bmatrix}$

B. $\begin{bmatrix} 25 & 40 & 80 \end{bmatrix} \begin{bmatrix} 16 & 18 & 14 \end{bmatrix}$

C. $\begin{bmatrix} 16 \\ 18 \\ 14 \end{bmatrix} \begin{bmatrix} 25 \\ 40 \\ 80 \end{bmatrix}$

D. $\begin{bmatrix} 16 \\ 18 \\ 14 \end{bmatrix} \begin{bmatrix} 25 & 40 & 80 \end{bmatrix}$

E. $\begin{bmatrix} 100 & 40 & 15 \end{bmatrix} \begin{bmatrix} 16 \\ 18 \\ 14 \end{bmatrix}$

Question 6

A colony of fruit bats feeds nightly at three different locations, *A*, *B* and *C*.

Initially, the number of bats from the colony feeding at each of the locations was as follows.

Location	Number of bats
A	1568
B	1105
C	894

The bats change feeding locations according to the following transition matrix *T*.

$$\begin{array}{cc} & \text{this night} \\ & \begin{array}{ccc} A & B & C \end{array} \\ T = & \begin{bmatrix} 0.8 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.1 \\ 0.1 & 0.3 & 0.7 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \end{array} \text{ next night}$$

If this pattern of feeding continues, the number of bats feeding at location *A* in the long term will be closest to

A. 1254

B. 1543

C. 1568

D. 1605

E. 1725

Question 7

Each year, a family always goes on its holiday to one of three places; Portland (P), Quambatook (Q) or Rochester (R).

They never go to the same place two years in a row. For example, if they went to Portland one year, they would not go to Portland the next year; they would go to Quambatook or Rochester instead.

A transition matrix that can be used to model this situation is

A.

$$\text{Next year} \quad \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \text{This year} \\ \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

B.

$$\text{Next year} \quad \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \text{This year} \\ \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0.9 & 0 & 0 \\ 0.1 & 0 & 1 \end{bmatrix} \end{array}$$

C.

$$\text{Next year} \quad \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \text{This year} \\ \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0 & 0.3 & 0.8 \\ 0.3 & 0.4 & 0.2 \\ 0.7 & 0.3 & 0 \end{bmatrix} \end{array}$$

D.

$$\text{Next year} \quad \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \text{This year} \\ \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0 & 0.2 & 0 \\ 0.3 & 0 & 0.8 \\ 0.5 & 0.6 & 0 \end{bmatrix} \end{array}$$

E.

$$\text{Next year} \quad \begin{array}{c} \\ P \\ Q \\ R \end{array} \begin{array}{c} \text{This year} \\ \begin{array}{ccc} P & Q & R \end{array} \\ \begin{bmatrix} 0 & 0 & 0.1 \\ 0.5 & 0 & 0.9 \\ 0.5 & 1 & 0 \end{bmatrix} \end{array}$$

Question 8

Kerry sat for a multiple-choice test consisting of six questions.

Each question had four alternative answers, *A*, *B*, *C* or *D*.

He selected *D* for his answer to the first question.

He then determined the answers to the remaining questions by following the transition matrix

$$\text{Next question} \quad \begin{array}{c c} & \text{This question} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

The answers that he gave to the six test questions, starting with *D*, were

A.

Question	1	2	3	4	5	6
Answer	*D*	*B*	*C*	*A*	*D*	*B*

B.

Question	1	2	3	4	5	6
Answer	*D*	*B*	*C*	*A*	*A*	*A*

C.

Question	1	2	3	4	5	6
Answer	*D*	*B*	*C*	*A*	*C*	*A*

D.

Question	1	2	3	4	5	6
Answer	*D*	*A*	*C*	*B*	*D*	*D*

E.

Question	1	2	3	4	5	6
Answer	*D*	*C*	*B*	*A*	*B*	*C*

Question 9

Matrix M is a 3×4 matrix.

Matrix P has five rows.

N is another matrix.

If the matrix product

$$M(NP) = \begin{bmatrix} 4 & 1 & 7 & 2 \\ 0 & 9 & 7 & 4 \\ 4 & 3 & 3 & 1 \end{bmatrix},$$

then the order of matrix N is

A. 3×5

B. 5×3

C. 4×5

D. 5×4

E. 5×5

END OF MULTIPLE-CHOICE QUESTION BOOK

FURTHER MATHEMATICS

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time. This formula sheet is provided for your reference.

Further Mathematics Formulas

Core: Data analysis

standardised score: $z = \dfrac{x - \bar{x}}{s_x}$

least squares line: $y = a + bx$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$

residual value: residual value = actual value – predicted value

seasonal index: seasonal index = $\dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Module 1: Number patterns

arithmetic series: $a + (a + d) + \ldots + (a + (n - 1)d) = \dfrac{n}{2}\left[2a + (n-1)d\right] = \dfrac{n}{2}(a + l)$

geometric series: $a + ar + ar^2 + \ldots + ar^{n-1} = \dfrac{a(1 - r^n)}{1 - r}, r \neq 1$

infinite geometric series: $a + ar + ar^2 + ar^3 + \ldots = \dfrac{a}{1 - r}, |r| < 1$

Module 2: Geometry and trigonometry

area of a triangle: $\dfrac{1}{2}bc\sin A$

Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{1}{2}(a + b + c)$

circumference of a circle: $2\pi r$

area of a circle: πr^2

volume of a sphere: $\dfrac{4}{3}\pi r^3$

surface area of a sphere: $4\pi r^2$

volume of a cone: $\dfrac{1}{3}\pi r^2 h$

volume of a cylinder: $\pi r^2 h$

volume of a prism: area of base × height

volume of a pyramid: $\dfrac{1}{3}$ area of base × height

Pythagoras' theorem: $c^2 = a^2 + b^2$

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $c^2 = a^2 + b^2 - 2ab \cos C$

Module 3: Graphs and relations

Straight line graphs

gradient (slope): $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

equation: $y = mx + c$

Module 4: Business-related mathematics

simple interest: $I = \dfrac{PrT}{100}$

compound interest: $A = PR^n$ where $R = 1 + \dfrac{r}{100}$

hire purchase: effective rate of interest $\approx \dfrac{2n}{n+1} \times$ flat rate

Module 5: Networks and decision mathematics

Euler's formula: $v + f = e + 2$

Module 6: Matrices

determinant of a 2 × 2 matrix: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

inverse of a 2 × 2 matrix: $A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ where $\det A \neq 0$

END OF FORMULA SHEET