



Resonance
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AIEEE 2012 : PAPER-1

Date : 29-04-2012

Duration : 3 Hours

Max. Marks : 360

IMPORTANT INSTRUCTIONS

1. Immediately fill the particulars on this page of the Test Booklet with Blue / Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are three parts in the question paper A, B, C consisting of **Mathematics, Physics and Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for each correct response.
6. Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question. $\frac{1}{4}$ (**one fourth**) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 6 above.
8. Use **Blue/Black Ball Point Pen only** for writing particulars/markings responses on **Side-1** and **Side-2** of the Answer Sheet. **Use of pencil is strictly prohibited.**
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, paper, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 3 pages at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
12. The CODE for this Booklet is C. Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. Do not fold or make any stray marks on the Answer Sheet.

Name of the Candidate (in Capital letters) : _____

Roll Number : in figures :

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 in words : _____

Examination Centre Number :

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Name of Examination Centre (in Capital letters) : _____

Candidate's Signature : _____ Invigilator's Signature : _____

PART-A (MATHEMATICS)

1. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

- (1) infinite number of real roots
- (2) no real roots
- (3) exactly one real root
- (4) exactly four real roots

Ans. (2)

Sol. Let $e^{\sin x} = t$

$$\Rightarrow t^2 - 4t - 1 = 0$$

$$\Rightarrow t = \frac{4 \pm \sqrt{16 + 4}}{2}$$

$$\Rightarrow t = e^{\sin x} = 2 \pm \sqrt{5}$$

$$\Rightarrow e^{\sin x} = 2 - \sqrt{5},$$

$$e^{\sin x} = 2 - \sqrt{5} < 0,$$

so rejected
hence no solution

$$e^{\sin x} = 2 + \sqrt{5}$$

$$\Rightarrow \sin x = \ln(2 + \sqrt{5}) > 1$$

so rejected

2. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{4}$

Ans. (3)

Sol. $\vec{c} = \hat{a} + 2\hat{b}$

$$\vec{d} = 5\hat{a} - 4\hat{b}$$

$$\vec{c} \cdot \vec{d} = 0$$

$$\Rightarrow (\hat{a} + 2\hat{b}) \cdot (5\hat{a} - 4\hat{b}) = 0$$

$$\Rightarrow 5 + 6\hat{a} \cdot \hat{b} - 8 = 0$$

$$\Rightarrow \hat{a} \cdot \hat{b} = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

3. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :

(1) $\frac{9}{7}$

(2) $\frac{7}{9}$

(3) $\frac{2}{9}$

(4) $\frac{9}{2}$

Ans. (3)

Sol. $V = \frac{4}{3}\pi r^3$ $4500\pi = \frac{4\pi r^3}{3}$

$$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt} \right)$$

$$45 \times 25 \times 3 = r^3$$

$$r = 15 \text{ m}$$

$$\text{after 49 min} = (4500 - 49.72)\pi = 972 \pi \text{ m}^3$$

$$972 \pi = \frac{4}{3} \pi r^3$$

$$r^3 = 3 \times 243 = 3 \times 3^5$$

$$r = 9$$

$$72 \pi = 4\pi \times 9 \times 9 \left(\frac{dr}{dt} \right)$$

$$\frac{dr}{dt} = \left(\frac{2}{9} \right)$$

4. **Statement-1 :** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \dots + (361 + 380 + 400)$ is 8000.

Statement-2 : $\sum_{k=1}^n (k^3 - (k-1)^3) = n^3$, for any natural number n .

- (1) Statement-1 is false, Statement-2 is true.
 (2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
 (3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
 (4) Statement-1 is true, statement-2 is false.

Ans. (2)

Sol. $T_n = (n-1)^2 + (n-1)n + n^2 = \frac{((n-1)^3 - n^3)}{(n-1) - n} = n^3 - (n-1)^3$

$$T_1 = 1^3 - 0^3$$

$$T_2 = 2^3 - 1^3$$

$\vdots$

$$T_{20} = 20^3 - 19^3$$

$$S_{20} = 20^3 - 0^3 = 8000$$

5. The negation of the statement

"If I become a teacher, then I will open a school", is :

- (1) I will become a teacher and I will not open a school.
 (2) Either I will not become a teacher or I will not open a school.
 (3) Neither I will become a teacher nor I will open a school
 (4) I will not become a teacher or I will open a school.

Ans. (1)

Sol. Let p : I become a teacher

q : I will open a school

Negation of $p \rightarrow q$ is $\sim(p \rightarrow q) = p \wedge \sim q$

i.e. I will become a teacher and I will not open a school.

6. If the integral $\int \frac{5 \tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$, then a is equal to :

(1) -1

(2) -2

(3) 1

(4) 2

Ans. (4)

Sol. $\int \frac{5 \tan x}{\tan x - 2} dx = \int \frac{5 \sin x}{\sin x - 2 \cos x} dx = \int \frac{(\sin x - 2 \cos x) + 2(\cos x + 2 \sin x)}{(\sin x - 2 \cos x)} dx$

$$= \int dx + 2 \int \frac{\cos x + 2 \sin x}{\sin x - 2 \cos x} dx = x + 2 \ln |(\sin x - 2 \cos x)| + k$$

$$\Rightarrow a = 2$$

7. **Statement-1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement-2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, ($m \neq 0$) is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the

ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4) Statement-1 is true, statement-2 is false.

Ans. (2)

Sol. Equation of tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{4} = 1$ is

$$y = mx \pm \sqrt{2m^2 + 4} \quad \dots(1)$$

equation of tangent to the parabola $y^2 = 16\sqrt{3}x$

$$\text{is } y = mx + \frac{4\sqrt{3}}{m} \quad \dots(2)$$

On comparing (1) and (2)

$$\frac{4\sqrt{3}}{m} = \pm \sqrt{2m^2 + 4}$$

$$\Rightarrow 48 = m^2(2m^2 + 4)$$

$$\Rightarrow m^4 + 2m^2 - 24 = 0$$

$$\Rightarrow m^2 = 4$$

$$\Rightarrow 2m^4 + 4m^2 - 48 = 0$$

$$\Rightarrow (m^2 + 6)(m^2 - 4) = 0$$

$$\Rightarrow m = \pm 2$$

$$\Rightarrow \text{equation of common tangents are } y = \pm 2x \pm 2\sqrt{3}$$

statement -1 is true.

statement-2 is obviously true.

8. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal

to :

$$(1) \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$(2) \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

$$(3) \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$$(4) \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

Ans. (4)

Sol. $A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad |A| = 1$

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$u_1 + u_2 = A^{-1} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

9. If n is a positive integer, then $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$ is :

- (1) an irrational number (2) an odd positive integer
(3) an even positive integer (4) a rational number other than positive integers

Ans. (1)

Sol. $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$
 $= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \dots]$
 $= \text{which is an irrational number}$

10. If 100 times the 100th term of an AP with non zero common difference equals the 50 times its 50th term, then the 150th term of this AP is :

- (1) - 150 (2) 150 times its 50th term
(3) 150 (4) zero

Ans. (4)

Sol. $100(a + 99d) = 50(a + 49d)$
 $2a + 198d = a + 49d$
 $a + 149d = 0$
 $T_{150} = a + 149d = 0$

11. In a ΔPQR , if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to :

- (1) $\frac{5\pi}{6}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Ans. (2)

Sol. $3 \sin P + 4 \cos Q = 6 \quad \dots(i)$
 $4 \sin Q + 3 \cos P = 1 \quad \dots(ii)$

Squaring and adding (i) & (ii) we get $\sin(P + Q) = \frac{1}{2}$

$$\Rightarrow P + Q = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

$$\Rightarrow R = \frac{5\pi}{6} \text{ or } \frac{\pi}{6}$$

If $R = \frac{5\pi}{6}$ then $0 < P, Q < \frac{\pi}{6}$

$$\Rightarrow \cos Q < 1 \text{ and } \sin P < \frac{1}{2}$$

$$\Rightarrow 3 \sin P + 4 \cos Q < \frac{11}{2}$$

$$\text{So } R = \frac{\pi}{6}$$

12. A equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

- (1) $x - 2y + 2z - 3 = 0$ (2) $x - 2y + 2z + 1 = 0$ (3) $x - 2y + 2z - 1 = 0$ (4) $x - 2y + 2z + 5 = 0$

Ans. (1)

Sol. Equation of parallel plane $x - 2y + 2z + d = 0$

$$\text{Now } \left| \frac{d}{\sqrt{1^2 + 2^2 + 2^2}} \right| = 1$$

$$d = \pm 3$$

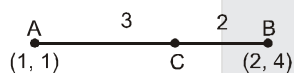
So equation required plane $x - 2y + 2z \pm 3 = 0$

13. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :

- (1) $\frac{29}{5}$ (2) 5 (3) 6 (4) $\frac{11}{5}$

Ans. (3)

Sol.



$$\therefore C \left(\frac{8}{5}, \frac{14}{5} \right)$$

Line $2x + y = k$ passes $C \left(\frac{8}{5}, \frac{14}{5} \right)$

$$\frac{2 \times 8}{5} + \frac{14}{5} = k$$

$$k = 6$$

14. Let $x_1, x_2, \dots, x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance

Statement-1 : Variance of $2x_1, 2x_2, \dots, 2x_n$ is $4\sigma^2$.

Statement-2 : Arithmetic mean $2x_1, 2x_2, \dots, 2x_n$ is $4\bar{x}$.

- (1) Statement-1 is false, Statement-2 is true.
 (2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
 (3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
 (4) Statement-1 is true, statement-2 is false.

Ans. (4)

Sol. A.M. of $2x_1, 2x_2, \dots, 2x_n$ is $\frac{2x_1 + 2x_2 + \dots + 2x_n}{n} = 2 \left(\frac{x_1 + x_2 + \dots + x_n}{n} \right) = 2\bar{x}$

So statement-2 is false

variance $(2x_i) = 2^2 \text{ variance } (x_i) = 4\sigma^2$

so statement-1 is true.

15. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation

$$\frac{dp(t)}{dt} = 0.5 p(t) - 450. \text{ If } p(0) = 850, \text{ then the time at which the population becomes zero is :}$$

- (1) $2 \ln 18$ (2) $\ln 9$ (3) $\frac{1}{2} \ln 18$ (4) $\ln 18$

Ans. (1)

Sol. $2 \frac{dp(t)}{900 - p(t)} = -dt$

$$-2 \ln (900 - p(t)) = -t + c$$

when $t = 0$, $p(0) = 850$

$$-2 \ln(50) = c$$

$$\therefore 2 \ln \left(\frac{50}{900 - p(t)} \right) = -t$$

$$900 - p(t) = 50 e^{t/2}$$

$$p(t) = 900 - 50 e^{t/2}$$

let $p(t_1) = 0$

$$0 = 900 - 50 e^{\frac{t_1}{2}}$$

$$\therefore t_1 = 2 \ln 18$$

16. Let $a, b \in \mathbb{R}$ be such that the function f given by $f(x) = \ln |x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement-1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement-2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement-1 is false, Statement-2 is true.

(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(4) Statement-1 is true, statement-2 is false.

Ans. (2)

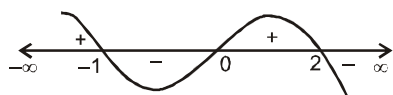
Sol. $f'(x) = \frac{1}{x} + 2bx + a$

at $x = -1$ $-1 - 2b + a = 0$
 $a - 2b = 1$... (i)

at $x = 2$ $\frac{1}{2} + 4b + a = 0$
 $a + 4b = -\frac{1}{2}$... (ii)

On solving (i) and (ii) $a = \frac{1}{2}$, $b = -\frac{1}{4}$

$$f'(x) = \frac{1}{x} - \frac{x}{2} + \frac{1}{2} = \frac{2 - x^2 + x}{2x} = \frac{-(x+1)(x-2)}{2x}$$

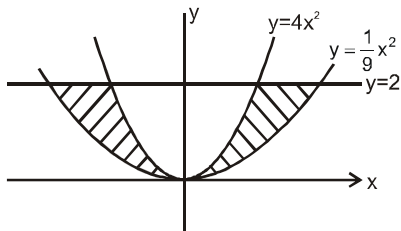


So maxima at $x = -1, 2$

17. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is :

- (1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$

Ans (3)



Sol.

$$y = 4x^2$$

$$y = \frac{1}{9}x^2$$

$$\text{Area} = 2 \int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2} \right) dy = 2 \left[\frac{5}{2} \frac{y^{3/2}}{3/2} \right]_0^2 = 2 \cdot \frac{5}{3} \cdot 2\sqrt{2} = \frac{20\sqrt{2}}{3}$$

18. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :

- (1) 880 (2) 629 (3) 630 (4) 879

Ans (4)

Sol. $(10 + 1)(9 + 1)(7 + 1) - 1 = 11 \cdot 10 \cdot 8 - 1 = 879$

19. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $f(x) = [x] \cos \left(\frac{2x-1}{2} \pi \right)$, where $[x]$ denotes the greatest integer function,

then f is :

- (1) continuous for every real x .
 (2) discontinuous only at $x = 0$.
 (3) discontinuous only at non-zero integral values of x .
 (4) continuous only at $x = 0$.

Ans. (1)

Sol. Doubtful points are $x = n, n \in \mathbb{I}$

$$\text{L.H.L.} = \lim_{x \rightarrow n^-} [x] \cos \left(\frac{2x-1}{2} \pi \right) = (n-1) \cos \left(\frac{2n-1}{2} \pi \right) = 0$$

$$\text{R.H.L.} = \lim_{x \rightarrow n^+} [x] \cos \left(\frac{2x-1}{2} \pi \right) = n \cos \left(\frac{2n-1}{2} \pi \right) = 0$$

$$f(n) = 0$$

Hence continuous

20. If the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then k is equal to :

- (1) -1 (2) $\frac{2}{9}$ (3) $\frac{9}{2}$ (4) 0

Ans. (3)

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$

$$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$$

$$\vec{a}(1, -1, 1); \quad \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\vec{b}(2, 3, 4)$$

$$\vec{c}(3, k, 0); \quad \vec{r} = \vec{c} + \mu \vec{d}$$

$$\vec{d}(1, 2, 1)$$

These lines will intersect if lines are coplaner

$\vec{a} - \vec{c}, \vec{b}$ & $\vec{d}$ are coplaner

$$\therefore [\vec{a} - \vec{c}, \vec{b}, \vec{d}] = 0$$

$$\begin{vmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow 2(-5) - (k+1)(-2) - 1(1) = 0$$

$$\Rightarrow 2(k+1) = 11$$

$$\Rightarrow k = \frac{9}{2}$$

21. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is :

- (1) $\frac{3}{8}$ (2) $\frac{1}{5}$ (3) $\frac{1}{4}$ (4) $\frac{2}{5}$

Ans. (2)

Sol. Let Event (Given : {1, 2, 3, ..., 8})

A : Maximum of three numbers is 6.

B : Minimum of three numbers is 3

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

22. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

- (1) either on the real axis or on a circle passing through the origin.
 (2) on a circle with centre at the origin.
 (3) either on the real axis or on a circle not passing through the origin.
 (4) on the imaginary axis.

Ans. (1)

Sol. $\frac{z^2}{z-1} = \frac{\bar{z}^2}{\bar{z}-1}$

$$\Rightarrow z \bar{z} z - z^2 = z \bar{z} \bar{z} - \bar{z}^2$$

$$\Rightarrow |z|^2 (z - \bar{z}) - (z - \bar{z}) (z + \bar{z}) = 0$$

$$\Rightarrow (z - \bar{z}) (|z|^2 - (z + \bar{z})) = 0$$

Either $z = \bar{z} \Rightarrow$ real axis

$$\text{or } |z|^2 = z + \bar{z} \Rightarrow z \bar{z} - z - \bar{z} = 0$$

represents a circle passing through origin.

- 23.** Let P and Q be 3×3 matrices $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :
(1) -2 (2) 1 (3) 0 (4) -1

Ans. (3)

Sol. Subtracting $P^3 - P^2Q = Q^3 - Q^2P$

$$P^2(P - Q) + Q^2(P - Q) = 0$$

$$(P^2 + Q^2)(P - Q) = 0$$

If $|P^2 + Q^2| \neq 0$ then $P^2 + Q^2$ is invertible

$$\Rightarrow P - Q = 0 \quad \text{contradiction}$$

$$\text{Hence } |P^2 + Q^2| = 0$$

- 24.** If $g(x) = \int_0^x \cos 4t \, dt$, then $g(x + \pi)$ equals

(1) $\frac{g(x)}{g(\pi)}$

(2) $g(x) + g(\pi)$

(3) $g(x) - g(\pi)$

(4) $g(x) \cdot g(\pi)$

Ans. (2 or 3)

Sol. $g(x + \pi) = \int_0^{x+\pi} \cos 4t \, dt = g(x) + \int_0^{\pi} \cos 4t \, dt$
 $= g(x) + g(\pi)$

$$\text{Here } g(\pi) = \int_0^{\pi} \cos 4t \, dt = 0$$

so answers are (2) or (3)

- 25.** The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(1) $\frac{10}{3}$

(2) $\frac{3}{5}$

(3) $\frac{6}{5}$

(4) $\frac{5}{3}$

Ans. (1)

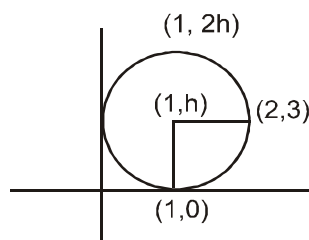
Sol. Now

$$h^2 = (1 - 2)^2 + (h - 3)^2$$

$$0 = 1 - 6h + 9$$

$$6h = 10$$

$$h = \frac{5}{3}$$



$$\text{Now diameter is } 2h = \frac{10}{3}$$

26. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X, Z \subseteq X$ and $Y \cap Z$ is empty, is :

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

Ans. (2)

Sol. Every element has 3 options. Either set Y or set Z or none
so number of ordered pairs $= 3^5$

27. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

(1) $4x^2 + y^2 = 4$ (2) $x^2 + 4y^2 = 8$ (3) $4x^2 + y^2 = 8$ (4) $x^2 + 4y^2 = 16$

Ans. (4)

Sol. $\Rightarrow$ Length of semi minor axis is $= 2$

Length of semi major axis is 4

then equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

$$x^2 + 4y^2 = 16$$

28. Consider the function, $f(x) = |x - 2| + |x - 5|, x \in \mathbb{R}$.

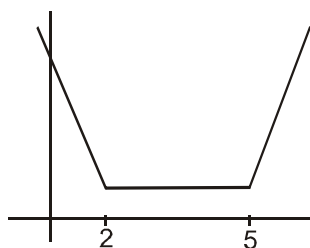
Statement-1 : $f'(4) = 0$

Statement-2 : f is continuous in $[2, 5]$, differentiable in $(2, 5)$ and $f(2) = f(5)$.

- (1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

Ans. (3)

Sol. $f(x) = 3 \quad 2 \leq x \leq 5$
 $f'(x) = 0 \quad 2 < x < 5$
 $f'(4) = 0$



29. A line is drawn through the point $(1, 2)$ to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is :

(1) $-\frac{1}{4}$ (2) -4 (3) -2 (4) $-\frac{1}{2}$

Ans. (3)

Sol. $(y - 2) = m(x - 1)$

$$OP = 1 - \frac{2}{m}$$

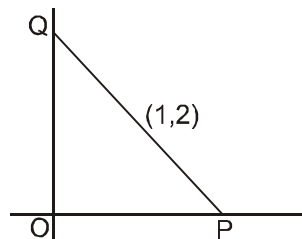
$$OQ = 2 - m$$

$$\text{Area of } \triangle POQ = \frac{1}{2} (OP)(OQ) = \frac{1}{2} \left(1 - \frac{2}{m} \right) (2 - m)$$

$$= \frac{1}{2} \left[2 - m - \frac{4}{m} + 2 \right]$$

$$= \frac{1}{2} \left[4 - \left(m + \frac{4}{m} \right) \right]$$

$$m = -2$$



- 30.** Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by :

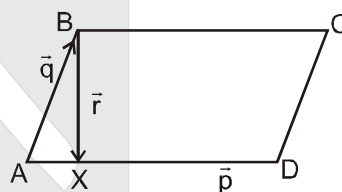
(1) $\vec{r} = 3\vec{q} - \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})} \vec{p}$ (2) $\vec{r} = -\vec{q} + \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}} \right) \vec{p}$ (3) $\vec{r} = \vec{q} - \left(\frac{\vec{p} \cdot \vec{q}}{\vec{p} \cdot \vec{p}} \right) \vec{p}$ (4) $\vec{r} = -3\vec{q} + \frac{3(\vec{p} \cdot \vec{q})}{(\vec{p} \cdot \vec{p})} \vec{p}$

Ans. (2)

Sol. $\overrightarrow{AX} = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}| |\vec{p}|} \vec{p} = \frac{\vec{p} \cdot \vec{q}}{|\vec{p}|^2} \vec{p}$

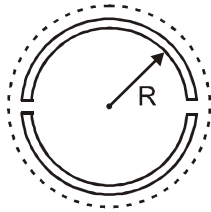
$$\overrightarrow{BX} = \overrightarrow{BA} + \overrightarrow{AX}$$

$$= -\vec{q} + \frac{\vec{p} \cdot \vec{q}}{|\vec{p}|^2} \vec{p}$$



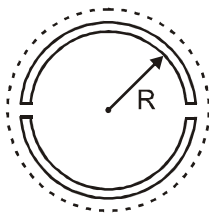
PART-B (PHYSICS)

31. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L . L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α , and its Young's modulus is Y , the force that one part of the wheel applies on the other part is :



- Ans. (1) $2\pi SY\alpha\Delta T$
(4)

Sol.



$$L \Rightarrow S$$

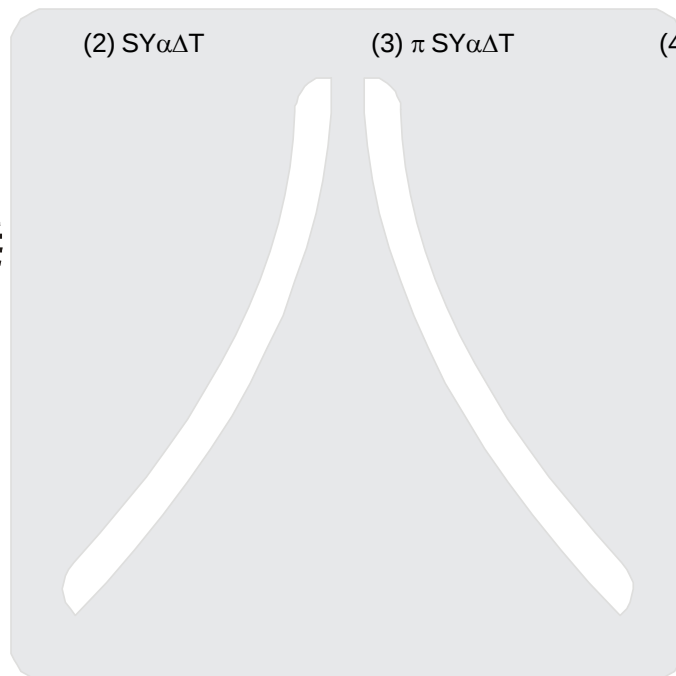
$$\Delta L = L \propto \Delta T$$

$$\frac{F}{A} = \frac{\Delta L}{L} Y$$

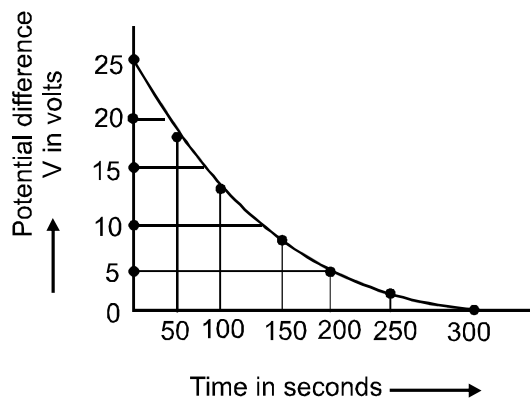
$$F = \alpha \Delta T Y S$$

$$\text{So, } T = 2F$$

$$T = 2\alpha \Delta T Y S$$



32. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ of this circuit lies between :



- (1) 150 sec and 200 sec
(3) 50 sec and 100 sec

- (2) 0 and 50 sec
(4) 100 sec and 150 sec

Ans. (4)

Sol. $Q = C\epsilon_0 e^{-t/CR}$

$$4\epsilon = 4\epsilon_0 e^{-t/\tau}$$

$$\epsilon = \epsilon_0 e^{-t/\tau}$$

When $t = 0 \Rightarrow \epsilon_0 = 25$

$$\epsilon = \epsilon_0 = 25$$

when $t = 200 \Rightarrow \epsilon = 5$

$$5 = 25 e^{-\frac{200}{\tau}}$$

$$\ln 5 = \frac{200}{\tau}$$

$$\tau = \frac{200}{\ln 5} = \frac{200}{\ln 10 - \ln 2}$$

$$= \frac{200}{\ln 10 - 0.693}$$

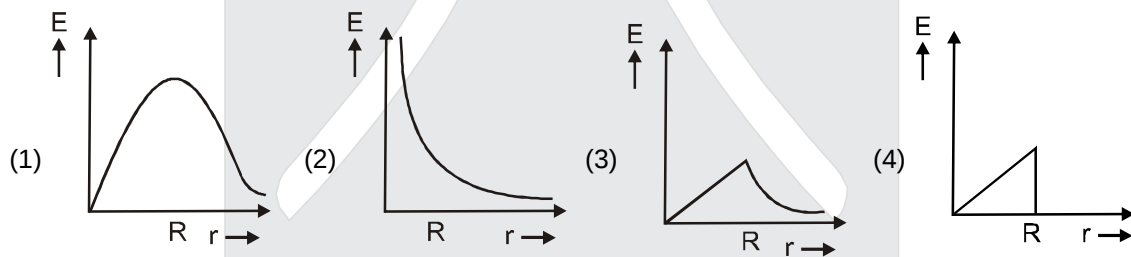
Alternative :

Time constant is the time in which 63% discharging is completed.

So remaining charge = $0.37 \times 25 = 9.25$ V

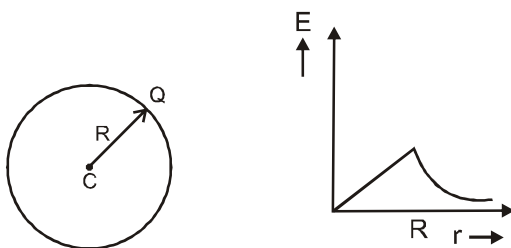
Which time in $100 < t < 150$ sec.

33. In a uniformly charged sphere of total charge Q and radius R , the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be :



Ans. (3)

Sol.



34. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then

(1) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

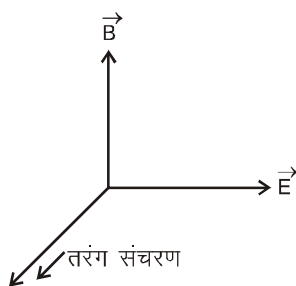
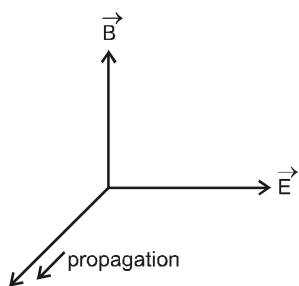
(2) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

(3) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

(4) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

Ans. (2)

Sol.



35. If a simple pendulum has significant amplitude (up to a factor of $1/e$ of original) only in the period between $t = 0$ to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :

(1) $\frac{0.693}{b}$

(2) b

(3) $\frac{1}{b}$

(4) $\frac{2}{b}$

Ans. (4)

Sol. $m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt}$

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

This has solution of type

$x = e^{\lambda t}$ substituting this

$$m\lambda^2 + b\lambda + k = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

on solving for x, we get

$$x = e^{-\frac{b}{2m}t} a \cos(\omega_1 t - \alpha)$$

$$\omega_1 = \sqrt{\omega_0^2 - \lambda^2} \quad \text{where } \omega_0 = \sqrt{\frac{k}{m}}$$

$$\lambda = +\frac{b}{2}$$

$$\text{So, average life} = \frac{2}{b}$$

36. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be :

(1) 2

(2) 3

(3) 5

(4) 6

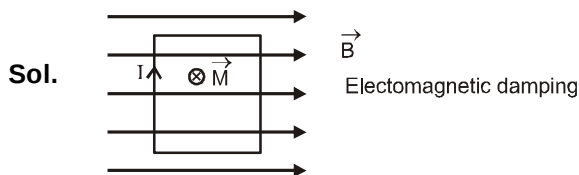
Ans. (4)

Sol. If $n = 4$

$$\text{lines} = \frac{n(n-1)}{2} = 6$$

37. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :
- (1) development of air current when the plate is placed.
 - (2) induction of electrical charge on the plate
 - (3) shielding of magnetic lines of force as aluminium is a paramagnetic material.
 - (4) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

Ans. (4)



38. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :

- (1) 6.4×10^{11} Joules
- (2) 6.4×10^8 Joules
- (3) 6.4×10^9 Joules
- (4) 6.4×10^{10} Joules

Ans. (4)

Sol.
$$W = 0 - \left(-\frac{GMm}{R} \right) = \frac{GMm}{R}$$

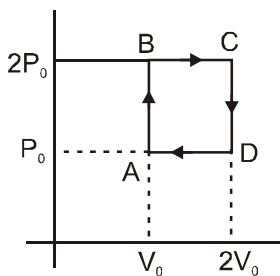
$$= gR^2 \times \frac{m}{R} = mgR$$

$$= 1000 \times 10 \times 6400 \times 10^3$$

$$= 64 \times 10^9 \text{ J}$$

$$= 6.4 \times 10^{10}$$

39. Helium gas goes through a cycle ABCDA (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly :
(Assume the gas to be close to ideal gas)



(1) 15.4%

(2) 9.1%

(3) 10.5%

(4) 12.5%

Ans. (1)

Sol.
$$\eta = \frac{p_0 V_0}{\frac{f}{2}(p_0 V_0) + \frac{f}{2}(2p_0)V_0 + 2p_0 V_0} = \frac{1}{\frac{3}{2} + 3 + 2}$$

$$= \frac{200}{13} = 15.4\%$$

40. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by :

(1) $\frac{I_m}{9}(4 + 5 \cos \phi)$ (2) $\frac{I_m}{3}\left(1 + 2 \cos^2 \frac{\phi}{2}\right)$ (3) $\frac{I_m}{5}\left(1 + 4 \cos^2 \frac{\phi}{2}\right)$ (4) $\frac{I_m}{9}\left(1 + 8 \cos^2 \frac{\phi}{2}\right)$

Ans. (4)

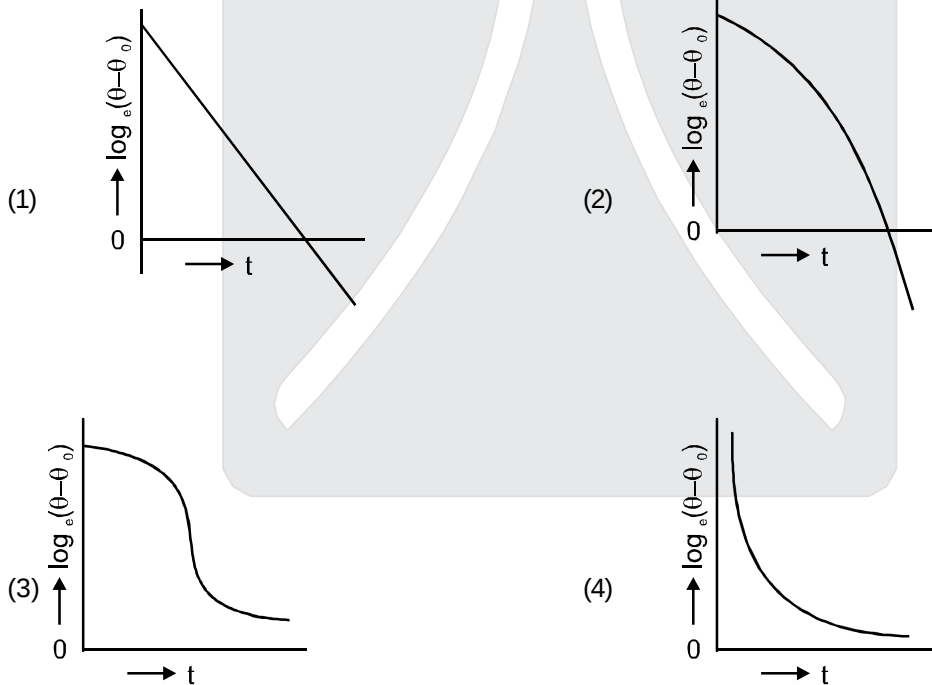
Sol. $I_m = I_0 + 4I_0 + 2\sqrt{I_0 \times 4I_0} \cos \phi$

$I_m = I_0 + 4I_0 + 4I_0 \cos \phi$

$= \frac{I_m}{9}(5 + 4 \cos \phi)$

$= \frac{I_m}{9}\left(1 + 8 \cos^2 \frac{\phi}{2}\right)$

41. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :



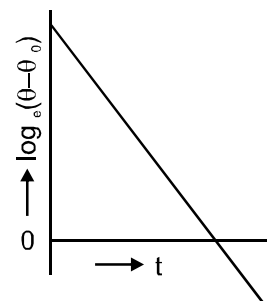
Ans. (1)

Sol. $\frac{d\theta}{dt} = -k(\theta - \theta_0)$

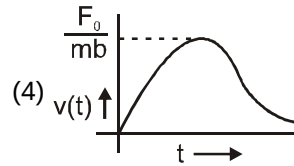
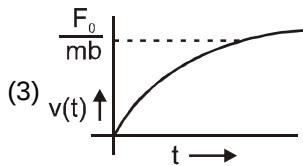
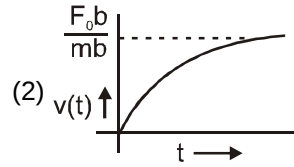
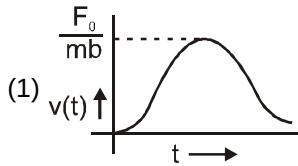
$\int_{\theta_0}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k \int_0^t dt$

$\ln(\theta - \theta_0) = -kt + C$

So graph is straight line.



42. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves ?



Ans. (3)

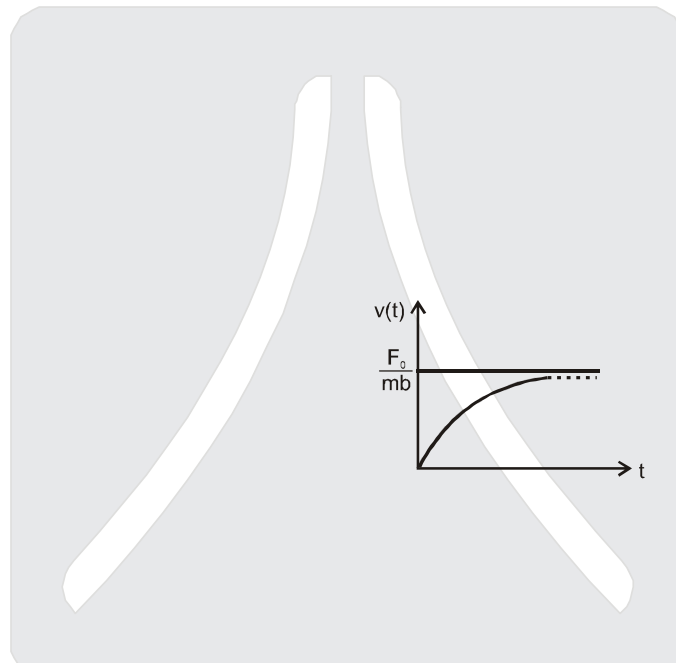
Sol. $F = ma = F_0 e^{-bt}$

$$\frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\int_0^v dv = \frac{F_0}{m} \int_0^t e^{-bt} dt$$

$$v = \frac{F_0}{m} \left[\frac{e^{-bt}}{-b} \right]_0^t$$

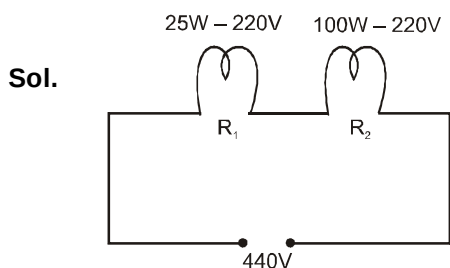
$$v = \frac{F_0}{mb} (1 - e^{-bt})$$



43. Two electric bulbs marked 25W – 220V and 100W – 220 V are connected in series to a 440 V supply. Which of the bulbs will fuse ?

(1) both (2) 100W (3) 25W (4) neither

Ans. (3)



$$\text{As } R_1 = \frac{220}{25} \times 220 \text{ and } R_2 = \frac{220}{100} \times 220$$

$$R = R_1 + R_2$$

$$= 220 \times 220 \left(\frac{1}{25} + \frac{1}{100} \right)$$

$$= 220 \times 220 \frac{1}{20}$$

$$\therefore I_{\text{live}} = \frac{440}{\frac{220 \times 220}{20}} = \frac{40}{220} \text{ A}$$

$\therefore$ 1st bulb (25 W) will fuse only

44. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is :

(1) 6% (2) zero (3) 1% (4) 3%

Ans. (1)

Sol. $R = \frac{V}{I} \Rightarrow \pm \frac{\Delta R}{R} = \pm \frac{\Delta V}{V} \pm \frac{\Delta I}{I}$
 $= 3 + 3 = 6\%$

45. A boy can throw a stone up to a maximum height of 10m. The maximum horizontal distance that the boy can throw the same stone up to will be :

(1) $20\sqrt{2}$ m (2) 10 m (3) $10\sqrt{2}$ m (4) 20m

Ans. (4)

Sol. $h_{\text{max}} = \frac{u^2}{2g} = 10$

$$u^2 = 200$$

$$R_{\text{max}} = \frac{u^2}{g} = 20\text{m}$$

46. This equation has statement 1 and Statement 2. Of the four choices given the Statements, choose the one that describes the two statements.

Statement 1 : Davisson-Germer experiment established the wave nature of electrons.

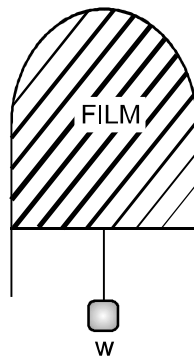
Statement 2 : If electrons have wave nature, they can interfere and show diffraction.

- (1) Statement 1 is false, Statement 2 is true.
 (2) Statement 1 is true, Statement 2 is false
 (3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for statement 1
 (4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

Ans. (3)

Sol. Both are true

47. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is :



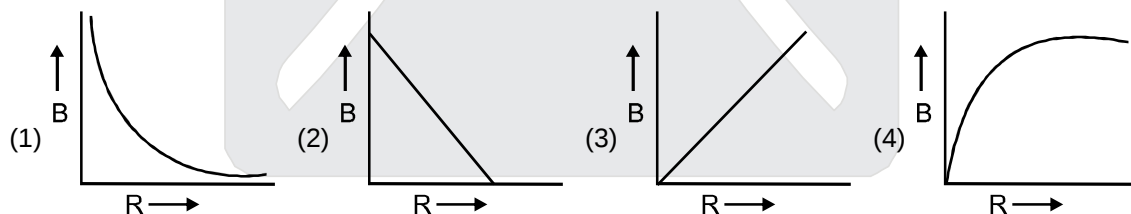
- (1) 0.0125 Nm^{-1} (2) 0.1 Nm^{-1} (3) 0.05 Nm^{-1} (4) 0.025 Nm^{-1}

Ans. (4)

Sol. $2TL = mg$

$$T = \frac{mg}{2L} = \frac{1.5 \times 10^{-2}}{2 \times 30 \times 10^{-2}} = \frac{1.5}{600} = 0.025 \text{ N/m}$$

48. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R . The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω . As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure :



Ans. (1)

Sol.
$$dB = \frac{\mu_0(dq)}{2r} \left(\frac{\omega}{2\pi} \right)$$

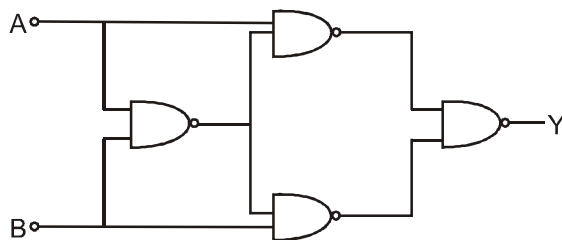
$$B = \int dB = \frac{\mu_0 \omega}{4\pi} \cdot \frac{Q}{\pi R^2} 2\pi \int_0^R \frac{rdr}{r}$$

$$B = \frac{\mu_0 \omega Q}{2\pi R^2} \cdot R$$

$$B = \frac{\mu_0 \omega Q}{2\pi R}$$

$$B \propto \frac{1}{R}$$

49. Truth table for system of four NAND gates as shown in figure is :



(1)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(3)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Ans. (1)

Sol.
$$y = \overline{(\overline{A \cdot A \cdot B})} \cdot \overline{(\overline{B \cdot A \cdot B})}$$

$$= \overline{(\overline{A \cdot A \cdot B})} \cdot \overline{(\overline{B \cdot A \cdot B})}$$

$$= A \cdot (\overline{A + B}) + B \cdot (\overline{A + B})$$

$$= A \cdot \overline{A} + A \cdot \overline{B} + B \cdot \overline{A} + B \cdot \overline{B}$$

$$y = 0 + A \cdot \overline{B} + B \cdot \overline{A} + 0$$

50. A radar has a power of 1kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is :
 (1) 80 km (2) 16 km (3) 40 km (4) 64 km

Ans. (1)

Sol.

$$r = R + h \cong R$$

$$x = \sqrt{(R+h)^2 - R^2}$$

$$= \sqrt{h^2 + 2hR}$$

$$x^2 = 25000 + 2 \cdot 500 \cdot 6.4 \cdot 100000$$

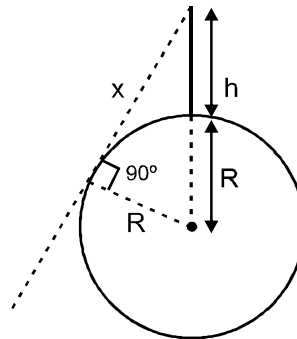
$$= 25000 + (64 \times 100000000)$$

$$= 10^4 (640025)$$

$$x^2 \cong 10^4 \cdot 640000$$

$$x = 8 \times 10^4 \text{ m}$$

$$= 80 \text{ km.}$$



- 51.** Assume that a neutron breaks into a proton and an electron. The energy released during this process is :
(mass of neutron = $1.6725 \times 10^{-27} \text{ kg}$, Mass of proton = $1.6725 \times 10^{-27} \text{ kg}$, mass of electron = $9 \times 10^{-31} \text{ kg}$)

(1) 0.73 MeV

(2) 7.10 MeV

(3) 6.30 MeV

(4) 5.4 MeV

Ans. (1)

Sol. ${}_0^1n^1 \rightarrow {}_1^1H^1 + {}_{-1}^0e^0 + \bar{\nu} + Q$

$$\Delta m = m_n - m_p - m_e$$

$$= (1.6725 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31}) \text{ kg}$$

$$= -9 \times 10^{-31} \text{ kg}$$

$$\text{Energy} = 9 \times 10^{-31} \times (3 \times 10^8)^2$$

$$= 0.511 \text{ MeV}$$

Which is nearly equal to 0.73 MeV

but as energy will be required.

since mass is increasing

so answer = -0.511 MeV

either (1) or bonus.

- 52.** A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :

(1) efficiency of carnot engine cannot be made larger than 50%

(2) 1200 K

(3) 750 K

(4) 600 K

Ans. (3)

Sol. For 1st case

$$\text{efficiency} = \eta = \left(1 - \frac{T_1}{T_2}\right) \times 100$$

$$\left(1 - \frac{T_1}{500}\right) \times 100 = 40$$

$$T_1 = 300 \text{ K}$$

for 2nd case

$$\eta = \left(1 - \frac{300}{T_2}\right) \times 100 = 60$$

$$T_2 = 750 \text{ K}$$

53. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

If two springs S_1 and S_2 of force constants k_1 and k_2 , respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2 .

Statement 1 : If stretched by the same amount, work done on S_1 , will be more than that on S_2

Statement 2 : $k_1 < k_2$

(1) Statement 1 is false, Statement 2 is true.

(2) Statement 1 is true, Statement 2 is false

(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for statement 1

(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

Ans. (1)

Sol. $k_1 x_1 = k_2 x_2 = F$

$$W_1 = \frac{1}{2} k_1 x_1^2 = \frac{(k_1 x_1)^2}{2k_1} = \frac{F^2}{2k_1}$$

$$\text{Similarly } W_2 = \frac{F^2}{2k_2} \Rightarrow W \propto \frac{1}{k}$$

$$W_1 > W_2 \Rightarrow k_1 < k_2 \text{ statement-2 is true.}$$

$$\text{Statement-1 } W_1 = \frac{1}{2} k_1 x^2$$

$$W_2 = \frac{1}{2} k_2 x^2$$

So, $W_2 > W_1$
Statement-1 is false.

54. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 , respectively. Their speeds are such that they make complete circles in the same time t . The ratio of their centripetal acceleration is :

(1) $m_1 r_1 : m_2 r_2$

(2) $m_1 : m_2$

(3) $r_1 : r_2$

(4) $1 : 1$

Ans. (3)

Sol. They have same ω .

centripetal acceleration $= \omega^2 r$

$$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$$

55. A cylindrical tube, open at both ends, has a fundamental frequency, f , in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now :

(1) f

(2) $f/2$

(3) $3f/4$

(4) $2f$

Ans. (1)

Sol. $f = \frac{v}{2\ell}$

now, it will act like one end open and other closed.

$$\text{so, } f_0 = \frac{v}{4\ell'} = \frac{v}{4 \cdot \frac{\ell}{2}} = \frac{v}{2\ell} = f$$

56. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film ?

(1) 7.2 m (2) 2.4 m (3) 3.2 m (4) 5.6 m

Ans. (4)

Sol. $\frac{1}{f} = \frac{1}{12} + \frac{1}{240} = \frac{20+1}{240}$

$$f = \frac{240}{21} \text{ m}$$

$$\text{shift} = 1 \left(1 - \frac{2}{3} \right) = \frac{1}{3}$$

$$\text{Now } v' = 12 - \frac{1}{3} = \frac{35}{3} \text{ cm}$$

$$\therefore \frac{21}{240} = \frac{3}{35} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5} \left(\frac{3}{7} - \frac{21}{48} \right)$$

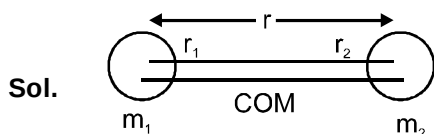
$$\frac{5}{u} = \left| \frac{144 - 147}{48 \times 7} \right|$$

$$u = 560 \text{ cm} = 5.6 \text{ m}$$

57. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r . If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by : (n is an integer)

(1) $\frac{(m_1 + m_2)^2 n^2 h^2}{2m_1^2 m_2^2 r^2}$ (2) $\frac{n^2 h^2}{2(m_1 + m_2)r^2}$ (3) $\frac{2n^2 h^2}{(m_1 + m_2)r^2}$ (4) $\frac{(m_1 + m_2)n^2 h^2}{2m_1 m_2 r^2}$

Ans. (4)



$$m_1 r_1 = m_2 r_2$$

$$r_1 + r_2 = r$$

$$\therefore r_1 = \frac{m_2 r}{m_1 + m_2}$$

$$r_2 = \frac{m_1 r}{m_1 + m_2}$$

$$\therefore \varepsilon = \frac{1}{2} I \omega^2$$

$$= \frac{1}{2} (m_1 r_1^2 + m_2 r_2^2) \cdot \omega^2 \quad \dots\dots\dots(i)$$

$$mvr = \frac{h}{2\pi} = I\omega$$

$$\omega = \frac{nh}{2\pi I}$$

$$\therefore \varepsilon = \frac{1}{2} I \cdot \frac{n^2 h^2}{4\pi^2 I^2}$$

$$= \frac{n^2 h^2}{8\pi^2} \frac{1}{(m_1 r_1^2 + m_2 r_2^2)}$$

$$= \frac{n^2 h^2}{8\pi^2} \frac{1}{m_1 \frac{m_2^2 r_0^2}{(m_1 + m_2)^2}} + m_2 \frac{m_1^2 r^2}{(m_1 + m_2)^2}$$

$$= \frac{n^2 h^2}{8\pi^2 r^2} \frac{(m_1 + m_2)^2}{m_1 m_2 (m_1 + m_2)} = \frac{(m_1 + m_2) n^2 h^2}{8\pi^2 r^2 m_1 m_2}$$

58. A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree

Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data :

(1) 58.59 degree

(2) 58.77 degree

(3) 58.65 degree

(4) 59 degree

Ans. (3)

Sol. 30 V.S.D. $\Rightarrow$ 29 M.S.D.

1 V.S.D. $\rightarrow \frac{29}{30}$ M.S.D.

$$= \frac{29}{30} \times 0.5$$

least count of vernier = 1 M.S.D. – 1 V.S.D.

$$= 0.5^\circ - \frac{29}{30} \times 0.5^\circ$$

$$= \frac{0.5^\circ}{30}$$

Reading of vernier = M.S. reading + V.S reading $\times$ least count

$$= 58.5^\circ + 9 \times \frac{0.5^\circ}{30}$$

$$= 58.65$$

59. This questions has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describe the two statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ . As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinite is zero.

Statement-1 : When a charge ' q ' is take from the centre of the surface of the sphere its potential energy

changes by $\frac{q\rho}{3\epsilon_0}$.

Statement-2 : The electric field at a distance r ($r < R$) from the centre of the sphere is $\frac{\rho r}{3\epsilon_0}$

- (1) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of statement-1.
 (2) Statement 1 is true Statement 2 is false.
 (3) Statement 1 is false Statement 2 is true.
 (4) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

Ans. (3)

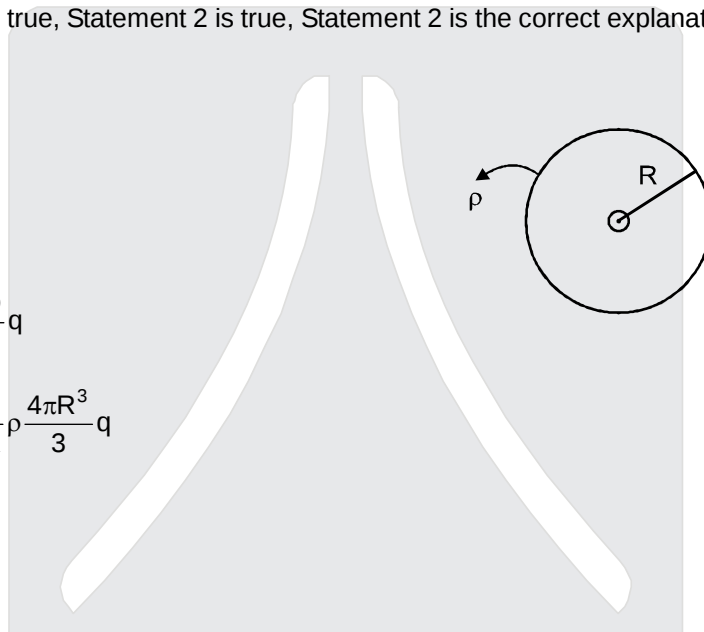
Sol. $U_c = \frac{3 KQ}{2 R} q$

$$U_s = \frac{KQ}{R} q$$

$$\therefore \Delta U = \frac{KQ}{2R} q$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{2R} \rho \frac{4\pi R^3}{3} q$$

$$= \frac{\rho R^2 q}{6\epsilon_0}$$



60. Proton, Deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p , r_d and r_α . Which one of the following relation is **correct**?

- (1) $r_\alpha = r_p = r_d$ (2) $r_\alpha = r_p < r_d$ (3) $r_\alpha > r_d > r_p$ (4) $r_\alpha = r_d > r_p$

Ans. (2)

Sol. $r = \frac{\sqrt{2mE}}{3q}$

$$r \propto \frac{\sqrt{m}}{q}$$

$$r_p = k \frac{\sqrt{m}}{q}$$

$$r_d = k \frac{\sqrt{2m}}{q}$$

$$r_\alpha = k \frac{\sqrt{4m}}{2q} = \frac{k\sqrt{m}}{q}$$

$$\therefore r_p = r_\alpha < r_d.$$

PART-C (CHEMISTRY)

61. Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide?

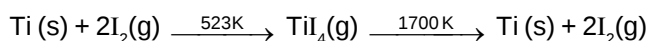
- (1) $[\text{Cr}(\text{en})_3]\text{Br}_3$ (2) $[\text{Cr}(\text{en})_2\text{Br}_2]\text{Br}$ (3) $[\text{Cr}(\text{en})\text{Br}_4]^-$ (4) $[\text{Cr}(\text{en})\text{Br}_2]\text{Br}$

Ans. (2)

Sol. $[\text{Cr}(\text{en})_2\text{Br}_2]\text{Br}$

dibromidobis(ethylenediamine)chromium(III) Bromide.

62. Which method of purification is represented by the following equation :



- (1) Zone refining (2) Cupellation (3) Polling (4) Van Arkel

Ans. (4)

Sol. The process is known as Van Arkel method.

63. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be :

- (1) 75 pm (2) 300 pm (3) 240 pm (4) 152 pm

Ans. (4)

Sol. For BCC structure

$$\sqrt{3} a = 4r$$

$$r = \frac{\sqrt{3}}{4} a = \frac{\sqrt{3}}{4} \times 351 = 152 \text{ pm.}$$

64. The molecule having smallest bond angle is :

- (1) NCl_3 (2) AsCl_3 (3) SbCl_3 (4) PCl_3

Ans. (3)

Sol. As electronegativity of central atom decreases, bond angle decreases.

(Hybridisation and number of lone pair on central atom are same in all options)

65. Which of the following compounds can be detected by Molisch's test :

- (1) Nitro compounds (2) Sugars (3) Amines (4) Primary alcohols

Ans. (2)

Sol. **Molisch's Test** : This is a general test for carbohydrates. One or two drops of alcoholic solution of α -naphthol is added to 2 ml glucose solution. 1 ml of conc. H_2SO_4 solution is added carefully along the sides of the test tube. The formation of a violet ring at the junction of two liquids confirms the presence of a carbohydrate or sugar.

66. The incorrect expression among the following is :

(1) $\frac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$

(2) In isothermal process, $w_{\text{reversible}} = -nRT \ln \frac{V_f}{V_i}$

(3) $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

(4) $K = e^{-\Delta G^\circ/RT}$

Ans. (3)

Sol. $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$$-RT \ln K = \Delta H^\circ - T\Delta S^\circ$$

$$\ln K = -\frac{\Delta H^\circ - T\Delta S^\circ}{RT}$$

67. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is :

- (1) 0.50 M (2) 1.78 M (3) 1.02 M (4) 2.05 M

Ans. (4)

Sol. $\text{Molarity} = \frac{\text{mols of solute}}{\text{volume of sol. (l)}}$

$$= \frac{120 \times 1.15}{60 \times 1120} \times 1000 = 2.05 \text{ M}$$

68. The species which can best serve as an initiator for the cationic polymerization is :

- (1) LiAlH_4 (2) HNO_3 (3) AlCl_3 (4) BaLi

Ans. (3)

Sol. AlCl_3 is a lewis acid and can be used to generate a cation.

69. Which of the following on thermal decomposition yields a basic as well as acidic oxide ?

- (1) NaNO_3 (2) KClO_3 (3) CaCO_3 (4) NH_4NO_3

Ans. (3)

Sol. $\text{CaCO}_3 \xrightarrow{\Delta} \text{CaO} + \text{CO}_2 \uparrow$
 Basic oxide Acidic oxide

70. The standard reduction potentials for Zn^{2+}/Zn , Ni^{2+}/Ni and Fe^{2+}/Fe are -0.76 , -0.23 and -0.44 V respectively. The reaction $\text{X} + \text{Y}^{2+} \rightarrow \text{X}^{2+} + \text{Y}$ will be spontaneous when :

- (1) $\text{X} = \text{Ni}$, $\text{Y} = \text{Fe}$ (2) $\text{X} = \text{Ni}$, $\text{Y} = \text{Zn}$ (3) $\text{X} = \text{Fe}$, $\text{Y} = \text{Zn}$ (4) $\text{X} = \text{Zn}$, $\text{Y} = \text{Ni}$

Ans. (4)

Sol. $\text{X} + \text{Y}^{2+} \rightarrow \text{X}^{2+} + \text{Y}$

For reaction to be spontaneous E° must be positive.

$$E^\circ_{\text{Zn} / \text{Zn}^{2+}} + E^\circ_{\text{Ni}^{2+} / \text{Ni}} = 0.76 + (-0.23) = +0.53 \text{ (positive)}$$

71. According to Freundlich adsorption isotherm which of the following is correct?

- (1) $\frac{x}{m} \propto p^0$ (2) $\frac{x}{m} \propto p^1$
 (3) $\frac{x}{m} \propto p^{1/n}$ (4) All the above are correct for different ranges of pressure

Ans. (4)

Sol. $\frac{x}{m} \propto P^{1/n}$ where $n \geq 1$

72. The equilibrium constant (K_c) for the reaction $\text{N}_2(\text{g}) + \text{O}_2(\text{g}) \rightarrow 2\text{NO}(\text{g})$ at temperature T is 4×10^{-4} . The

value of K_c for the reaction $\text{NO}(\text{g}) \rightarrow \frac{1}{2} \text{N}_2(\text{g}) + \frac{1}{2} \text{O}_2(\text{g})$ at the same temperature is :

- (1) 0.02 (2) 2.5×10^2 (3) 4×10^{-4} (4) 50.0

Ans. (4)

Sol. $\text{N}_2 + \text{O}_2 \rightarrow 2\text{NO} \quad K = 4 \times 10^{-4}$

$$\text{NO} \rightarrow \frac{1}{2} \text{N}_2 + \frac{1}{2} \text{O}_2 \quad K' = \frac{1}{\sqrt{K}} = \frac{1}{\sqrt{4 \times 10^{-4}}} = 50$$

73. The compressibility factor for a real gas at high pressure is :
 (1) $1 + RT/pb$ (2) 1 (3) $1 + pb/RT$ (4) $1 - pb/RT$

Ans. (3)

Sol. $\left(P + \frac{a}{V^2}\right)(V - b) = RT$

at high pressure $\frac{a}{V^2}$ can be neglected

$$PV - Pb = RT$$

$$PV = RT + Pb$$

$$\frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

$$Z = 1 + \frac{Pb}{RT} ; \quad Z > 1 \text{ at high pressure}$$

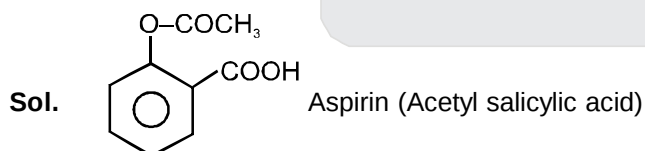
74. Which one of the following statements is correct?
 (1) All amino acids except lysine are optically active
 (2) All amino acids are optically active
 (3) All amino acids except glycine are optically active
 (4) All amino acids except glutamic acids are optically active

Ans. (3)

Sol. Only glycine : $\begin{array}{c} \text{H} \\ | \\ \text{H}-\text{C}-\text{COOH} \\ | \\ \text{NH}_2 \end{array}$ is optically inactive amino acid.

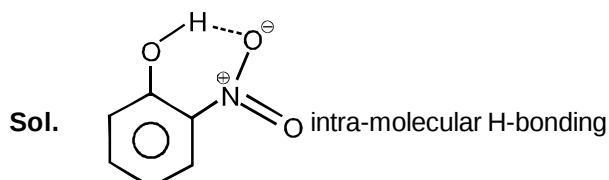
75. Aspirin is known as :
 (1) Acetyl salicylic acid (2) Phenyl salicylate
 (3) Acetyl salicylate (4) Methyl salicylic acid

Ans. (1)



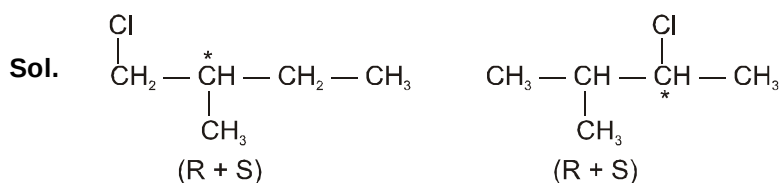
76. Ortho-Nitrophenol is less soluble in water than p- and m- Nitrophenols because :
 (1) o-Nitrophenol is more volatile steam than those of m- and p-isomers.
 (2) o-Nitrophenol shows Intramolecular H-bonding
 (3) o-Nitrophenol shows intermolecular H-bonding
 (4) Melting point of o-Nitrophenol is lower than those of m- and p-isomers.

Ans. (2)



77. How many chiral compounds are possible on monochlorination of 2- methyl butane ?
 (1) 8 (2) 2 (3) 4 (4) 6

Ans. (3)

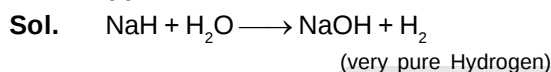


Four monochloro derivatives are chiral.

78. Very pure hydrogen (99.9) can be made by which of the following processes ?

- | | |
|------------------------------------|--|
| (1) Reaction of methane with steam | (2) Mixing natural hydrocarbons of high molecular weight |
| (3) Electrolysis of water | (4) Reaction of salts like hydrides with water |

Ans. (4)



79. The electrons identified by quantum numbers n and ℓ :

- | | |
|-----------------------|-----------------------|
| (a) $n = 4, \ell = 1$ | (b) $n = 4, \ell = 0$ |
| (c) $n = 3, \ell = 2$ | (d) $n = 3, \ell = 1$ |

can be placed in order of increasing energy as :

- | | |
|-----------------------------|-----------------------------|
| (1) $(c) < (d) < (b) < (a)$ | (2) $(d) < (b) < (c) < (a)$ |
| (3) $(b) < (d) < (a) < (c)$ | (4) $(a) < (c) < (b) < (d)$ |

Ans. (2)

Sol. (a) 4 p (b) 4 s (c) 3 d (d) 3 p
Acc. to $(n + \ell)$ rule, increasing order of energy $(d) < (b) < (c) < (a)$

80. For a first order reaction $(A) \rightarrow \text{products}$ the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is :

- | | |
|---|---|
| (1) $1.73 \times 10^{-5} \text{ M/min}$ | (2) $3.47 \times 10^{-4} \text{ M/min}$ |
| (3) $3.47 \times 10^{-5} \text{ M/min}$ | (4) $1.73 \times 10^{-4} \text{ M/min}$ |

Ans. (2)

Sol. $k = \frac{1}{40} \ln \frac{0.1}{0.025} = \frac{1}{40} \ln 4$

$$R = k[A]^1 = \frac{1}{40} \ln 4(.01) = \frac{2 \ln 2}{40} (.01) = 3.47 \times 10^{-4}$$

81. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect ?

- (1) Ferrous oxide is more basic in nature than the ferric oxide.
- (2) Ferrous compounds are relatively more ionic than the corresponding ferric compounds
- (3) Ferrous compounds are less volatile than the corresponding ferric compounds
- (4) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

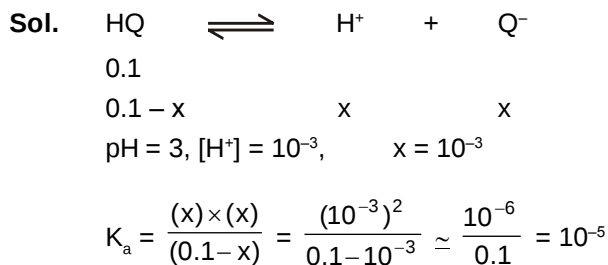
Ans. (4)

Sol. Fe^{3+} is easily hydrolysed than Fe^{2+} due to more positive charge.

82. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, K_a of the acid is :

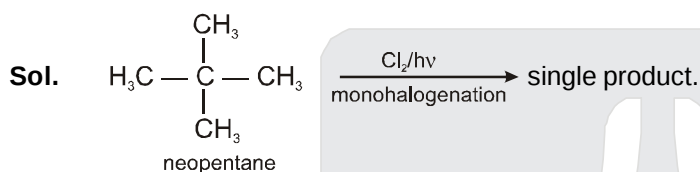
- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| (1) 3×10^{-1} | (2) 1×10^{-3} | (3) 1×10^{-5} | (4) 1×10^{-7} |
|------------------------|------------------------|------------------------|------------------------|

Ans. (3)



- 83.** Which branched chain isomer of the hydrocarbon with molecular mass 72u gives only one isomer of mono substituted alkyl halide ?
 (1) Tertiary butyl chloride (2) Neopentane
 (3) Isohexane (4) Neohexane

Ans. (2)



- 84.** K_f for water is $1.86 \text{ K kg mol}^{-1}$. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($\text{C}_2\text{H}_6\text{O}_2$) must you add to get the freezing point of the solution lowered to -2.8°C ?
 (1) 72 g (2) 93 g (3) 39 g (4) 27 g

Ans. (2)

Sol. $\Delta T_f = i \times k_f \times m$

$$2.8 = 1 \times 1.86 \times \frac{x}{62 \times 1}$$

$$x = \frac{2.8 \times 62}{1.86} = 93 \text{ gm}$$

- 85.** What is DDT among the following :
 (1) Greenhouse gas (2) A fertilizer
 (3) Biodegradable pollutant (4) Non-biodegradable pollutant

Ans. (4)

Sol. DDT is a non-biodegradable pollutant.

- 86.** The increasing order of the ionic radii of the given isoelectronic species is :
 (1) Cl^- , Ca^{2+} , K^+ , S^{2-} (2) S^{2-} , Cl^- , Ca^{2+} , K^+ (3) Ca^{2+} , K^+ , Cl^- , S^{2-} (4) K^+ , S^{2-} , Ca^{2+} , Cl^-

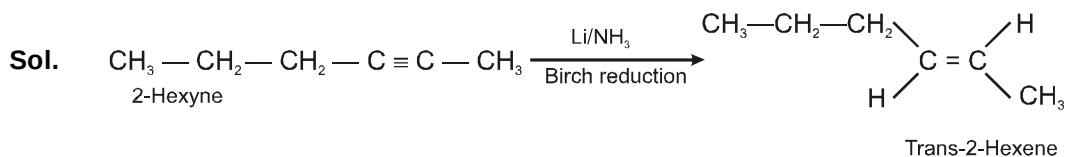
Ans. (3)

Sol. Order of ionic radii $\text{Ca}^{2+} < \text{K}^+ < \text{Cl}^- < \text{S}^{2-}$

in isoelectronic species as $\frac{Z}{e}$ increases size decreases.

- 87.** 2-Hexyne gives trans -2-Hexene on treatment with :
 (1) Pt/H_2 (2) Li / NH_3 (3) Pd/BaSO_4 (4) LiAlH_4

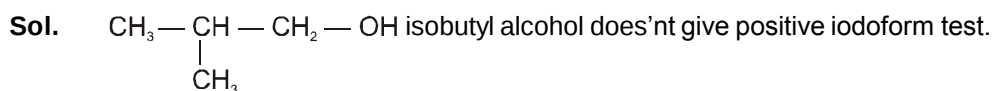
Ans. (2)



88. Iodoform can be prepared from all except :

- (1) Ethyl methyl ketone (2) Isopropyl alcohol
(3) 3-Methyl-2-butanone (4) Isobutyl alcohol

Ans. (4)



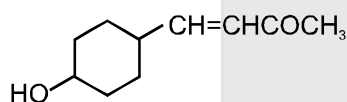
89. In which of the following pairs the two species are not isostructural ?

- (1) CO_3^{2-} and NO_3^- (2) PCl_4^+ and SiCl_4 (3) PF_5 and BrF_5 (4) AlF_6^{3-} and SF_6

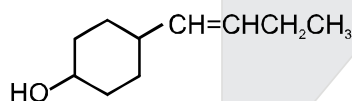
Ans. (3)

Sol. PF_5 trigonal bipyramidal
 BrF_5 square pyramidal (distorted)

90. In the given transformation, which the following is the most appropriate reagent ?

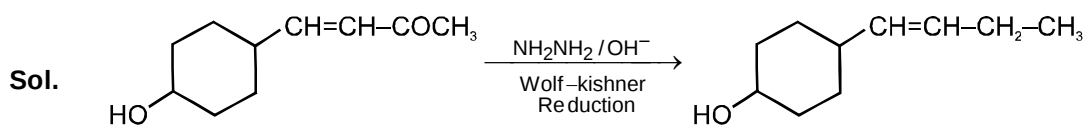


Reagent $\rightarrow$



- (1) $\text{NH}_2\text{NH}_2, \text{OH}^-$ (2) Zn-Hg/HCl (3) Na, Liq, NH_3 (4) NaBH_4

Ans. (1)



$-\text{OH}$ group and alkene are acid-sensitive groups so clemmensen reduction can not be used.