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AIEEE- 2012 Physics

PART A - PHYSICS

1. Two electric bulbs marked 25 W – 220 V and 100 W – 220 V are connected in series to a 440 V supply.
(1) 100 W (2) 25 W (3) neither (4) both

1. **(2)** $R_1 = \frac{(220)^2}{25}$ $R_2 = \frac{(220)^2}{100}$ $i = \frac{440}{(220)^2\left(\frac{1}{25}+\frac{1}{100}\right)} = \frac{2}{220}.\frac{100}{5} = \frac{2}{11}A$

$\therefore$ $P_1 = \left(\frac{2}{11}\right)^2.\frac{(220)^2}{25} = 64\ W > 25\ W$ $P_2 = \left(\frac{2}{11}\right)^2.\frac{(220)^2}{100} = 16\ W$

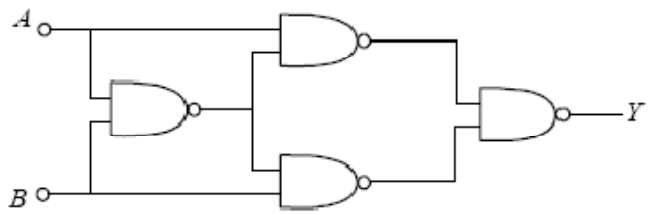
$\therefore$ Bulb of 25 W – 220 V will fuse.

2. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be :
(1) 10 m (2) $10\sqrt{2}$ m (3) 20 m (4) $20\sqrt{2}$ m

2. **(3)** $u^2 = 2.10.10 \Rightarrow u = 10\sqrt{2}$

$\therefore$ $R = \frac{u^2\sin(2\times 45^\circ)}{g} = \frac{100\times 2}{10} = 20\text{m}$

3. Truth table for system of four NAND gates as shown in figure is :



(1)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

3. **(3)** $Y = \overline{\{\overline{A}.(A+B)\}+\{\overline{B}.(A+B)\}} = A.B+\overline{A}.\overline{B}$

A	B	$\overline{A}$	$\overline{B}$	Y
0	0	1	1	1
0	1	0	1	0
1	0	0	1	0
1	1	0	0	1

4. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement – 1 : Davisson – Germer experiment established the wave nature of electrons.

Statement – 2 : If electrons have wave nature, they can interfere and show diffraction.

(1) Statement 1 is true, Statement 2 is false.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

(4) Statement 1 is false, Statement 2 is true.

4. **(2)** C. J. Davisson and L. H. Germer tested the wave nature of electron. If it is wave then it should show diffraction and interference.

5. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I wahen they interefere at phase difference ϕ is given by:

(1) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$ (2) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$

(3) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$ (3) $\frac{I_m}{9}(4+5\cos\phi)$

5. **(2)** $I \propto A^2$, $I_1 = I_0$, $I_2 = 4I_0$
$I = I_0 + 4I_0 + 2.\sqrt{I_0}\ \sqrt{(4I_0)}\cos\phi$
$= I_0[1 + 4\cos^2(\phi/2)]$ $= (I_m/5)[1 + 4\cos^2(\phi/2)]$

6. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between $t = 0$s to $t = \tau$s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds:
(1) b (2) $(1/b)$ (3) $(2/b)$ (4) $(0.693/b)$

6. **(2)** $m(dv/dt) = -Kx - bv$, $\Rightarrow A = A_0 e^{-bt}$ ...(1)
In case of damping.
Given, $A = A_0 e^{-t/\tau}$ ...(2)
Comparing (**1**) & (2)
Average Life = $(1/b)$

7. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are streched by the same force, it is found that more work is done on spring S_1 and on spring S_2.
Statement – 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.
Statement – 2 : $k_1 < k_2$.
(1) Statement 1 is true, Statement 2 is false.
(2) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.
(4) Statement 1 is false, Statement 2 is true.

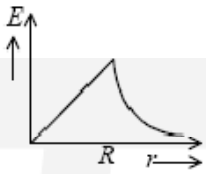
7. **(4)** $x_1 = (F/K_1)$, $x_2 = (F/K_2)$ $W_1 = (1/2)K_1x_1^2 = (F^2/2K_1)$
$W_2 = [F^2/2K_2]$
$\Rightarrow$ Statement - 1 :- $W_1 = (1/2)K_1x^2$
$W_2 = (1/2)K_2x^2$ (False)
Statement - 2 :- True $\because$ $W_1 > W_2$ $\Rightarrow K_1 < K_2$

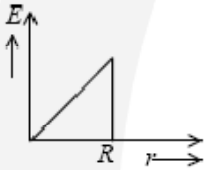
8. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film?
(1) 2.4 m (2) 3.2 m (3) 5.6 m (4) 7.2 m

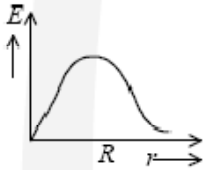
8. **(3)** $\Delta t = [1 - (1 / 15)] \times 1 = (1 / 3)$ cm $(1 / v) - (1 / u) = (1 / f) \Rightarrow (1 / 12) - (1 / -240) = (1 / f)$
$\Rightarrow f = (240 / 21)$ cm Again $(1 / v) - (1 / u) = (1 / f) \quad \Rightarrow (3 / 35) - (1 / -u) = (21 / 240)$
$(1 / u) = (3 / 35) - (21 / 240)$
$u = [(35 \times 240) / (3 \times 240 - 21 \times 35)] = [8400 / (720 - 735)] = (8400 / -15)$
$= -560$ cm $= -5.6$ m.

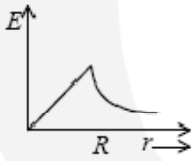
9. In a uniformly chared sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be:

(1) 

(2) 

(3) 

(4) 

9. **(2)** 

$E = (KQr / R^3) \quad (r \le R)$
$E = (KQ / r^2) \quad (r \ge R)$

10. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to:
(1) Induction of electrical charge on the plate.
(2) Shielding of magnetic lines of force as aluminium is a paramagnetic material.
(3) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.
(4) Development of air current when the plate is placed.

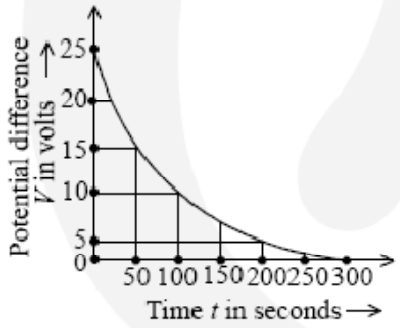
10. **(3)** According to Lenz's Law, eddy current is developed on plate.

11. A spectrometer gives the following reading when used to measure the angle of prism.
Main scale read : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data :
(1) 58.77 degree (2) 58.65 degree (3) 59 degree (4) 58.59 degree

11. **(2)** $1VSD = \frac{29 \times 0.5^\circ}{30}$, 1 MSD = 0.5°
L.C. = 1 MSD – 1 VSD = (0.5° / 30) = (**1** / 60)°
∴ reading MSR + n × L.C = [58.5 + (9 ×1 / 60)] = 58.65

12. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by: (n is an integer)
(1) $\frac{n^2h^2}{2(m_1+m_2)r^2}$ (2) $\frac{2n^2h^2}{(m_1+m_2)r^2}$ (3) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (4) $\frac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$

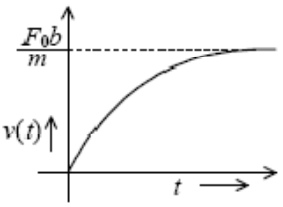
12. **(3)** $E = (1/2)\,\mu r^2 . \omega^2$ ..... (1)
and $L = \mu\omega r^2 = nh$ ..... (2) where $\mu = [m_1m_2 / (m_1+m_2)]$
using (1) & (2) $E = \frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ where $\boldsymbol{h}$ stands for $(\boldsymbol{h} / 2\pi)$

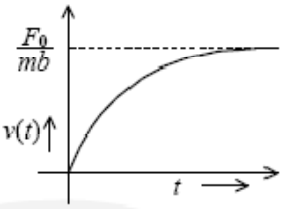
13.


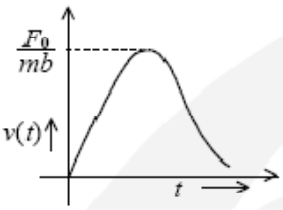
The figure shows an experimental plot for discharging of a capacitor in an R–C circuit. The time constant τ of this circuit lies between :
(1) 0 and 50 sec (2) 50 sec and 100 sec
(3) 100 sec and 150 sec (4) 150 sec and 200 sec

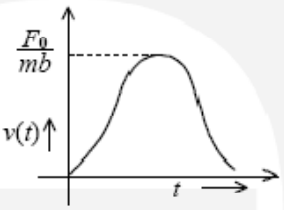
13. **(3)** $V = V_0e^{-t/\tau} = 25\,e^{-t/\tau}$ for $v = 12.5$, $t = 75$
$\Rightarrow (t/\tau) = \ln 2 \quad \Rightarrow t = (t / \ln 2) = (75 / 0.693) \approx 110$

14. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x direction. Its speed $v(t)$ is depicted by which of the following curves ?

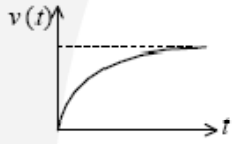
(1) 

(2) 

(3) 

(4) 

14. **(2)** $a(t) = [F(t) / m] = [F_0 / m)\, e^{-bt}$
$\Rightarrow \quad V(t) = V_0 + \int_0^t a(t)\, dt \;=\; (F_0 / m) \int_0^t e^{-bt} = (F_0 / mb)\,(1 - e^{-bt})$
where F_0 / mb is the peak value.



15. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respecitively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is
(1) $m_1 : m_2$ (2) $r_1 : r_2$ (3) 1 : 1 (4) $m_1 r_1 : m_2 r_2$

15. **(2)** $(a_1 / a_2) = (\omega^2 r_1 / \omega^2 r_2)$
$\because \quad \omega = (2\pi / t)$ (same for both)

16. A radar has a power of 1 kW and is operating at a fequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is:
(1) 16 km (2) 40 km (3) 64 km (4) 80 km

16. **(4)** $OA = \sqrt{(OO_1)^2 - (O_1A)^2}$
$= \sqrt{(R+h)^2 - R^2}$
$= 8 \times 10^4$ m = 80 km

17. Assume that neutron breaks into a proton and an electron. The energy released during this process is:
(Mass of neutron = 1.6725×10^{-27} kg
Mass of proton = 1.6725×10^{-27} kg
Mass of electron = 9×10^{-31} kg)
(1) 7.10 MeV (2) 6.30 MeV (3) 5.4 MeV (4) 0.73 MeV

17. **(No Option is correct)**
From data $m_n = m_p$. So reaction $n \rightarrow p + e$ is not possible without energy input.

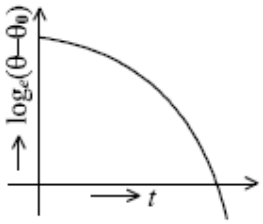
18. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinity is zero.
Statement – 1 : When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy charges by $(q\rho / 3\varepsilon_0)$.
Statement – 2 : The electric field at a distance r ($r < R$) from the centre of the sphere is $(qr / 3\varepsilon_0)$.
(1) Statement 1 is true, Statement 2 is false.
(2) Statement 1 is false, Statement 2 is true.
(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1.
(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

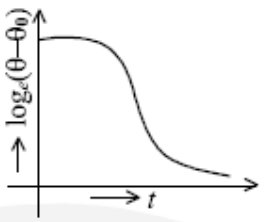
18. **(2)** $\Delta V = (q\rho / 3\varepsilon_0)$ (dimensionally wrong)

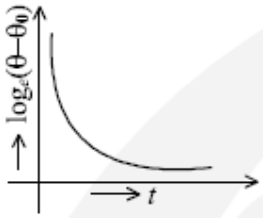
from gauss therom $E(4\pi r^2) = \dfrac{\rho(\frac{4}{3}\pi r^3)}{\varepsilon_0}$

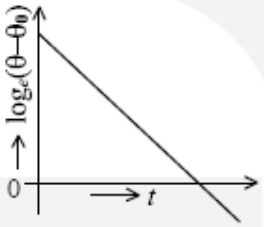
$\Rightarrow \quad E = (\rho r / 3\varepsilon_0)$

19. A liquid in a breaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is:

(1) 

(2) 

(3) 

(4) 

19. **(4)** $-(d\theta / dt) = k(\theta - \theta_0)$

$$\Rightarrow \quad \int \frac{d\theta}{\theta - \theta_0} = \int -kdt$$

$$\Rightarrow \quad \log_e(\theta - \theta_0) = -kt + C$$

20. Resistance of given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is:
(1) Zero (2) 1% (3) 3% (4) 6%

20. **(4)** $R = (V / I) \Rightarrow (dR / R) = [(dV / V) + (dI / I)] = (3 + 3)\% = 6\%$

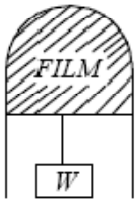
21. The mass of a spaceship is 100 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 ms^{-2} and 6400 km respectively. The required energy for this work will be:
(1) 6.4×10^8 Joules (2) 6.4×10^9 Joules
(3) 6.4×10^{10} Joules (4) 6.4×10^{11} Joules

21. **(3)** $E = mgR = 6.4 \times 10^{10}$ Joules

22. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now
(1) $(f / 2)$ (2) $(3f / 4)$ (3) $2f$ (4) f

22. **(4)** Tube open at both ends : $(\lambda / 2) = L$, $\lambda = 2L = (C / f)$
Tube portion dipped in water : $(\lambda' / 4) = L / 2$, $\lambda' = 2L = (C / f')$
$\therefore \quad f' = f$

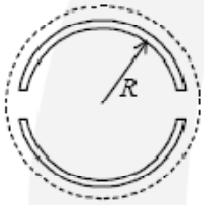
23. A thin liquid film formed between *U*–shaped wire and a light slide supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is:



(1) $0.1\ Nm^{-1}$ (2) $0.05\ Nm^{-1}$ (3) $0.025\ Nm^{-1}$ (**4**) $0.0125\ Nm^{-1}$

23. **(3)** $2.S.L = W$
$S = (W / 2L) = [(1.5 \times 10^{-2}N) / (2 \times 0.3\ m)] = 0.025\ Nm^{-1}$

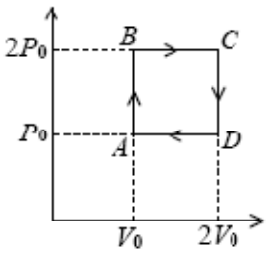
24. A wooden wheel of radius *R* is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area *S* and length *L*. *L* is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Youngs' modulus is *Y*, the force that one part of the wheel applies on the other part is:



(1) $SY\alpha\Delta T$ (2) $\pi\ SY\alpha\Delta T$ (3) $2SY\alpha\Delta T$ (**4**) $2\ \pi\ SY\alpha\Delta T$

24. **(1)** Thermal stress = $Y . (\Delta L / L) = Y\alpha\Delta\theta$
Force = $Y\alpha\Delta\theta . S$

25. Helium gas goes through a cycle *ABCDA* (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly:
(Assume the gas to be close to ideal gas)



(1) 9.1% (2) 10.5% (3) 12.5% (4) 15.4%

25. **(4)** efficiency, $\eta = (\Delta W / \Delta Q_{+ve})$
= [Area under pv–diagram / $(\Delta Q_{AB} + \Delta Q_{BC})$]

$$\eta = \frac{P_0V_0}{\frac{3}{2}nR(T_B - T_A) + 4P_0V_0} = \frac{P_0V_0}{\frac{3}{2}P_0V + 4P_0V_0}$$

$= (1 / 6.5) \approx 15.4\%$

26. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be:
(1) 3 (2) 5 (3) 6 (4) 2

26. **(3)** No. of spectral line in emission spectra = $[n(n-1)/2] = 6$

27. Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct ?
(1) $r_\alpha = r_p < r_d$ (2) $r_\alpha > r_d > r_p$ (3) $r_\alpha = r_d > r_p$ (4) $r_\alpha = r_p = r_d$

27. **(1)** $r = (mv / qB) = \dfrac{\sqrt{2km}}{qB}$

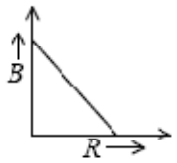
$\therefore \quad r \propto (\sqrt{m} / q)$

$m_\alpha = 4\ m_p$ $\qquad q_\alpha = 2\ q_p$

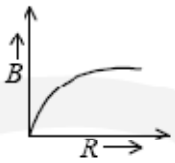
$m_d = 2\ m_p$ $\qquad q_d = q_p$

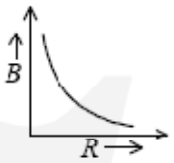
$\Rightarrow \quad r_\alpha = r_p < r_d$

28. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotatioin a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure.

(1) 

(2) 

(3) 

(4) 

28. **(4)** 

$$B = \int_0^R \frac{\mu_0}{2x} \cdot \frac{\frac{Q}{\pi R^2} \cdot 2\pi x dx}{T} = \frac{\mu_0 Q}{R^2} \cdot \frac{R}{T} = \frac{\mu_0 Q}{RT} = \frac{\mu_0 Q \omega}{2\pi R}$$

29. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarizatioin is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then :

(1) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

(2) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{E} \times \vec{B}$

(3) $\vec{X} \parallel \vec{E}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

(4) $\vec{X} \parallel \vec{B}$ and $\vec{k} \parallel \vec{B} \times \vec{E}$

29. **(1)** e.m. wave are polarized along $\vec{E}$ – vector
and, wave velocity is along $\vec{E} \times \vec{B}$ vector.

30. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :

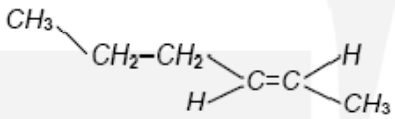
(1) 1200 K
(2) 750 K
(3) 600 K
(4) Efficiency of Carnot engine cannot be made larger than 50%

30. **(2)** $\eta = [1 - (T_{sink} / T_{source})]$
$\therefore \quad 0.4 = [1 - (T_{sink} / 500)] \quad \Rightarrow \quad T_{sink} = 300\ k$
Also, $0.6 = [1 - (300 / T_{source})] \quad \Rightarrow \quad T_{source} = 750\ k$

AIEEE- 2012 Chemistry

PART B - CHEMISTRY

31. 2- Hexyne gives trans -2- Hexene on treatment with :
(1) Li / NH_3 (2) $Pd / BaSO_4$ (3) $LiAlH_4$ (4) Pt / H_2

31. **(1)** $CH_3-CH_2-CH_2-C\equiv C-CH_3 \xrightarrow{Li / NH_3}$ 

32. Which of the following on thermal decomposition yields a basic as well as an acidic oxide ?
(1) $KClO_3$ (2) $CaCO_3$ (3) NH_4NO_3 (4) $NaNO_3$

32. **(2)** $CaCO_3 \xrightarrow{\Delta} \underset{\text{(Basic)}}{CaO} + \underset{\text{(acidic)}}{CO_2}$

33. Which one of the following statements is correct ?
(1) All amino acids are optically active
(2) All amino acids except glycine are optically active
(3) All amino acids except glutamic acid are optically active
(4) All amino acids except lysine are optically active

33. **(2)** Structure of glycine is $\begin{matrix} CH_2-COOH \\ | \\ NH_2 \end{matrix}$

34. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g / mL. The molarity of this solution is :
(1) 1.78 M (2) 1.02 M (3) 2.05 M (4) 0.50

34. **(3)** Mass of solution = 1000 + 120 = 1120 g
Volume of solution = (1120 / 1.15) g / cc

$\therefore \quad M = \frac{120 \times 1.15 \times 1000}{60 \times 1120} = 2.05(M)$

35. The incorrect expression among the following is :

(1) In isothermal process,

$$W_{\text{reversible}} = -nRT\,1n\frac{V_f}{V_i}$$

(2) $1nK = \frac{\Delta H° - T\Delta S°}{RT}$

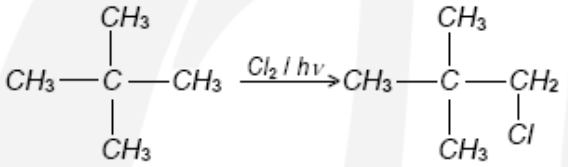
(3) $K = e^{-\Delta G°/RT}$

(4) $\frac{\Delta G_{\text{system}}}{\Delta S_{\text{total}}} = -T$

35. **(2)** $\Delta G° = -RT \ln K$

36. Which branched chain isomer of the hydrocarbon with molecular mass 72 u gives only one isomer of mono substituted alkyl halide ?

(1) Neopentane (2) Isohexane (3) Neohexane (4) Tertiary butyl chloride

36. **(1)**  (only one isomer)

37. According to Freundlich adsorption isotherm, which of the following is correct ?

(1) $\frac{x}{m} \propto p^1$

(2) $\frac{x}{m} \propto p^{1/n}$

(3) $\frac{x}{m} \propto p^0$

(4) All the above are correct for different ranges of pressure.

37. **(4)** At low pressure $\frac{x}{m} \propto p^1$

At moderate pressure $\frac{x}{m} \propto p^{1/n}$

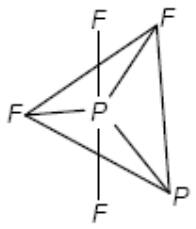
At high pressure $\frac{x}{m} \propto p^0$ (i.e. independent of pressure)

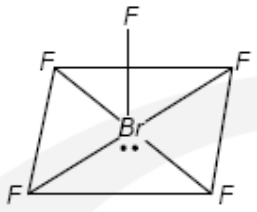
38. In which of the following pairs the two species are not isostructural ?

(1) PCl_4^+ and $SiCl_4$ (2) PF_5 and BrF_5

(3) AlF_6^{3-} and SF_6 (4) CO_3^{2-} and NO_3^-

38. **(2)**

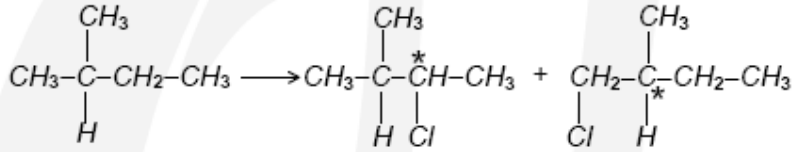




39. How many chiral compounds are possible on monochlorination of 2-methyl butane ?

(1) 2 (2) 4 (3) 6 (4) 8

39. **(2)**



and their enantiomers

40. The increasing order of the ionic radii of the given isoelectronic species is :

(1) S^{2-} , Cl^-, Ca^{2+}, K^+ (2) Ca^{2+}, K^+, Cl^-, S^{2-}

(3) K^+, S^{2-}, Ca^{2+}, Cl^- (4) Cl^-, Ca^{2+}, K^+, S^{2-}

40. **(2)** Greater the positive charge, smaller is the size of the ion. Greater the negative charge larger is the size of the ion for isoelectronic species.

41. The compressibility factor for a real gas at high pressure is :

(1) 1 (2) $1 + pb / RT$ (3) $1 - pb / RT$ (4) $1 + RT / pb$

41. **(2)** $\left(P+\frac{a}{V^2}\right)(v-b)=RT$

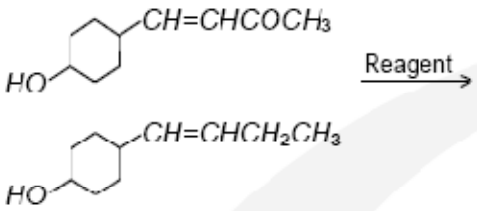
$P(V-b)=RT$ $\frac{a}{V^2}$ can be neglected

$\therefore \quad \frac{PV}{RT} = 1 + \frac{Pb}{RT}$

42. Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide ?
(1) $[Cr(en)_2Br_2]Br$ (2) $[Cr(en)Br_4]^-$ (3) $[Cr(en)Br_2]Br$ (4) $[Cr(en)_3]Br_3$

42 **(1)** As per the IUPAC convention

43. In the given transformation, the following is the most appropriate reagent ?



(1) $Zn - Hg / HCl$ (2) Na, Liq. NH_3 (3) $NaBH_4$ (4) $NH_2NH_2, \overset{\ominus}{O}H$

43. **(4)** It should be basic reagent.

44. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be :
(1) 300 pm (2) 240 pm (3) 152 pm (4) 75 pm

44. **(3)** $4r = \sqrt{3} \times a$

$$r = \frac{\sqrt{3} \times 351}{4} = 152\text{pm}$$

45. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to – 2.8° C ?
(1) 93 g (2) 39 g (3) 27 g (4) 72 g

45. **(1)** $2.8 = \frac{1.86 \times w}{62}$

46. The molecule having smallest bond angle is :
(1) $AsCl_3$ (2) $SbCl_3$ (3) PCl_3 (4) NCl_3

46. **(2)** Lower is the electronegativity of the central atom smaller is the bond angle.

47. What is DDT among the following :
(1) A fertilizer (2) Biodegradable pollutant
(3) Non-biodegradable pollutant (4) Greenhouse gas

47. **(3)** DDT (dichloro diphenyl trichloro ethane) is banned because it is a non-biodegradable pollutant.

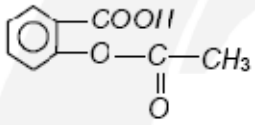
48. The *pH* of a 0.1 molar solution of the acid *HQ* is 3. The value of the ionization constant, *Ka* of this acid is :
(1) 1×10^{-3} (2) 1×10^{-5} (3) 1×10^{-7} (4) 3×10^{-1}

48. **(2)** $C\alpha = 10^{-3}$ $\therefore \alpha = 10^{-2}$
$k = C\alpha^2 = 0.1 \times (10^{-2})^2 = 1 \times 10^{-5}$

49. Very pure hydrogen (99.9%) can be made by which of the following processes ?
(1) Mixing natural hydrocarbons of high molecular weight
(2) Electrolysis of water
(3) Reaction of salt like hydrides with water
(4) Reaction of methane with steam

49. **(2)** Very pure hydrogen (99.9%) can be made by electrolysis of water.

50. Aspirin is known as :
(1) Phenyl salicylate (2) Acetyl salicylate
(3) Methyl salicylic acid (4) Acetyl salicylic acid

50. **(4)** 

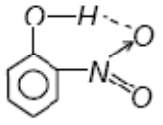
51. Which of the following compounds can be detected by Molisch's test ?
(1) Sugars (2) Amines
(3) Primary alcohols (4) Nitro compounds

51. **(1)** Molisch's test is used for the detection of carbohydrates. In which a small amount of α - napthol is added to the compound and then conc. sulphuric acid is dropped carefully, if the compound is carbohydrate, then a violet ring is obtained at the junction of the two liquids.

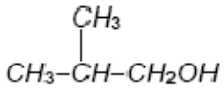
52. The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are –0.76, – 0.23 and –0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when :
(1) $X = Ni$, $Y = Zn$ (2) $X = Fe$, $Y = Zn$ (3) $X = Zn$, $Y = Ni$ (4) $X = Ni$, $Y = Fe$

52. **(3)** E_{cell} should be positive. Zinc will have higher tendency to get oxidised and *Ni* have higher tendency to get reduced.

53. Ortho-Nitrophenol is less soluble in water than *p*–and *m*–Nitrophenols because :
(1) *o*–Nitrophenol shows Intramolecular *H*–bonding.
(2) *o*–Nitrophenol shows Intermolecular *H*–bonding.
(3) Melting point of *o*–Nitrophenol is lower than those of *m*–and *p*–isomers.
(4) *o*–Nitrophenol is more volatile in steam than those of *m*– and *p*–isomers.

53. **(1)** 

54. Iodoform can be prepared from all except :
(1) Isopropyl alcohol (2) 3–Methyl–2–butanone
(3) Isobutyl alcohol (4) Ethyl methyl ketone

54. **(3)** 

It is primary alcohol. Ethyl alcohol among 1° alcohols gives chloroform reaction.

55. The species which can best serve as an initiator for the cationic polymerization is :
(1) HNO_3 (2) $AlCl_3$ (3) $BuLi$ (4) $LiAlH_4$

55. **(2)** The initiator for a cationic polymerization should be a Lewis acid like $AlCl_3$ or BF_3

56. The equilibrium constant (K_c) for the reaction $N_2(g) + O_2(g) - 2NO(g)$ at temperature T is 4×10^{-4}. The value of K_c for the reaction, $NO(g) \rightarrow 1/2N_2(g) + 1/2O_2(g)$ at the same temperature is:
(1) 2.5×10^2 (2) 4×10^{-4} (3) 50.0 (4) 0.02

56. **(3)** $\sqrt{\frac{1}{k}} = \frac{1}{\sqrt{4 \times 10^{-4}}} = 50$

57. For a first order reaction, (A) → products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is :
(1) 3.47×10^{-4} M/min (2) 3.47×10^{-5} M/min
(3) 1.73×10^{-4} M/min (4) 1.73×10^{-5} M/min

57. **(1)** $k = \frac{2.3}{40} 2\ln 2 = \frac{0.693}{20}$

$R = \frac{0.693}{20}[.01] = 3.47 \times 10^{-4}$

58. Which method of purification is represented by the following equation :

$Ti(s) + 2I_2(g) \xrightarrow{523K} TiI_4(g) \xrightarrow{1700K} Ti(s) + 2I_2(g)$

(1) Cupellation (2) Poling (3) Van Arkel (4) Zone refining

58. **(3)** It is Van Arkel process.

59. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is incorrect?
(1) Ferrous compounds are relatively more ionic than the corresponding ferric compounds
(2) Ferrous compounds are less volatile than the corresponding ferric compounds
(3) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds
(4) Ferrous oxide is more basic in nature than the ferric oxide.

59. **(3)** Lower the oxidation state, more is the ionic character and more is the basic character.

60. The electrons identified by quantum numbers n and l :

(a) $n = 4, l = 1$ (b) $n = 4, l = 0$ (c) $n = 3, l = 2$ (d) $n = 3, l = 1$

Can be placed in order of increasing energy as :

(1) (d) < (b) < (c) < (a) (2) (b) < (d) < (a) < (c)

(3) (a) < (c) < (b) < (d) (4) (c) < (d) < (b) < (a)

60. **(1)** Based on Aufbau's principle.

AIEEE- 2012 Mathematics

PART C - MATHEMATICS

61. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is

(1) 3^5 (2) 2^5 (3) 5^3 (4) 5^2

61. **(1)** Ordered pair of $(Y, Z) = \sum_{r=0}^{5} {}^5C_r \cdot 2^{5-r} = 3^5$.

62. The population $p(t)$ at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is:

(1) $\ln 9$ (2) $(1/2) \ln 18$ (3) $\ln 18$ (4) $2 \ln 18$

62. **(4)** $\frac{dp(t)}{dt} = 0.5p(t) - 450$

$$\int \frac{dp(t)}{0.5p(t) - 450} = \int dt$$

$2 \ln |(1/2)\, p(t) - 450| = t + c$

$\because \quad p(0) = 850$

$2 \ln |425 - 450| = c$

$2 \ln 25 = c$

$2 \ln |0.5\, p(t) - 450| = t + 2 \ln 25$

Puting $p(t) = 0$

$2 \ln 450 = t + 2 \ln 25$

$t = 2 \ln (450/25) = 2 \ln 18$.

63. If $f : R \to R$ is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where $[x]$ denotes the greatest integer function, then f is

(1) discontinuous only at $x = 0$.
(2) discontinuous only at non - zero integral values of x.
(3) continuous only at $x = 0$.
(4) continuous for every real x.

63. **(4)** For $x = 0$, LHL = RHL = 0

For $x \in I$, since $2x - 1$ is odd integer, so LHL = RHL = 0

So, function is continuous $\forall\, x \in R$.

64. Let P and Q be 3×3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to

(1) 1 (2) 0 (3) -1 (4) -2

64. **(2)** $P^3 = Q^3$ and $Q^2P = P^2Q$

$\therefore \quad (P^2 + Q^2)P = (Q^2 + P^2)Q \quad \therefore \quad (P^2 + Q^2)(P - Q) = O$

$\therefore \quad |P^2 + Q^2||P - Q| = 0 \quad \Rightarrow \quad |P^2 + Q^2| = 0 \quad \because P \neq Q.$

65. If the integral $\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k$ then a is equal to

(1) -2 (2) 1 (3) 2 (4) -1

65. **(3)** $\int \frac{5\sin x}{\sin x - 2\cos x}dx$

Let $5\sin x = A(\sin x - 2\cos x) + B(\cos x + 2\sin x)$

$\Rightarrow \quad A + 2B = 5$

$-2A + B = 0$

$\Rightarrow \quad A = 1, B = 2$

$\therefore \quad I = \int \frac{(\sin x - 2\cos x) + 2(\cos x + 2\sin x)}{\sin x - 2\cos x} = x + 2\ln|\sin x - 2\cos x| + k.$

66. If $g(x) = \int_0^x \cos 4t\, dt$, then $g(x + \pi)$ equals

(1) $g(x) + g(\pi)$ (2) $g(x) - g(\pi)$ (3) $g(x) \cdot g(\pi)$ (4) $\frac{g(x)}{g(\pi)}$

66. **(1, 2)** $g(x) = \left[\frac{\sin 4t}{4}\right]_0^x = \frac{1}{4}\sin 4x$

$$g(x + \pi) = \frac{1}{4}\sin 4(x + \pi) = \frac{1}{4}\sin 4x$$

$g(\pi) = 0$

$\Rightarrow \quad g(x + \pi) = g(x) \pm g(\pi).$

67. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is

(1) $x - 2y + 2z + 1 = 0$ (2) $x - 2y + 2z - 1 = 0$

(3) $x - 2y + 2z + 5 = 0$ (4) $x - 2y + 2z - 3 = 0$

67. **(4)** Equation of parallel plane is $x - 2y + 2z + \lambda = 0$

above plane is at unit distance from origin.

$$\frac{|\lambda|}{\sqrt{1+4+4}} = 1$$

$|\lambda| = 3 \quad \Rightarrow \quad \lambda = \pm 3.$

68. A spherical balloon is filled with 4500 π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72 π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is
(1) 7 / 9 (2) 2 / 9 (3) 9 / 1 (4) 9 / 7

68. **(2)** $(4/3)\pi r^3 = 4500\pi \Rightarrow r = 15$
Now $dV/dt = -72\pi = 4\pi r^2 . (dr/dt)$
$\Rightarrow -18 = r^2 . (dr/dt) \Rightarrow -18dt = r^2\, dr$
$\Rightarrow -\int_0^{49} 18dt = \int_{15}^{r} r^2 dr \Rightarrow r = 9$ (at the end of 49 minute)
$\therefore dr/dt = -18/r^2 = -18/81 = -2/9.$

69. If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals
(1) 5 (2) 6 (3) 11 / 5 (4) 29 / 5

69. **(2)** Since, $P\left(\frac{8}{5}, \frac{14}{5}\right)$ divides (1,1) & (2, 4) into 3 : 2 ratio.

Hence, $k = \frac{30}{5} = 6$, as P lies on the line given.

70. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is
(1) $\pi/2$ (2) $\pi/3$ (3) $\pi/4$ (4) $\pi/6$

70. **(2)** Since, $\overline{c}.\overline{d} = 0 \Rightarrow 5\overline{a}.\overline{a} + 10\overline{a}.\overline{b} - 4\overline{a}.\overline{b} - 8\overline{b}.\overline{b} = 0$
$\Rightarrow 5 - 8 + 6\cos\theta = 0 \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = (\pi/3)$, where θ is the angle between $\hat{a}$ and $\hat{b}$.

71. **Statement-1:** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement-2: If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

71. **(1)** Since, $y = mx \pm \sqrt{2m^2 + 4}$ is a tangent to the given ellipse for all m, and

$y = m'x + \frac{4\sqrt{3}}{m'}$ is a tangent to the given parabola.

Hence, $m = m'$ & $2m^2 + 4 = \frac{48}{m^2} \Rightarrow m^4 + 2m^2 = 24$

$\therefore\ m = \pm 2 \Rightarrow$ common tangents are $y = \pm 2x \pm 2\sqrt{3}$.

72. Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is

(1) 1 / 5 (2) 1 / 4 (3) 2 / 5 (4) 3 / 8

72. **(1)** A : event that 3 numbers selected with max. as 6, $n(A) = {}^5C_2$

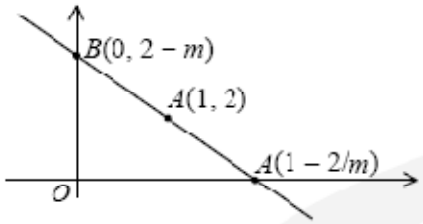
B : event that 3 numbers selected with max. as 6 and min. as 3, $n(B) = {}^2C_1$

$\therefore\quad P(B / A) = {}^2C_1 / {}^5C_2 = 1 / 5.$

73. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is

(1) -4 (2) -2 (3) $-1/2$ (4) $-1/4$

73. **(2)**



$$A = -\frac{1}{2}\frac{(m-2)^2}{m}$$

$$\frac{dA}{dm} = -\frac{1}{2}\frac{(m-2)(m+2)}{m^2}$$

Put $\frac{dA}{dm} = 0$, $m = 2, -2$

$\left(\frac{d^2A}{dm^2}\right)_{at\ m=-2} > 0$ $\therefore$ m is least when $m = -2$.

74. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is

(1) 629 (2) 630 (3) 879 (4) 880

74 **(3)** Number of ways = $(10 + 1)(9 + 1)(7 + 1) - 1 = 879$

75. **Statement-1:** The sum of the series
$1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \ldots\ldots + (361 + 380 + 400)$ is 8000.

Statement-2: $\sum_{k=1}^{n} (k^3 - (k-1)^3) = n^3$, for any natural number n.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement 1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

75. **(1)** $(1) = (1 - 0)(1^2 + 1.0 + 0^2) = (1^3 - 0^3)$
$(1 + 2 + 4) = (2 - 1)(2^2 + 1.0 + 1^2) = (2^3 - 1^3)$
$(4 + 6 + 9) = (3 - 2)(3^2 + 3.2 + 2^2) = (3^3 - 2^3)$
.
.
.
$(361 + 380 + 400) = (20 - 19)(20^2 + 20.19 + 19^2) = (20^3 - 19^3)$
$\Rightarrow$ Required sum $= (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + \ldots\ldots + (20^3 - 19^3)$
$= 8000$

Also, $\sum_{k=1}^{n} k^3 - (k-1)^3 = \sum_{k=1}^{n} (3k^2 - 3k + 1) = \frac{n(n+1)(2n+1)}{2} - \frac{3n(n+1)}{2} + n = n^3$

Both statements are correct and statement - 2 is the correct explanation of statement - 1.

76. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to

(1) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (3) $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

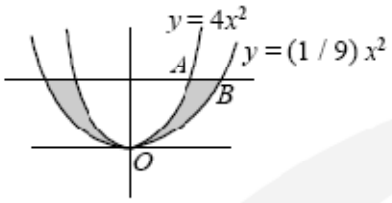
76. **(3)** Let $u_1 + u_2 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ Now, $Au_1 + Au_2 = A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{bmatrix} x \\ 2x + y \\ 3x + 2y + z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \Rightarrow x = 1, y = -1, z = -1 \quad \therefore \quad u_1 + u_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

77. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is

(1) $\frac{10\sqrt{2}}{3}$ (2) $\frac{20\sqrt{2}}{3}$ (3) $10\sqrt{2}$ (4) $20\sqrt{2}$

77. **(2)**



Required area = $2\int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}}\right) dy = \frac{20\sqrt{2}}{3}$.

78. Let $x_1, x_2, \ldots\ldots, x_n$ be n observations and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement-1: Variance of $2x_1, 2x_2, \ldots\ldots, 2x_n$ is $4\sigma^2$.

Statement-2: Arithmetic mean of $2x_1, 2x_2, \ldots\ldots, 2x_n$ is $4\bar{x}$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.

(3) Statement-1 is true, Statement-2 is false.

(4) Statement-1 is false, Statement-2 is true.

78. **(3)** **Statement-1:** Variance of $2x_1, 2x_2, \ldots\ldots, 2x_n =$

$$\frac{1}{n}\sum (x_i - \bar{x})^2 = \frac{1}{n}\sum (2x_i - 2\bar{x})^2 = 4\frac{1}{n}\sum (x_i - \bar{x})^2 = 4\sigma^2$$

Statement-2: A.M. = $\frac{2(x_1 + x_2 + \ldots\ldots + x_n)}{n} = 2\bar{x}$

∴ Statement - 1 is true and Statement - 2 is false.

79. If n is a positive integer, then $(\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n}$ is
(1) an odd positive integer
(2) an even positive integer
(3) a rational number other than positive integers
(4) an irrational number

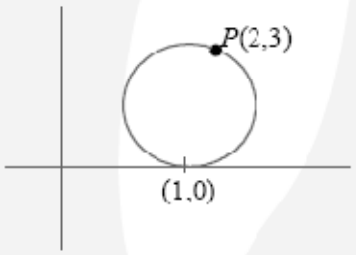
79. **(4)** $\because \quad (\sqrt{3}+1)^{2n} - (\sqrt{3}-1)^{2n} = 2\{{}^{2n}C_1 (\sqrt{3})^{2n-1} + {}^{2n}C_3 (\sqrt{3})^{2n-3} + \ldots\ldots\}$
$= 2 \times \sqrt{3}\{{}^{2n}C_1 (\sqrt{3})^{2n-2} + {}^{2n}C_3 (\sqrt{3})^{2n-4} + \ldots\ldots\}$
$= \sqrt{3} \times$ even number
= an irrational number.

80. If 100 times the 100^{th} term of an A.P. with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is
(1) 150 times its 50^{th} term (2) 150
(3) zero (4) -150

80. **(3)** Let a be the first terms and d be the common difference
100. $t_{100} = 50\ t_{50}$; $t_{150} = ?$
$\Rightarrow 2[a + 99\ d] = a + 49\ d \quad \Rightarrow \quad a + 149\ d = 0 \qquad \Rightarrow t_{150} = 0$

81. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :
(1) 3 / 5 (2) 6 / 5 (3) 5 / 3 (4) 10 / 3

81. **(4)**



Let required equation be $x^2 + y^2 + 2gx + 2fy + c = 0$
$g^2 = c$ ..... (i)
$1 + 2g + g^2 = 0$
$(g + 1)^2 = 0$
$g = -1$ ..... (ii)
It passes through (2, 3)
$\therefore [c = 1]$
$4 + 9 + 4g + 6f + c = 0$
$13 - 4 + 6f + 1 = 0 \quad \therefore f = -(5 / 3).$

82. Let $a, b, \in R$ be such that the function f given by $f(x) = \ln|x| + bx^2 + ax, x \neq 0$ has extreme values at $x = -1$ and $x = 2$.
Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
(2) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is false.
(4) Statement 1 is false, Statement 2 is true.

82. **(1)** $f'(x) = (1/x) + 2bx + a$
Now, $f'(-1) = 0 \Rightarrow a - 2b - 1 = 0$ .... (i)
and $f'(2) = 0 \Rightarrow 2a + 8b + 1 = 0$ ..... (ii)
Solve equation (i) & (ii) we get $a = (1/2)$, $b = -(1/4)$

$$f''(x) = -\frac{1}{x^2} + 2b = -\frac{1}{x^2} - \frac{1}{2}$$

$$f''(-1) = -\frac{3}{2} < 0 \text{ and } f''(2) = -\frac{3}{4} < 0$$

Hence at $x = -1$ and $x = 2$ has local maximum.
Both statements are correct and statement - 2 is the correct explanation of statement - 1.

83. Let $ABCD$ be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is the given by :

(1) $\vec{r} = -\vec{q} + \left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$

(2) $\vec{r} = \vec{q} + \left(\frac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$

(3) $\vec{r} = -3\vec{q} + \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

(4) $\vec{r} = 3\vec{q} + \frac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

83 **(1)** $\overline{AK} - \lambda\overline{p}$, where 'k' is the foor of perpendicular from B on AD
Using Δ law, $\lambda\overline{p} - \overline{r} = \overline{q}$

$$\therefore \overline{r} = \lambda\overline{p} - \overline{q} \;\&\; \overline{r}.\overline{p} = 0 \Rightarrow \lambda = \frac{\overline{p}.\overline{q}}{\overline{p}.\overline{p}} \qquad \therefore \overline{r} = \left(\frac{\overline{p}.\overline{q}}{\overline{p}.\overline{p}}\right)\overline{p} - \overline{q}$$

84. If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ and $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then k is equal to :

(1) 2 / 9 (2) 9 / 2 (3) 0 (4) – 1

84. **(2)** Any point on line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ is $(2\lambda + 1, 3\lambda - 1, 4\lambda + 1)$

and on 2nd line is $(\mu + 3, 2\mu + k, \mu)$

For lines to intersect

$2\lambda + 1 = \mu + 3$ ..... (i)

$3\lambda - 1 = 2\mu + k$ .... (ii)

$4\lambda + 1 = \mu$ ..... (iii)

Solving (i) and (iii)

$\lambda = -3/2, \mu = -5$

$\therefore$ from equation (ii) $k = 9/2$.

85. An ellipse is drawn by taking a diameter of the circle $(x-1)^2 + y^2 = 1$ as its semiminor axis and a diameter of the circle $x^2 + (y-2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinates axes, then the equation of the ellipse is :

(1) $x^2 + 4y^2 = 8$ (2) $4x^2 + y^2 = 8$ (3) $x^2 + 4y^2 = 16$ (4) $4x^2 + y^2 = 4$

85. **(3)** $(x-1)^2 + y^2 = 1$ $r_1 = 1$ $\therefore$ $b = 2$

$x^2 + (y-2)^2 = 4$ $r_2 = 2$ $\therefore$ $a = 4$

$\therefore$ Equation of ellipse : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\Rightarrow$ $x^2 + 4y^2 = 16$.

86. The negation of the statement " If I become a teacher, then I will open a school", is :

(1) Either I will not become a teacher or I will not open a school.

(2) Neither I will become a teacher not I will open a school.

(3) I will not become a teacher or I will open a school.

(4) I will become a teacher and I will not open a school.

86. **(4)** $\sim (p \Rightarrow q) \equiv p \wedge \sim q$

$\therefore$ Negattion of statement is : "I will become a teacher and I will not open a school".

87. Consider the function $f(x) = |x-2| + |x-5|$, $x \in R$
Statement 1 : $f'(4) = 0$
Statement 2 : f is continuous in $[2, 5]$, differentiable in $(2, 5)$ and $f(2) = f(5)$
(1) Statement 1 is true, Statement 2 is true; Statement 2 is a correct explanation for Statement 1.
(2) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for Statement 1.
(3) Statement 1 is true, Statement 2 is false.
(4) Statement 1 is false, Statement 2 is true.

87. **(2)** $f(x) = |x-2| + |x-5|$, $x \in R$
when $2 < x < 5$ $\quad f(x) = x - 2 - x + 5 = 3$
$\therefore f'(x) = 0$
$\therefore f'(4) = 0$
$f(2) = 3$, $f(5) = 3$.
$\therefore$ Statement 1 is true, Statement 2 is true; Statement 2 is **not** the correct explanation for Statement 1.

88. If $z \neq 1$ and $\dfrac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :
(1) on a circle with centre at the origin
(2) either on the real axis or on a circle not passing through the origin
(3) on the imaginary axis
(4) either on the real axis or on a circle passing through the origin.

88. **(4)** If $z = x + iy$

$$\frac{z^2}{z-1} = \frac{(x+iy)^2}{(x-1)+iy} = \frac{(x^2-y^2+2ixy)\{(x-1)-iy\}}{(x-1)^2+y^2}$$

$$\text{Im}\left(\frac{z^2}{z-1}\right) = 0 \quad \Rightarrow \quad -y(x^2-y^2) + 2xy(x-1) = 0$$

$$\Rightarrow \quad y(x^2+y^2-2x) = 0 \quad \Rightarrow \quad y = 0 \text{ or } (x-1)^2 + y^2 = 1.$$

89. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has
(1) no real roots. (2) exactly one real root
(3) exactly four real roots (4) infinite number of real roots.

89. **(1)** $e^{\sin x} - e^{-\sin x} = 4$ .... (i)
$\Rightarrow (e^{\sin x} + e^{-\sin x})^2 = 20$
$\Rightarrow e^{\sin x} + e^{-\sin x} = 2\sqrt{5}$ .....(ii)
from (i) and (ii) $e^{\sin x} = 2 + \sqrt{5} \quad \Rightarrow \quad \sin x = \ln(2 + \sqrt{5}) > 1$. (Not possible)

90. In a ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$, then the angle R equal to :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{4}$

90. **(1)** After squaring up both the equations and then add leads to

$$\sin(P+Q) = \frac{1}{2} \Rightarrow P+Q = \frac{\pi}{6} \text{ or } \frac{5\pi}{6} \Rightarrow R = \frac{5\pi}{6} \text{ or } \frac{\pi}{6}$$

But in the second equation, $4\sin Q + 3\cos P = 1$, equation holds iff P is obtuse.

Hence $R = \frac{\pi}{6}$.