Mathematics

1. Consider 5 independent Bernulli's trails each with probability of sucess p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(1) $\left(\frac{11}{12}, 1\right]$ **(2)** $\left(\frac{1}{2}, \frac{3}{4}\right]$

(3) $\left(\frac{3}{4}, \frac{11}{12}\right]$ **(4)** $\left(0, \frac{1}{2}\right]$

Ans: **[4]** P (at least one failure) =1 - P (all success)

$$1 \geq 1 - p^5 \geq \frac{31}{32}$$

$$\Rightarrow 0 \leq p^5 \leq 1 - \frac{31}{32} \leq \frac{1}{32}$$

$$\Rightarrow 0 \leq p^5 \leq \left(\frac{1}{2}\right)^5$$

$$\Rightarrow p \in \left[0, \frac{1}{2}\right]$$

2. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is

(1) 132 **(2)** 144

(3) – 132 **(4)** – 144

Ans: **[4]** Coeff of x^7 in the expansion of $\left((1-x)(1-x^2)\right)^6$

Coeff of x^7 in the expansion of $(1-x)^6(1-x^2)^6$

Coeff of x^7 in the expansion of

$$\left({}^6C_0 - {}^6C_1x + {}^6C_2x^2 - ... + {}^6C_6x^6\right).\left({}^6C_0 - {}^6C_1x^2 + {}^6C_2x^4 - ... + {}^6C_6x^{12}\right)$$

$$= \left(-{}^6C_0\right)\cdot\left(-{}^6C_5\right) + {}^6C_2\left(-{}^6C_3\right) + \left(-{}^6C_3\right)\left(-{}^6C_1\right)$$

$$= 36 + (-300) + 120$$

$$= -144$$

3. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(1) equals $\frac{1}{\sqrt{2}}$ **(2)** does not exist

(3) equals $\sqrt{2}$ **(4)** equals $-\sqrt{2}$

Ans: **[2]** $\lim_{x\to 2}\frac{\sqrt{1-\cos 2(x-2)}}{x-2}$

$$= \lim_{x\to 2}\frac{\sqrt{2}\left|\sin(x-2)\right|}{x-2}$$

$$\text{LHL} = = \lim_{x\to 2^-}\frac{\sqrt{2}\left|\sin(x-2)\right|}{x-2} = \lim_{x\to 2^-}\frac{-\sqrt{2}\sin(x-2)}{x-2}$$

$$= -\sqrt{2}$$

$$\text{RHL} = \lim_{x\to 2^+}\frac{\sqrt{2}\left|\sin(x-2)\right|}{x-2} = \lim_{x\to 2^+}\frac{\sqrt{2}\sin(x-2)}{x-2} = \sqrt{2}$$

$\text{LHL} \neq \text{RHL}$

The limit does not exist

4. Let R be the set of real numbers

Statement -1

$A = \{(x, y) \in R \times R : y - x \text{ is an integer}\}$ is an equivalence relation on R.

Statement -2

$B = \{(x, y) \in R \times R : x - \alpha y$ for some rational number α } is an equivalence relation on *R.*

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: **[4]** Statement-1 $(x, y) \in A \Rightarrow (x - x)$ integer

$x - x = 0\ \forall x \Rightarrow$ A is reflexive

$(y - x)$ integer $\Rightarrow$ $(x - y)$ integer

$\therefore$ A is symmetric

$(y - x)$ integer and $(z - y)$ integer

$\therefore$ $(y - x) + (z - y) = (z - x)$ integer

Hence $xAz \Rightarrow$ A in transitive

$\therefore$ A is equivalance

Statement-2 $(x, y) \in B \Rightarrow$ $x = \alpha y$, for α, some rational

$(0, 1) \in B$ as $0 = 0.1, 0$ is a rational no.

but $(1, 0) \notin B$ as $1 = \alpha . 0$ there is no rational α existing.

Hence B is not symmetric

5. Let α, β be real and z be complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re $z = 1$, then it is necessary that

(1) $\beta \in (1, \infty)$ **(2)** $\beta \in (0, 1)$

(3) $\beta \in (-1, 0)$ **(4)** $|\beta| = 1$

Ans: **[1]** The coefficients of quadratic equation

$$z^2 + \alpha z + \beta = 0$$

are real. Therefore complex roots exists in conjugate pair. Let

$$z_1 = 1 + iy,\ z_2 = 1 - iy$$

$$z_1 + z_2 = -\alpha \Rightarrow \alpha = -2$$

$$\beta = z_1 z_2 = 1 + y^2 \in (1, \infty)$$

[For $\beta = 1$, roots are equal]

6. $\dfrac{d^2x}{dy^2}$ equal

(1) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$ **(2)** $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

(3) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ **(4)** $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

Ans: **[1]** $\dfrac{d^2x}{dy^2} = \dfrac{d}{dy}\left(\dfrac{dx}{dy}\right) = \dfrac{d}{dy}\left(\dfrac{d}{dy/dx}\right)$

$$= \frac{d}{dy}\left(\frac{d}{dy/dx}\right)\frac{dx}{dy}$$

$$= -\frac{1}{\left(\frac{dy}{dx}\right)^2} \cdot \frac{d^2y}{dx^2} \cdot \frac{dx}{dy}$$

$$\frac{d^2x}{dy^2} = -\frac{\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^3}$$

7. The number of values of k for which the linear equations

$$4x + ky + 2z = 0$$

$$kx + 4y + z = 0$$

$$2x + 2y + z = 0$$

possess a non-zero solutoin is

(1) zero **(2)** 3

(3) 2 **(4)** 1

Ans: **[3]** For non-zero

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ $k^2 - 6k + 8 = 0$

$\Rightarrow$ $(k-2)(k-4) = 0 \Rightarrow k = 2, 4$

$\therefore$ Number of solution = 2

8. **Statement -1**

The point A (1, 0, 7) is the mirror image of the point B(1, 6, 3) in the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$

Statement -2

The line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ bisects the line segment joining A (1, 0, 7) and B (1, 6, 3)

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: **[3]** mid pt of AB $\equiv$ $(2, 3, 5)$ lies on the line

dr of AB $\equiv$ $(0, 6, -4)$

dr of given $\equiv$ $(1, 2, 3)$

$0(1) + 6(2) + (-4)(3) = 0$

$\therefore$ AB is perpendicular to the given line

$\therefore$ B is mirror image of A

Statement (1) is true .

Statmeent (2) is also true But not sufficient.

9. Consider the following statements

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the statment "Suman is brilliant and dishonest if and only if sumna is rich" can be expressed as

(1) $\sim (P \wedge \sim R) \leftrightarrow Q$ **(2)** $\sim P \wedge (Q \leftrightarrow \sim R)$

(3) $\sim (Q \leftrightarrow (P \wedge \sim R))$ **(4)** $\sim Q \leftrightarrow \sim P \wedge R$

Ans: **[3]** Given Statement is equivalent to

$$(P \wedge \sim R) \leftrightarrow Q$$

Negation is $\sim \left[(P \wedge \sim R) \leftrightarrow Q \right]$

10. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectivley. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement -1

The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

Statement -2

In any triangle, bisector an angle divides the trianle into two similar triangles.

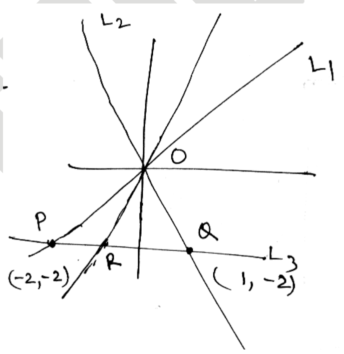
(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: **[4]**



$$\frac{PR}{PQ} = \frac{OP}{OQ}$$

$$= \frac{2\sqrt{2}}{\sqrt{5}}$$

Statement (1) is ture

Statment (2) is False

11. A man saves Rs 200 in each of the first three months of his services. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service wil be Rs. 11040 after

(1) 21 months **(2)** 18 months

(3) 19 months **(4)** 20 months

Ans: **[1]** Let number of months = n

Total saving = 200 + 200 + $\left(200 + 240 + 280 + \ldots \text{ to } (n-2) \text{ terms}\right)$

$\Rightarrow 11040 = 400 + \frac{n-2}{2}\left(2(200) + (n-3)40\right)$

$\Rightarrow 10640 = (n-2)\left(200 + (n-3)20\right)$

$\Rightarrow 532 = (n-2)(10 + n - 3)$

$\Rightarrow = (n-2)(n+7)$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n-21)(n+26) = 0$

$\Rightarrow n = 21 \quad (\because n > 0)$

12. Equations of the ellipse whose axes are the axes of coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{2/5}$ is

(1) $5x^2 + 3y^2 - 32 = 0$ **(2)** $3x^2 + 5y^2 - 32 = 0$

(3) $5x^2 + 3y^2 - 48 = 0$ **(4)** $3x^2 + 5y^2 - 15 = 0$

Ans: **[2]** $e^2 = 1 - \frac{b^2}{a^2}$

$\Rightarrow \frac{b^2}{a^2} = 1 - e^2 = 1 - \frac{2}{5} = \frac{3}{5}$

$\Rightarrow a^2 : b^2 = 5 : 3$

Equation of ellipse

$$\frac{x^2}{5} + \frac{y^2}{3} = k \quad \ldots (1)$$

(1) passes through $(-3, 1)$

$\therefore k = \frac{(-3)^2}{5} + \frac{(1)^2}{3} = \frac{9}{5} + \frac{1}{3} = \frac{32}{15}$

Equation of ellipse

$$\frac{x^2}{5} + \frac{y^2}{3} = \frac{32}{15}$$

$\Rightarrow 3x^2 + 5y^2 = 32$

13. If $A=\sin^2 x+\cos^4 x$ then for all real x.

(1) $\frac{3}{4}\le A\le\frac{13}{16}$ **(2)** $\frac{3}{4}\le A\le 1$

(3) $\frac{13}{16}\le A\le 1$ **(4)** $1\le A\le 2$

Ans: **[2]** $A=\sin^2 x+\cos^4 x$

$=\cos^4 x-\cos^2 x+1$

$=\left(\cos^2 x-\frac{1}{2}\right)^2+\frac{3}{2}$

Now $0\le\left(\cos^2 x-\frac{1}{2}\right)^2\le\frac{1}{4}$

$\Rightarrow \quad \frac{3}{4}\le A\le 1$

14. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}\,dx$ is

(1) $\log 2$ **(2)** $\pi\log 2$

(3) $\frac{\pi}{8}\log 2$ **(4)** $\frac{\pi}{2}\log 2$

Ans: **[2]** $I=\int_0^1 \frac{8\log(1+x)}{1+x^2}\cdot dx$

$x=\tan\theta$

$dx=\sec^2\theta$

$=\int_0^{\pi/4}\frac{8\log(1+\tan\theta)}{1+\tan^2\theta}.\sec^2\theta\,dx$

also $I=\int_0^{\pi/4} 8\log\left[1+\tan\theta\left(\frac{\pi}{4}-\theta\right)\right].d\theta$

$=\int_0^{\pi/4} 8\log\left[1+\frac{(1-\tan\theta)}{1+\tan\theta}\right]\cdot d\theta$

$=\int_0^{\pi/4} 8\log\left(\frac{2}{1+\tan\theta}\right)\cdot d\theta \quad \ldots (2)$

using equaiton (1) and (2)

$2I=\int_0^{\pi/4} 8\left[\log(1+\tan\theta)+\log\left(\frac{2}{1+\tan\theta}\right)\right]\cdot d\theta$

$=\int_0^{\pi/4} 8\log 2\cdot d\theta=8\log 2\left(\frac{\pi}{4}\right)$

$I=\pi\log 2$

15. If the angle between the line $x=\frac{y-1}{2}=\frac{z-3}{\lambda}$ and the plane $x+2y+3z=4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$

(1) $5/3$ **(2)** $2/3$

(3) $3/2$ **(4)** $2/5$

Ans: **[2]** Let angle between line and plane is θ then

dr's of line : $(1, 2, \lambda)$

dr's of normal plane $(1, 2, 3)$

so, $\cos(90-\theta)=\frac{1\cdot 1+2.2+\lambda\cdot 3}{\sqrt{\lambda^2+4+1}\cdot\sqrt{9+4+1}}$

$\sin\theta=\frac{3\lambda+5}{\sqrt{14}\sqrt{\lambda^2+5}}$

given $\cos\theta=\sqrt{\frac{5}{14}}\Rightarrow\sin\theta=\frac{3}{\sqrt{14}}$

Hence $\frac{3\lambda+5}{\sqrt{14}\sqrt{\lambda^2+5}}=\frac{3}{\sqrt{14}}$

$\Rightarrow \quad 9(\lambda^2+5)=(3\lambda+5)^2$

$\Rightarrow \quad 30\lambda=20$

$\Rightarrow \quad \lambda=\frac{2}{3}$

16. For $x\in\left(0,\frac{5\pi}{2}\right)$, define

$f(x)=\int_0^x\sqrt{t}\ \sin t\,dt$

Then f has

(1) local maximum at π and local minimum at 2π

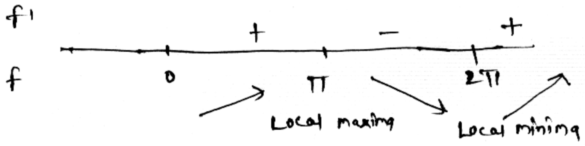
(2) lcoal maximum at π and 2π

(3) lcoal minimum at π and 2π

(4) local minimum at π and local maximum at 2π

Ans: **[1]** $f(x)=\int_0^x\sqrt{t}\sin t\,dt$

applying newton leibnitz formula



$f'(x) = (\sqrt{x} \cdot \sin x) \cdot 1 - 0$

$\Rightarrow \quad f'(x) = \sqrt{x} \cdot \sin x$

Hence $f(x)$ is maximum at $x = \pi$

and local minima at $x = 2\pi$

17. The domain of the function

(1) $f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is **(2)** $(-\infty, \infty) - \{0\}$

(3) $(0, \infty)$ **(4)** $(-\infty, 0)$

Ans: **[4]** $f(x) = \dfrac{1}{\sqrt{|x| - x}}$

Now $|x| - x > 0$

case 1 $\rightarrow$ $x > 0$ $\quad x - x > 0$

$0 > 0 \Rightarrow x = \phi$

case 2 $x < 0$

$-x - x > 0$

$\Rightarrow \quad -2x > 0$

$x < 0 \Rightarrow x \in R$

Hence $x \in (-\infty, 0)$

18. If the mean deviation about the median of the numbers a, 2a, . . ., 50 a is 50, then $|a|$ equals

(1) 5 **(2)** 2

(3) 3 **(4)** 4

Ans: **[4]** $x_i = a, 2a \ \ldots \ 25a, 26a, \ldots$ $50a$

Median $= \dfrac{25a + 26a}{2} = 25.5a$

Now $(x_i - \bar{x}) = 24.5|a|, 23.5|a| \ \ldots \ 0.5|a|, 0.5|a| \ \ldots$

$23.5|a|, 24.5|a|$

$\sum |x_i - \bar{x}| = 2(0.5|a| + |a| + 1.5|a| + \ldots + 24.5|a|)$

$= 2 \times \dfrac{25}{2}\left[0.5|a| + 24.5|a|\right]$

$= 625|a|$

Mean deviation $= \dfrac{\sum |x_i - \bar{x}|}{n}$

$= \dfrac{625|a|}{50} = 50$

$\Rightarrow \quad |a| = 4$

19. If $\vec{a} = \dfrac{1}{\sqrt{10}}\left(3\hat{i} + \hat{k}\right)$ and $\vec{b} = \dfrac{1}{7}\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$, then the value of $\left(2\vec{a} - \vec{b}\right) \cdot \left[\left(\vec{a} \times \vec{b}\right)\left(\vec{a} + 2\vec{b}\right)\right]$ is

(1) 3 **(2)** -5

(3) -3 **(4)** 5

Ans: **[2]** $|\vec{a}| = 1, |\vec{b}| = 1, \vec{a}.\vec{b} = 0$

$(2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})\right]$

$= (2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \times \vec{b}) \times \vec{a} + 2(\vec{a} \times \vec{b}) \times \vec{b}\right]$

$= (2\vec{a} - \vec{b}) \cdot \left[(\vec{a} \cdot \vec{a})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{a} + 2\left\{(\vec{a} \cdot \vec{b})\,\vec{b} - (\vec{b} \cdot \vec{b})\vec{a}\right\}\right]$

$= (2\vec{a} - \vec{b}) \cdot (\vec{b} - 2\vec{a})$

$= -4|\vec{a}|^2 - 4|\vec{b}|^2$

$= -4 - 1 = -5$

20. The value of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x} & , x < 0 \\ q & , x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}} & , x > 0 \end{cases}$$

is continuous for all x in R, are,

(1) $p = \dfrac{1}{2}, q = \dfrac{3}{2}$ **(2)** $p = \dfrac{1}{2}, q = -\dfrac{3}{2}$

(3) $p = \dfrac{5}{2}, q = \dfrac{3}{2}$ **(4)** $p = -\dfrac{3}{2}, q = \dfrac{1}{2}$

Ans: [4] $q = \lim_{x \to 0^+} f(x)$

$$= \lim_{x \to 0^+} \frac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}$$

$$= \lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$$= \lim_{x \to 0} \frac{(1+x) - 1}{x} \cdot \frac{1}{\sqrt{1+x} + 1}$$

$$= \frac{1}{2}$$

$$q = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(p+1)x + \sin x}{x}$$

$$= \lim_{x \to 0^-} \left[\frac{\sin(p+1)x}{(p+1)x} \cdot (p+1) + \frac{\sin x}{x} \right]$$

$$= (p+1) + 1 = p + 2$$

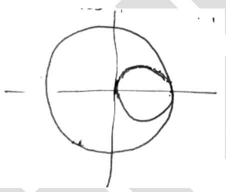
$$\Rightarrow \quad p = q - 2 = \frac{1}{2} - 2 = -\frac{3}{2}$$

21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if

(1) $|a| = 2c$ **(2)** $2|a| = c$

(3) $|a| = c$ **(4)** $a = 2c$

Ans: [3]



Diameter of $x^2 + y^2 = ax$ = Radius of $x^2 + y^2 = c^2$

22. Let be the purchase value of an equaipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential euqation $\frac{dV(t)}{dt} = -k(T - t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is

(1) e^{-kT} **(2)** $T^2 - \frac{1}{k}$

(3) $I - \frac{kT^2}{2}$ **(4)** $I - \frac{k(T-t)^2}{2}$

Ans: [3] $\frac{dV}{dt} = -kT + kt$

$$\int_I^{V(T)} dV = \int_0^T (-kT + kt) dt$$

$$\Rightarrow \quad V(T) - I = \left(-kTt + k\frac{t^2}{2} \right) \Big|_0^T$$

$$= -kT^2 + k\frac{T^2}{2} = -\frac{kT^2}{2}$$

$$\therefore \quad V(T) = I - \frac{kT^2}{2}$$

23. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(1) $P(C|D) = \frac{P(D)}{P(C)}$ **(2)** $P(C|D) = P(C)$

(3) $P(C|D) \geq P(C)$ **(4)** $P(C|D) < P(C)$

Ans: [3] $P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)}$ $(\because \ C \subset D)$

$\because \quad O \leq P(D) \leq 1$

$\Rightarrow \quad P\left(\frac{C}{D}\right) \geq P(C)$

24. Let A and B be two symmetric matrices of order 3

Statement -1

A (BA) and (AB) A are symmetric matrices

Statement -2

AB is symmetric matrix if matrix multiplication of A with B commutative.

(1) Statement -1 is false, Statement -2 is true

(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1

(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1

(4) Statement -1 is true, Statement-2 is false

Ans: [3] $A(BA) = (AB)A = ABA$

$$(ABA)^T = A^T B^T A^T = ABA$$

$\therefore$ statement 1 is TRUE

Given $AB = BA$

$$(AB)^T = B^T A^T = BA = AB$$

$\therefore$ AB is symmetric

statement 2 is TRUE but NOT a correct explanationof statement 1.

25. If $\omega(\neq 1)$ is a cube root of unity, and $(1+\omega)^7 = A + B\omega$. Then (A, B) equals

(1) (–1, 1) **(2)** (0, 1)
(3) (1, 1) **(4)** (1, 0)

Ans: [3] $A + B\omega = (1+\omega)^7 = (-\omega^2)^7$

$= -\omega^{14} = -\omega^2 = 1 + \omega$

$\Rightarrow A = 1,\ B = 1$

26 **Statement -1**
The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.
Statement -2
The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$

(1) Statement -1 is false, Statement -2 is true
(2) Statement -1 is true, Statement -2 is true; Statement -2 is a correct explanation for statement-1
(3) Statement-1 is true, Statement -2 is true, Statement-2 is not correct explanation for Statement-1
(4) Statement -1 is true, Statement-2 is false

Ans: [3] Statement 1 number of way's of distributing identical things into m person when each receive at least one

$= {}^{n-1}C_{m-1}$

hence total ways $= {}^{10-1}C_{4-1} = {}^9C_3$

Statement 2 is correct but not correct explanation of staement 1.

27 The shortest distance between line $y - x = 1$ and curve $x = y^2$ is

(1) $\frac{4}{\sqrt{3}}$ **(2)** $\frac{\sqrt{3}}{4}$

(3) $\frac{3\sqrt{2}}{8}$ **(4)** $\frac{8}{3\sqrt{2}}$

Ans: [3] Let a point $P(t^2, t)$ on the parabola. Distance of the line $y = x + 1$ from P

$$= \frac{|t^2 - t + 1|}{\sqrt{2}} = \frac{t^2 - t + 1}{\sqrt{2}} \qquad \because\ t^2 - t + 1 > 0$$

$$= \frac{\left(t - \frac{1}{2}\right)^2 + \frac{3}{4}}{\sqrt{2}}$$

$$\geq \frac{\frac{3}{4}}{\sqrt{2}} \qquad \text{(when } t = \frac{1}{2}\text{)}$$

$$= \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$$

28. The area of the region enclosed by trhe curves $y = x, x = e, y = (1/x)$ and the positive x-axis is

(1) 5/2 square units
(2) 1/2 square units
(3) 1 square units
(4) 3/21 square units

Ans: [4] Area $= \int_0^1 x\,dx + \int_1^e \frac{1}{x}\,dx$

$$= \left(\frac{x^2}{2}\right)_0^1 + (\ln x)_1^e = \frac{1}{2} + 1 = \frac{3}{2}$$

29. If $\frac{dy}{dx} = y + 3 > 0$ and $y(0) = 2$, the $y(\ln 2)$ is equal to:

(1) –2 **(2)** 7
(3) 5 **(4)** 13

Ans: [2] $\frac{dy}{dx} = y + 3 \qquad \Rightarrow \frac{dy}{y+3} = dx$

$\Rightarrow \ln(y+3) = x + C \qquad \Rightarrow y + 3 = C \cdot e^x$

Now $y(0) = 2 \qquad \Rightarrow 2 + 3 = C \qquad \Rightarrow C = 5$

$\therefore\ y = 5e^x - 3 \qquad \Rightarrow y(\ln 2) = 5e^{\ln 2} - 3 = 7$

30. The vectors $\vec{a}$ and $\vec{b}$ are no perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying : $\vec{b} \times \vec{c} = \vec{b} \times \vec{d}$ and $\vec{a} \cdot \vec{d} = 0$. Then the vector $\vec{d}$ is equal to:

(1) $\vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ **(2)** $\vec{b} - \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

(3) $\vec{c} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$ **(4)** $\vec{b} + \left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

Ans: [1] $\vec{b} \times \vec{c} = \vec{b} \times \vec{d} \qquad \vec{b} \times (\vec{c} - \vec{d}) = \vec{0}$

$\Rightarrow \vec{c} - \vec{d} = \lambda\vec{b} \qquad \Rightarrow \vec{d} = \vec{c} - \lambda\vec{b}$

Now $\vec{a} \cdot \vec{d} = 0 \qquad \Rightarrow \vec{a}\cdot\vec{c} - \lambda(\vec{a}\cdot\vec{b}) = 0$

$\Rightarrow \lambda = \frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}} \qquad \therefore\ \vec{d} = \vec{c} - \left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$

Part -B Chemistry

31. In context of the lanthanoids, which of the following statements is not correct?

(1) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series.

(2) There is a gardual decrease in the radii of the members with increasing atomic number in the series.

(3) All the members exhibit +3 oxidation state.

(4) Because of similar properties the separation of lanthanoids is not easy

Sol. **[1]** +3 oxidation state is predominant in lanthanoids.

32. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing form one of the face centred points, the formula of the compound is

(1) A_2B_5 **(2)** A_2B

(3) AB_2 **(4)** A_2B_3

Sol. **[1]** A : B

1 : $5 \times \frac{1}{2} = \frac{5}{2}$

2 : 5 i.e., A_2B_5

33. The magnetic moment (spin only) of $[NiCl_4]^{4-}$ is

(1) 1.41 BM **(2)** 1.82 BM

(3) 5.46 BM **(4)** 2.82 BM

Sol. **[4]** Outer orbital complex, 2 unpaired electrons

$\mu_{spin} = \sqrt{2(2+2)} = \sqrt{8} = 2.82$ B.M.

34. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is wrong?

(1) The complex gives which precipitate with silver nitrate solution.

(2) The complex involves d^2sp^3 hybridisation and is octahedral in shape.

(3) The complex is paramagnetic

(4) The complex is an outer orbital complex.

Sol. **[4]** It is an inner orbital complex.

35. The rate of a chemical reaction doubles for every $10°C$ rise of temperature. If the temperature is sraised by $50°C$ the rate of the reaction increases by aobut:

(1) 64 times **(2)** 10 times

(3) 24 times **(4)** 32 times

Sol. **[4]** Temperature increased by $50°C$, so change in rate constant will be $(2)^5 \Rightarrow 32$

36. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

(1) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

(2) a and b for Cl_2 > a and b for C_2H_6

(3) a and b for Cl_2 < a and b for C_2H_6

(4) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

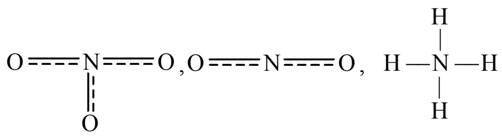
Sol. **[2]** For easy to liquify value of a and b both should be greater.

37. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively:

(1) sp^2, sp^3, sp **(2)** sp, sp^2, sp^3

(3) sp^2, sp, sp^3 **(4)** sp, sp^3, sp^2

Sol. **[3]**



38. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it form freezing at $-6°C$ will be: (K_f foer water $= 1.86$ K kg mol^{-1} and molar mass of ethylene glycol $= 62$ g mol^{-1})

(1) 304.60 g **(2)** 804.32 g

(3) 204.30 g **(4)** 400.00 g

Sol. **[2]** $\Delta T_f = 6 \Rightarrow \Delta T_f = k_f m$

$6 = 1.86 \times \frac{W}{62 \times 4} \Rightarrow w = \frac{6 \times 62 \times 4}{1.86} = 800$

Any mass which is greater than 800 g will not allow solution to freeze at $-6°C$.

39. The outer electron configuration of Gd (Atomic No : 64) is

(1) $4f^7\,5d^1\,6s^2$ **(2)** $4f^3\,5d^5\,6s^2$

(3) $4f^8\,5d^0\,6s^2$ **(4)** $4f^4\,5d^4\,6s^2$

Sol. **[1]** $[Xe]4f^7\,5d^1\,6s^2$

40. The structure of IF_7 is

(1) pentagonal bipyramid
(2) square pyramid
(3) trigonal bipyramid
(4) octahedral

Sol. **[1]** sp^3d^3 Hybridization

41. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of:

(1) an acetylenic triple bond
(2) two ethylenic double bonds
(3) a vinyl group
(4) an isopropyl group

Sol. **[3]** Since formaldehyde is formed on ozonolysis, presence of vinyl is indicated.

42. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (i) by the expression:

(1) $\alpha = \dfrac{x+y+1}{i-1}$ **(2)** $\alpha = \dfrac{i-1}{(x+y-1)}$

(3) $\alpha = \dfrac{i-1}{x+y-1}$ **(4)** $\alpha = \dfrac{x+y-1}{i-1}$

Sol. **[2]** For dissociation

$$A_x B_y \longrightarrow xA^{y+} + yB^{x-}$$

After dissociation $1-\alpha$ $\quad x\alpha$ $\quad y\alpha$

$$\text{Van't Hoff factor } (i) = \frac{\text{no. of molse after dissociation}}{\text{no. of molse before dissociation}}$$

$$i = \frac{1+(x+y-1)\alpha}{1} \Rightarrow i-1 = (x+y-1)\alpha$$

$$\Rightarrow \alpha = \frac{i-1}{x+y-1}$$

43. A gas absorbs a photon of 355 nm and emists at two wavelengths. If one of the emissions is at 680 nm, the other is at:

(1) 518 nm **(2)** 1035 nm
(3) 325 nm **(4)** 743 nm

Sol. **[4]** According to energy conservation

$$\frac{hc}{\lambda} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\Rightarrow \frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \qquad \Rightarrow \frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\Rightarrow \lambda_2 = 743\text{ nm}$$

44. Identify the compound that exhibits tautomerism.

(1) Phenol **(2)** 2-Butene
(3) Lactic acid **(4)** 2-Pentanone

Sol. **[4]** $CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_3$ (keto form) $\rightleftharpoons$

$CH_3-\overset{\overset{OH}{|}}{C}=CH-CH_2-CH_3$ (enol form).

45. The entropy change involved int he isothermal reversible expansion of 2 moles of an ideal gas form a volume of 10 dm^3 to a volume of 100 dm^3 at $27°C$ is

(1) $42.3\text{ J mol}^{-1}\text{ K}^{-1}$ **(2)** $38.3\text{ J mol}^{-1}\text{ K}^{-1}$

(3) $35.8\text{ J mol}^{-1}\text{ K}^{-1}$ **(4)** $32.3\text{ J mol}^{-1}\text{ K}^{-1}$

Sol. **[2]** Entropy change for isothermal process

$$\Delta S = nR\ln\frac{V_f}{V_i} \qquad = 2\times\frac{25}{3}\ln\frac{100}{10}$$

$$\Rightarrow 2\times\frac{25}{3}\times 2.303 = 38.3\text{ J mol}^{-1}\text{ K}^{-1}$$

46. Silver Mirror test is given by which one of the following compounds?

(1) Benzophenone **(2)** Acetaldehyde
(3) Acetone **(4)** Formaldehyde

Sol. **[2, 4]** Silver miror test is the characteristic test of aldehydes.

47. Trichloroacetaldehyde was subjected to Cannizszaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is:

(1) Chloroform **(2)** 2, 2, 2-Trichloroethanol
(3) Trichloromethanol **(4)** 2, 2, 2-Trichloropropanol

Sol. **[2]** Cannizaro reaction

$$CCl_3\,CHO \xrightarrow{NaOH} CCl_3COO^-Na^+ + \underset{2,2,2-\text{Trichloroethanol}}{CCl_3CH_2OH}$$

48. The reduction potential of hydrogen half cell will be negative if:

(1) $p(H_2) = 2$ atm and $[H^+] = 2.0$ M

(2) $p(H_2) = 1$ atm and $[H^+] = 2.0$ M

(3) $p(H_2) = 1$ atm and $[H^+] = 1.0$ M

(4) $p(H_2) = 2$ atm and $[H^+] = 1.0$ M

Sol. **[4]** $2H^+ + 2e^- \longrightarrow H_2$

$$E_{cell} = E^o_{cell} - \frac{0.0591}{2}\log\frac{P_{H_2}}{(H^+)^2}$$

$$E_{cell} = 0 - \frac{0.0591}{2}\log\frac{P_{H_2}}{(H^+)^2}$$

for $E_{cell} < 0 \Rightarrow P_{H_2} = 2$ atm, and $(H^+) = 1.0$ M

49. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is:

(1) 2, 4, 6-Tribromophenol
(2) 2-Bromophenol
(3) 3-Bromophenol
(4) 4-Bromophenol

Sol. **[1]** $KBrO_3$ and KBr mixture gives Br_2. Phenol reacts with bromine water to give 2, 4, 6-tribromophenol

50. Among the following the maximum covalent character is shown by the compound:

(1) $MgCl_2$ **(2)** $FeCl_2$
(3) $SnCl_2$ **(4)** $AlCl_3$

Sol. **[4]** Smaller the size of cation more will be the covalent character.

51. Boron cannot form which one of the following anions?

(1) BO_2^- **(2)** BF_6^{3-}
(3) BH_4^- **(4)** $B(OH)_4^-$

Sol. **[2]** Boron cannot expand its octet. Hence it does not form BF_6^{3-}

52. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is

(1) Ethyl ethanoate **(2)** Diethyl ether
(3) 2-Butanone **(4)** Ethyl chloride

Sol. **[1]** $CH_3COCl + C_2H_5ONa \longrightarrow CH_3COOC_2H_5$

53. Which of the following reagents may be used to distinguish between phenol and benzoic acid

(1) Neutral $FeCl_3$ **(2)** Aqueous NaOH
(3) Tollen's reagent **(4)** Molisch reagent

Sol. **[1]** With neutral $FeCl_3$ phenol gives violet colour where as benzoic acid gives buff colored precipitate

54. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm the value of K is

(1) 0.18 atm **(2)** 1.8 atm
(3) 3 atm **(4)** 0.3 atm

Sol. **[2]** $CO_2(g) + C(s) \longrightarrow 2CO(g)$

0.5 –x 2x
0.2 0.6

$0.5 - x + 2x = 0.8$

$\Rightarrow 0.5 + x = 0.8 \qquad \Rightarrow x = 0.3$

$\Rightarrow K = \frac{(0.6)^2}{0.2} = 1.8$

55. The strongest acid amongst the following compounds is

(1) $ClCH_2CH_2CH_2COOH$

(2) CH_3COOH

(3) HCOOH

(4) $CH_3CH_2CH(Cl)CO_2H$

Sol. **[4]** Due to lesser distance from $-COOH$ group electron withdrawing inductive effect of $-Cl$ group is more hence most acidic.

56. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?

(1) $K_2O < Na_2O < Al_2O_3 < MgO$

(2) $Al_2O_3 < MgO < Na_2O < K_2O$

(3) $MgO < K_2O < Al_2O_3 < Na_2O$

(4) $MgO < K_2O < MgO < Al_2O_3$

Sol. **[2]** In a group on moving down basic nature of oxide increases while in a period left to right basic nature decreases.

57. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH is supplied. What is the mole fraction of methyl alcohol in the solution?

(1) 0.050 **(2)** 1.100

(3) 0.190 **(4)** 0.086

Sol. **[4]** 5.2 mol (CH_3OH) in 1000 g water that is 55.55 mols of water.

mol fraction of $CH_3OH = \frac{5.2}{5.2 + 55.55} = 0.086$

58. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA?

(1) 4th **(2)** 1st

(3) 2nd **(4)** 3rd

Sol. **[3]** DNA contain 2-deoxy ribose sugar.

59. Which of the following statement is wrong?

(1) N_2O_4 has two resonance structures.

(2) The stability of hydrides increases form NH_3 to BiH_3 in group 15 of the periodic table.

(3) Nitrogen cannot form $d\pi - p\pi$ bond.

(4) Single N– N bond is weaker than the single $P-P$ bond.

Sol. **[2]** The order of stability of hydrides of 15 group is as follows

$NH_3 > PH_3 > AsH_3 > SbH_3 > BiH_3$

60. Which of the following statements regarding sulphur is incorrect?

(1) The oxidation state of sulphur is never less than +4 in its compounds.

(2) S_2 molecular is paramagnetic

(3) The vapour at $200°C$ consists mostly of S_8 rings.

(4) At $600°C$ the gas mainly consists of S_2

Sol. **[1]** Sulphur can also show +2, 0 and –2 oxidation states .

PART C — PHYSICS

61. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively:

(1) 310 K and 248 K **(2)** 372 K and 310 K
(3) 372 K and 330 K **(4)** 330 K and 268 K

Sol: [2] From $\eta = 1 - \frac{T_2}{T_1}$,

As on reducing T_2, efficiency is increasing means $T_1 > T_2$.

$1 - \frac{T_2}{T_1} = \frac{1}{6}$

and $1 - \frac{T_2 - 62}{T_1} = \frac{1}{3}$

Solving we get $T_2 = 310K$ and $T_1 = 372\,\mathrm{K}$

62. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about is axis of rotationis $10\,\mathrm{kg\,m^2}$, the number of rotations made by the pulley before its direction of motion if reversed, is:

(1) more than 9
(2) less than 3
(3) more than 3 but less than 6
(4) more than 6 but less than 9

Sol: [3] $\alpha = \frac{\tau}{I} = \frac{(20t - 5t^2)(2)}{10}$

or $\int_0^\omega d\omega = \int_0^t (4t - t^2)\,dt$ $\qquad \omega = 2t^2 - \frac{t^3}{3}$

The direction of motion reverses when ω becomes zero.

$\Rightarrow \quad \omega = 2t^2 - \frac{t^3}{3} = 0 \qquad \Rightarrow \qquad t = 6\,\mathrm{s}$

$\omega = \frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3}$

$\Delta\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt = 36\,\mathrm{rad}$

So number of rotations $= \frac{36}{2\pi}$ which is lesser loesser than 6 greater than 3.

63. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixtures is:

(1) $\frac{n_1^2 T_1^2 + n_2^2 T_2^2 + n_3^2 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$

(2) $\frac{(T_1 + T_2 + T_3)}{3}$

(3) $\frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$

(4) $\frac{n_1 T_1^2 + n_2 T_2^2 + n_3 T_3^2}{n_1 T_1 + n_2 T_2 + n_3 T_3}$

Sol: [3] From conservation of internal energy

$n_1 C_v T_1 + n_2 C_v T_2 + n_3 C_v T_3 = (n_1 + n_2 + n_3) T C_v$

$\Rightarrow \quad T = \frac{n_1 T_1 + n_2 T_2 + n_3 T_3}{n_1 + n_2 + n_3}$

64. A boat is moving due east in a region where the earth's magnetic field is $5.0 \times 10^{-5}\,\mathrm{N\,A^{-1}\,m^{-1}}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is $1.50\,\mathrm{ms^{-1}}$, the magnitude of the induced emf in the wire of aerial is:

(1) 0.15 mV **(2)** 1 mV
(3) 0.75 mV **(4)** 0.50 mV

Sol: [1] As $\vec{B} \perp \vec{v} \perp \vec{\ell}$ so

motional emf is

$\xi = Bv\ell = 5 \times 10^{-5} \times 1.5 \times 2 = 0.15 \times 10^{-3}\,\mathrm{V}$

65. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a dimater of the disc to reach its other end. During the journey of the insect, the angular speed of the the disc:

(1) first increases and then decreases
(2) remains unchanged
(3) continuously decreases
(4) continuously increases

Sol: [1] As moment of inertia first decreases then increases so from angular momentum conservation, angular speed first increases then decreases.

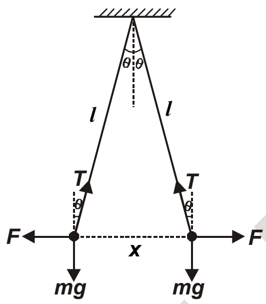
PHYSICS

66. Two identical charged spheres suspneded from a common point by two massless strings of length ℓ are initially a distance $d(d << \ell)$ a part because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them,

(1) $v \propto x$ **(2)** $v \propto x^{-\frac{1}{2}}$

(3) $v \propto x^{-1}$ **(4)** $v \propto x^{+\frac{1}{2}}$

Sol: [2] For the figure shown, in equilibrium.



$$\tan\theta = \frac{kq^2/x^2}{mg}$$

For $d << \ell, \tan\theta = \frac{x/2}{\ell}$

so $\frac{x}{2\ell} = \frac{kq^2}{mgx^2}$

or $x^3 = \left(\frac{2k\ell}{mg}\right)q^2$

or $x^{3/2} = \sqrt{\frac{2k\ell}{mg}}q$

Differentiating w.r.t. time we have

$$\frac{3}{2}x^{\frac{1}{2}}v = \sqrt{\frac{2k\ell}{mg}}\frac{dq}{dt} = \text{constant}$$

$\Rightarrow \quad v \propto x^{-\frac{1}{2}}$

67. 100 g of water is heated fvrom 30 °C to 50 °C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J / kg / K):

(1) 2.1 kJ **(2)** 4.2 kJ

(3) 84 kJ **(4)** 8.4 kJ

Sol: [4] On ignoring expansion,

$Q = \Delta U$

or $\Delta U = ms\Delta T = 0.1 \times 20 \times 4184 = 8.4\,\text{kJ}$

68. The half life oa radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is

(1) 28 min **(2)** 7 min

(3) 14 min **(4)** 20 min

Sol: [4] As the amount present is reducing by half amount in the given time interval so the time interval must be equal to half life which is 20 min.

69. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is:

(1) 122.4 eV **(2)** 12.1 eV

(3) 36.3 eV **(4)** 108.8 eV

Sol: [4] The energy required

$$E = 13.6z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For $z = 3$, and $n_1 = 1$, $n_2 = 3$ we get

$E = 108.8\,\text{eV}$

70. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance fromt he centre ; a, b, are constants. Then the charge density inside the ball is

(1) $-6a\varepsilon_0$ **(2)** $-24\pi a\varepsilon_0 r$

(3) $-6a\varepsilon_0 r$ **(4)** $-24\pi a\varepsilon_0$

Sol: [1] From $E = -\frac{dV}{dr}$ we have $E = -2ar$

As the electric field is directly proportional to r so the charge density must be constant. For a unfirom charge distrubition from $E = \frac{\rho r}{3\varepsilon_0}$ we get

$$\frac{\rho r}{3\varepsilon_0} = -2ar$$

$\Rightarrow \quad \rho = -6a\varepsilon_0$

71. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution $= 0.03\,\text{Nm}^{-1}$):

(1) 0.4π mJ **(2)** 4π mJ

(3) 0.2π mJ **(4)** 2π mJ

Sol: [1] Work done in increasing radius from r_1 to r_2

$$W = 8\pi S\left(r_2^2 - r_1^2\right) \approx 0.4\pi\,\text{mJ}$$

72. A resistor 'R'and 2 μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. $(\log_{10} 2.5 = 0.4)$

(1) $3.3 \times 10^7\,\Omega$ **(2)** $1.3 \times 10^4\,\Omega$

(3) $1.7 \times 10^5\,\Omega$ **(4)** $2.7 \times 10^6\,\Omega$

Sol: [4] For RC growth circuit,

$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

so $$120 = 200\left(1 - e^{-\frac{t}{RC}}\right)$$

or $$\frac{t}{RC} = \ell n\frac{5}{2}$$

$$R = \frac{t}{C\ell n\frac{5}{2}} = \frac{t}{C \times 2.3\log_{10}\frac{5}{2}}$$

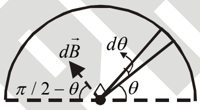
$$= \frac{5}{2 \times 10^{-6} \times 2.3 \times 0.4} \approx 2.7 \times 10^6\,\Omega$$

73. A current I flows in an infinitely long wire with cross section in the form of a semicircular ring of radius R. The magnitude of the magnetic induction along its axis is:

(1) $\frac{\mu_0 I}{4\pi R}$ **(2)** $\frac{\mu_0 I}{\pi^2 R}$

(3) $\frac{\mu_0 I}{2\pi^2 R}$ **(4)** $\frac{\mu_0 I}{2\pi R}$

Sol: [2] Consider an element which is a wire of width $Rd\theta$ along the length of the cylinder. The current through the element is



$$\left(\frac{I}{\pi R}\right)(Rd\theta) = \frac{I}{\pi}d\theta .$$

The magnetic field due to the element is

$$\frac{\mu_0\left(\frac{I}{\pi}d\theta\right)}{2\pi R}$$

From symmetry of figure the net field is

$$B = \int_0^{\pi}\frac{\mu_0\left(\frac{I}{\pi}d\theta\right)}{2\pi R}\sin\theta = \frac{\mu_0 I}{\pi^2 R}$$

74. An object, moving with a speed of 6.25 m / s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, woudl be:

(1) 8 s **(2)** 1 s
(3) 2 s **(4)** 4 s

Sol: [3] $\frac{dv}{dt} = -2.5\sqrt{v}$ or $\int_{6.25}^{0}\frac{dv}{\sqrt{v}} = \int_0^t -2.5dt$

$\Rightarrow$ $t = 2\,\text{s}$

75. ***Direction:***

*The question has paragraph fllowed by two statements, **Statement-1** and **Statement-2**. Of the given four alternatives after the statements, choose the one that describes the statements.*

A thin air film is formed by putting the convex surface of a plen-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film.

Statement - 1:

When light reflects from the air-lgass plate interface, the reflected wave surfers a phase change of π.

Statement - 2:

The centre of the interference pattern is dark.

(1) Statement - 1 is flase, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is flase.
(3) Statement - 1 is true, Statement - 2 is true and Statement - 2 is the correct explanation of Statement - 1
(4) Statement - 1 is true, Statement - 2 is true and Statement - 2 is **not** the correct explantion of Statement - 1.

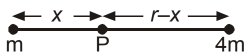
Sol: [3] Statement 1 and 2 both are true and statement 2 is a correct explantion of statement-1

76. Two bodies of masses m and 4m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is:

(1) $-\frac{9Gm}{r}$ **(2)** zero

(3) $-\frac{4Gm}{r}$ **(4)** $-\frac{6Gm}{r}$

Sol: [1]



For field at P to be zero

$$\frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$\Rightarrow$ $x = \frac{r}{3}$

potential at P

$$V = \left(\frac{-Gm}{r/3}\right) + \left(\frac{-G \times 4m}{2r/3}\right) = -\frac{9Gm}{r}$$

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77. This equation has Statement - 1 and Statement - 2. Of the four choices given afterthe statements, choose the one that best decrribes the two statements.

Statement - 1:

Sky wave signals are used for long distance radio communication. These signlas are in gneeral, less table than ground wave signals.

Statement - 2:

The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement - 1 is flase, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is flase.
(3) Statement - 1 is true, Statement - 2 is true and Statement - 2 is the correct explanation of Statement - 1
(4) Statement - 1 is true, Statement - 2 is true and Statement - 2 is **not** the correct explantion of Statement - 1.

Sol: [3] Statement - 1 is true, Statement - 2 is true and Statement - 2 is the correct explanation of Statement - 1

78. A fully charged capacitor C with iniital charge q_0 is connected to a coil of self inductance L at $t = 0$. The imt at which the enrgy is stored equally between the electric and the magnetic fiedls is:

(1) $\sqrt{LC}$ **(2)** $\pi\sqrt{LC}$

(3) $\frac{\pi}{4}\sqrt{LC}$ **(4)** $2\pi\sqrt{LC}$

Sol: [3] $U_B = U_e = U_{net}/2$

so $U_e = \frac{q^2}{2C} = \frac{1}{2}\left(\frac{q_0^2}{2C}\right)$

$q = \pm\frac{q_0}{\sqrt{2}}$

From $q = q_0\cos\omega t$ for $q = \frac{q_0}{\sqrt{2}}$ we get

$t = \frac{T}{8} = \frac{2\pi/\omega}{8} = \frac{\pi}{4} \times \sqrt{LC}$

79. This equation has Statement - 1 and Statement - 2. Of the four choices given afterthe statements, choose the one that best decrribes the two statements.

Statement - 1:

A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surfaceis doubled, both the K_{max} and V_0 are also doubled.

Statement - 2:

The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement - 1 is flase, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is flase.
(3) Statement - 1 is true, Statement - 2 is true and Statement - 2 is the correct explanation of Statement - 1
(4) Statement - 1 is true, Statement - 2 is true and Statement - 2 is **not** the correct explantion of Statement - 1.

Sol: [1] $K_{max} = hv - \phi$

On doubling v, K_{max} and hence V_0 become more than double so statement 1 is false.

80. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is $0.4\,\text{ms}^{-1}$. The dimaeter of the water stream at a distance 2×10^{-1} m below the tap is close to:

(1) 3.6×10^{-3} m **(2)** 5.0×10^{-3} m

(3) 7.5×10^{-3} m **(4)** 9.6×10^{-3} m

Sol: [1] If v_2 be the speed at depth h below the tap, then

$v_2 = \sqrt{v_1^2 + 2gh}$

From equation of continuty

$v_1 d_1^2 = v_2 d_2^2$

or $d_2 = \sqrt{\frac{v_1}{v_2}} \times d_1 = \sqrt{\frac{v_1}{\sqrt{v_1^2 + 2gh}}}\, d_1 \approx 3.6 \times 10^{-3}$ m

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81. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is:

(1) $\left(\frac{M+m}{M}\right)^{\frac{1}{2}}$ **(2)** $\frac{M}{M+m}$

(3) $\frac{M+m}{M}$ **(4)** $\left(\frac{M}{M+m}\right)^{\frac{1}{2}}$

Sol: [1] From momentum conservation

$$M(\omega_1 A_1) = (M+m)(\omega_2 A_2)$$

or $$\frac{A_1}{A_2} = \left(\frac{M+m}{M}\right)\left(\frac{\omega_2}{\omega_1}\right)$$

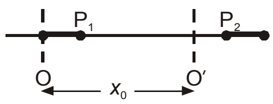
$$= \left(\frac{M+m}{M}\right)\frac{\left(\sqrt{\frac{k}{M+m}}\right)}{\left(\sqrt{\frac{k}{M}}\right)} = \sqrt{\frac{M+m}{M}}$$

82. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the *ax*-axis. Their mean positionis separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is

(1) $\frac{\pi}{6}$ **(2)** $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ **(4)** $\frac{\pi}{4}$

Sol: [3] The positions of two particles w.r.t. a common origion O are



$x_1 = A\sin\omega t$

$x_2 = x_0 + A\sin(\omega t + \phi)$

The separation between them is

$$\Delta x = x_2 - x_1 = x_0 + A[\sin(\omega t + \phi) - \sin\omega t]$$

$$= x_0 + A[2\cos(\omega t + \phi/2)\sin\phi/2]$$

As the maximum separation is $x_0 + A$ so the value of $\sin(\phi/2)$ must be half. For that

$\phi = \frac{\pi}{3}$.

83. If a wire is stretched to make it 0.1% longer, its resitance will

(1) decrease by 0.05% **(2)** increase by 0.05%
(3) increase by 0.2% **(4)** decrease by 0.2%

Sol: [3] Resistance of a wire $R = \frac{\rho\ell}{A} = \frac{\rho\ell^2}{A\ell} = \frac{\rho\ell^2}{V}$

Hence volume of the wire remains same thus for small changes in length, fractional change in resistance

$$\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell} = 0.2\%$$

84. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is *v*, the total area around the fountain that gets wet is:

(1) $\pi\frac{v^2}{g^2}$ **(2)** $\pi\frac{v^2}{g}$

(3) $\pi\frac{v^4}{g^2}$ **(4)** $\frac{\pi}{2}\frac{v^4}{g^2}$

Sol: [3] Maximum area over which water sprinkels $= \pi R_{max}^2$

where R_{max} in the maximum possible range

$$R_{max} = \frac{v^2}{g}$$

and thus maximum area is $\frac{\pi v^4}{g^2}$

85. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed *v* and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by:

(1) $\frac{(\gamma-1)}{2R}Mv^2K$ **(2)** $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$

(3) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$ **(4)** $\frac{\gamma Mv^2}{2R}K$

Sol: [1] As it is suddenly brought to rest, the kinetic associated with the motion of centre of mass (ordered kinetic energy) will be abosrbed by the gas and is now seen as the inerease in the disordered kinetic energy of gas and results in increase in the temperature of the gas then

$$\frac{1}{2}mv^2 = nC_v\Delta T \quad \Rightarrow \quad \frac{1}{2}mv^2 = \left(\frac{m}{M}\right)\left(\frac{R}{\gamma-1}\right)\Delta T$$

So $\Delta T = \frac{(\gamma-1)}{2M}Rv^2$

(where m is the mass of gas enclosed in the container)

PHYSICS

86. A screw gauge gives the following reading when used to measure the diameter of a wire
Main scale reading: 0 mm.
Circular scale reading: 52 dividison
Given tghat 1 mm on main scale corresponds to 100 dividions of the circular scale.
The dimater of wire from the above data is
(1) 0.005 cm **(2)** 0.52 cm
(3) 0.052 cm **(4)** 0.026 cm

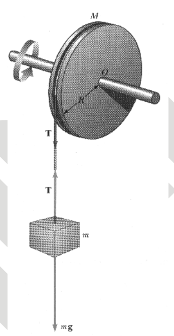
Sol: [3] Here least count of screw gauge

$$\frac{\text{pitch}}{\text{Number of disions on circular scale}} = 0.01\,\text{mm}$$

So the given reading is $= 52 \times LC = 0.52\,\text{mm} = 0.052\,\text{cm}$

87. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform ciruclar disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(1) $\frac{g}{3}$ **(2)** $\frac{3}{2}g$
(3) g **(4)** $\frac{2}{3}g$



Sol: [4] $mg - T = ma$ . . . (i)

$$TR = \left(\frac{mR^2}{2}\right)\alpha = \left(\frac{mR^2}{2}\right)\left(\frac{a}{R}\right) \text{ . . . (ii)}$$

Solving we get, $a = \frac{2g}{3}$

88. The transverse displacement $y(x,t)$ of a wave on a string is given by

$$y(x,t) = e^{-\left(ax^2 + bt^2 + 2\sqrt{ab}xt\right)}$$

This represents a:

(1) standing wave of frequency $\frac{1}{\sqrt{b}}$

(2) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$

(3) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(4) standing wave of fcrequency $\sqrt{b}$

Sol: [3] The given expression can be wrfitten as

$$y(x,t) = e^{-\left(\sqrt{a}x + \sqrt{b}t\right)^2}$$

is a travelling wave in $-x$ direction with speed $\sqrt{\frac{b}{a}}$.

89. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15 m / s. The speed of the image of the second car as seen in the mirror of the first one is:

(1) 15 m / s **(2)** $\frac{1}{10}$ m / s

(3) $\frac{1}{15}$ m / s **(4)** 10 m / s

Sol: [3] Differentiating the mirror equation w.r.t. time

$$-\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

or $$\frac{dv}{dt} = \frac{v^2}{u^2}\frac{du}{dt} = \left(\frac{f}{u-f}\right)^2 \frac{du}{dt}$$

$$= \left(\frac{20}{-280-20}\right)^2 \times 15 = \frac{1}{15}\,\text{m/s}$$

90. Let the x - z plane be the boundary between two transparewnt media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10k$ is incident on the plane of separation. The angle of refraction in medium 2 is:
(1) 75° **(2)** 30°
(3) 45° **(4)** 60°

Sol: [3] **In our opinion this question can be solved only if we consider the boundary between the two transparent media to *x-y* plane rather than *x-z* plane given wronlgy in question.**

If θ_1 be the angle of incidnet ray with the normal z-axis then using the direction cosine formula with z-axis we have

$$\cos\theta_1 = \frac{10}{\sqrt{\left(6\sqrt{3}\right)^2 + \left(8\sqrt{3}\right)^2 + (-10)^2}} = \frac{10}{20} = \frac{1}{2}$$

$\Rightarrow$ $\theta_1 = 60^\circ$

From snell's law

$$\left(\sqrt{2}\right)\sin 60^\circ = \left(\sqrt{3}\right)\sin\theta_2$$

$$\sin\theta_2 = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $\theta_2 = \frac{\pi}{4}$