Part A - PHYSICS

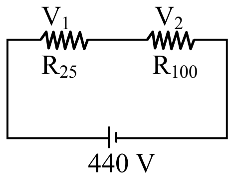
Q.1 Two electric bulbs marked 25 W - 220 V and 100 W - 220 V are connected in series to a 440 V supply. Which of the bulbs will fuse ?

(1) 100 W (2) 25 W (3) neither (4) both

Ans.[2]

Sol. We have $R = \frac{V^2}{P}$

$R_{25} > R_{100}$



$V_1 > 220$ V

hence 25 W bulb gets fused.

Q.2 A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be -

(1) 10 m (2) $10\sqrt{2}$ m (3) 20 m (4) $20\sqrt{2}$ m

Ans.[3]

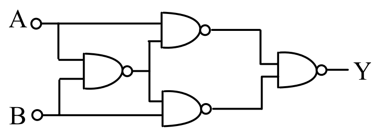
Sol. $H = \frac{u^2 \sin^2 \theta}{2g}$ $(\theta = 90^\circ)$

$$\frac{u^2}{2g} = 10 \text{ m}$$

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$R_{max} = \frac{u^2}{g} = 20 \text{ m}$$

Q.3 Truth table for system of four NAND gates as shown in figure is-



(1)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(2)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(3)

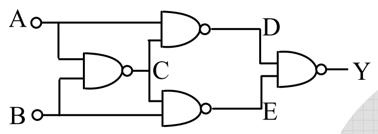
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ans.[4]

Sol.



A	B	C	$D = \overline{A.C}$	$E = \overline{C.B}$	Y
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	1	1	0

Q.4 This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1 : Davisson : Germer experiment established the wave nature of electrons.

Statement 2 : If electrons have wave nature, they can interfere and show diffraction.

(1) Statement 1 is true, Statement 2 is false

(2) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1

(3) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

(4) Statement 1 is false, Statement 2 is true.

Ans. **[2]**

Sol. Devision germer experiment show's wave nature of particle.

Q.5 In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by -

(1) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$

(2) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$

(3) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

(4) $\frac{I_m}{9}(4+5\cos\phi)$

Ans.[3]

Sol. Let $A_1 = A_0$

Then $A_2 = 2A_{-0}$

Intensity $I \propto A^2$

Hence $I_1 = I_0$

$I_2 = 4I_0$

We have

$$I = I_0 + 4I_0 + 2\sqrt{I_0 \times 4I_0}\ \cos\phi$$

For I_{max}, $\cos\phi = 1$

Hence

$$I_m = 9I_0$$

or $I_0 = \dfrac{I_m}{9}$

When phase difference is ϕ then

$$I = I_0 + 4I_0 + 2\sqrt{4{I_0}^2}\ \cos\phi = 5I_0 + 4I_0 \cos\phi$$

$$I = \frac{I_m}{9}(1 + 8\cos^2\frac{\phi}{2})$$

Q.6 If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0 s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds-

(1) b (2) $\dfrac{1}{b}$ (3) $\dfrac{2}{b}$ (4) $\dfrac{0.693}{b}$

Ans. **[3]**

Sol. In damped oscillation
we have, amplitude

$$A = A_0 e^{\frac{-b}{2m}t}$$

at $t = T$, $A = \dfrac{A_0}{e}$

$$\frac{A_0}{e} = A_0 e^{\frac{-b}{2m}t}$$

$$\frac{b}{2m}T = 1$$

$T = \dfrac{2}{b}$ (Taking m = 1)

Q.7 This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement 1 : If stretched by the same amount, work done on S_1, will be more than that on S_2.

Statement 2 : $k_1 < k_2$

(1) Statement 1 is true, Statement 2 is false

(2) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1

(3) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

(4) Statement 1 is false, Statement 2 is true.

Ans.[4]

Sol. $F = k_1x_1$

$x_1 = \frac{F}{k_1}$

$\frac{1}{2}k_1.\frac{F^2}{k_1^2} = W_1$

$\frac{1}{2}k_2.\frac{F^2}{k_2^2} = W_2$

$W_1 > W_2$

$k_1 < k_2$ ......(1)

$W_1' = \frac{1}{2}k_1x_0^2,\ W_2' = \frac{1}{2}k_2x_0^2$

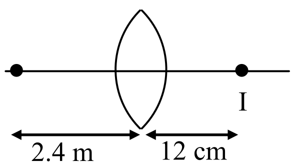
$\boxed{W_2' > W_1'}$

Q.8 An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film ?

(1) 2.4 m (2) 3.2 m (3) 5.6 m (4) 7.2 m

Ans. **[2]**

Sol.



$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{12} - \frac{1}{-240} = \frac{1}{f}$

$\frac{1}{f} = \frac{20+1}{240} \Rightarrow f = \frac{240}{21} = \frac{80}{7}$

After slab is inserted normal shifting of slab, $\Delta t = t\left(1-\frac{1}{\mu}\right)$

$$\Delta t = 1\left(1-\frac{2}{3}\right) = \frac{1}{3}\text{ cm}$$

To form the image at same position

due to lens only image should be formed at, $v' = \left(12-\frac{1}{3}\right) = \frac{35}{3}$ cm

$$f = \frac{80}{7}$$

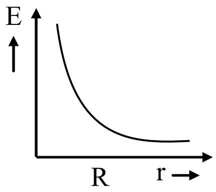
$$\frac{3}{35} - \frac{1}{u} = \frac{7}{80}$$

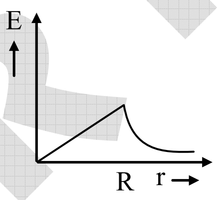
$$\frac{1}{u} = \frac{3}{35} - \frac{7}{80} = \frac{240-245}{2800} = \frac{-5}{2800} = -560\text{ cm}$$

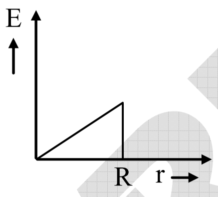
$= 5.6$ m

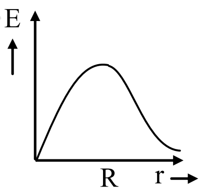
hence shifting of object = 5.6 – 2.4 = 3.2 m

Q.9 In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be-

(1) 

(2) 

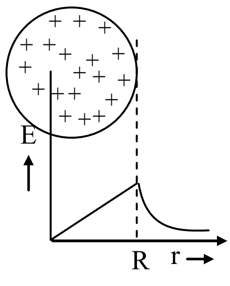
(3) 

(4) 

Ans. [2]

Sol. For a uniformly charged sphere electric field

$$E = \begin{cases} \dfrac{1}{4\pi\epsilon_0}\dfrac{Qr}{R^3}, & r < R \\ \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r^2}, & r > R \end{cases}$$



Q.10 A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to-

(1) induction of electrical charge on the plate

(2) shielding of magnetic lines of force as aluminium is a paramagnetic material

(3) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

(4) development of air current when the plate is placed

Ans. **[3]**

Sol. Due to change in magnetic flux, eddy current flows on the surface of aluminium plate which acts as electromagnetic damping.

Q.11 A spectrometer gives the following reading when used to measure the angle of a prism.

Main scale reading : 58.5 degree

Vernier scale reading : 09 divisions

Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data :

(1) 58.77 degree (2) 58.65 degree

(3) 59 degree (4) 58.59 degree

Ans.[2]

Sol. L.C. = 1 M.S.D. – 1 V.S.D.

$$= 0.5^\circ - \left(\frac{29}{30}\right) \times 0.5$$

$$= \frac{1}{300} \times 0.5^\circ$$

$$= \frac{1}{60} \text{ (degree)}$$

$\therefore$ V.S.R. = No. of division × LC = $9 \times \frac{1}{60} = 0.15$

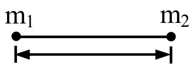
Reading = M.S.R. + V.S.R. = 58.5 + 0.15 = 58.65°

Q.12 A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by : (n is an integer)

(1) $\frac{n^2h^2}{2(m_1+m_2)r^2}$ (2) $\frac{2n^2h^2}{(m_1+m_2)r^2}$

(3) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (4) $\frac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$

Ans. **[3]**

Sol.


Reduced mass = $\frac{m_1m_2}{m_1+m_2}$

$$L = \frac{nh}{2\pi}$$

so rotational kinetic energy = $\frac{L^2}{2I}$

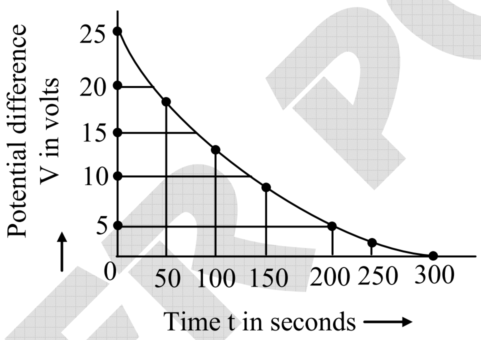
$$= \frac{n^2h^2}{4\pi^2\left[2\times\frac{m_1m_2}{m_1+m_2}\right]r^2}$$

$$= \frac{n^2h^2(m_1+m_2)}{2\,m_1m_2\,r^2}$$

In question instead of h, $\hbar$ should be given :

$$\hbar = \frac{h}{2\pi}$$

Q.13 The figure shows an experimental plot for discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between -



(1) 0 and 50 sec
(2) 50 sec and 100 sec
(3) 100 sec and 150 sec
(4) 150 sec and 200 sec

Ans. **[3]**

Sol. $V = V_0e^{-t/RC}$

At $t = \tau$

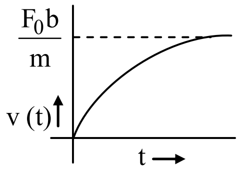
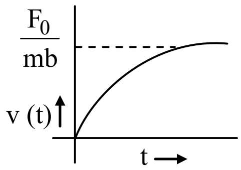
$V = \frac{V_0}{e}$

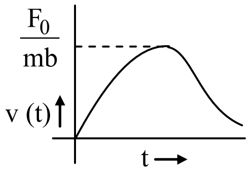
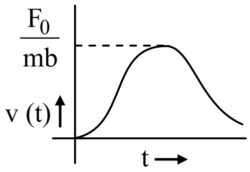
$V = 0.37\ V_0$

$V = 0.37 \times 25 = 9.25$ Volt

Thus voltage will occur at time 100 & 150 sec.

Q.14 A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x-direction. Its speed v(t) is depicted by which of the following curves ?

(1) 
(2) 

(3) 
(4) 

Ans. **[2]**

Sol. $\frac{mdv}{dt} = F_0e^{-bt}$

$\frac{dv}{dt} = \frac{F_0}{m}e^{-bt}$

$v = \frac{F_0}{mb}[-e^{-bt}]_0^t$

$v = \frac{F_0}{mb}[1 - e^{-bt}]$

Q.15 Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is-

(1) $m_1 : m_2$ (2) $r_1 : r_2$

(3) 1 : 1 (4) $m_1 r_1 : m_2 r_2$

Ans. **[2]**

Sol. $\frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

Q.16 A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is-

(1) 16 km (2) 40 km (3) 64 km (4) 80 km

Ans. **[4]**

Sol. $d = \sqrt{2hR}$

$= \sqrt{2 \times 500 \times 6.4 \times 10^6}$ m

= 80,000 m

= 80 km

Q.17 Assume that a neutron breaks into a proton and an electron. The energy released during this process is :

(Mass of neutron = 1.6725×10^{-27} kg, Mass of proton = 1.6725×10^{-27} kg, Mass of electron = 9×10^{-31} kg)

(1) 7.10 MeV (2) 6.30 MeV (3) 5.4 MeV (4) 0.73 MeV

Ans. []

Sol. Energy released $= (9 \times 10^{-31})(3 \times 10^{8})^2$

$= 81 \times 10^{-15}$ J

$$= \frac{81\times10^{-15}}{1.6\times10^{-13}} \text{ MeV}$$

$$= \left(\frac{0.81}{1.6}\right) \text{ MeV}$$

Q.18 This question has Statement-1 and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

An insulating solid sphere of radius R has a uniformly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point out side the sphere. The electric potential at infinity is zero.

Statement 1 : When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\in_0}$.

Statement 2 : The electric field at a distance r (r < R) from the centre of the sphere is $\frac{\rho r}{3\in_0}$.

(1) Statement 1 is true, Statement 2 is false

(2) Statement 1 is false, Statement 2 is true.

(3) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation for Statement 1

(4) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1

Ans. [2]

Sol. At internal point

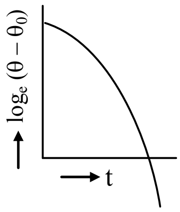
$$E = \frac{1}{4\pi\in_0}\frac{Q}{R^3}r$$

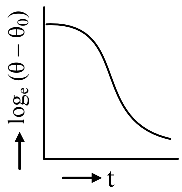
$$E = \frac{\rho r}{3\in_0}$$

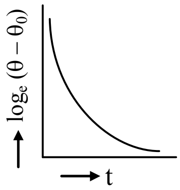
Charge in electric potential energy $|\Delta U| = q\int_0^R E\,dr$

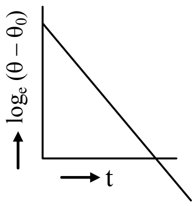
$$|\Delta U| = \frac{qR^2\rho}{6\in_0}$$

Q.19 A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is-

(1) 

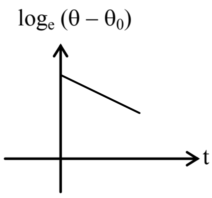
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



We have

$$\frac{d\theta}{dt} = -b(\theta - \theta_0)$$

$$\int_{\theta_1}^{\theta} \frac{d\theta}{(\theta - \theta_0)} = -\int_0^t b\, dt$$

$\ln(\theta - \theta_0) - \ln\theta_1 = -bt$

$\ln(\theta - \theta_0) = -bt + \ln\theta_1$

Q.20 Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3 % each, then error in the value of resistance of the wire is-

(1) zero (2) 1 % (3) 3 % (4) 6 %

Ans. **[4]**

Sol. $R = \frac{V}{I}$

$$\frac{dR}{R}\times 100 = \frac{dV}{V}\times 100 + \frac{dI}{I}\times 100$$

$$= 3\,\% + 3\,\% = 6\,\%$$

Q.21 The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be-

(1) 6.4×10^8 Joules (2) 6.4×10^9 Joules

(3) 6.4×10^{10} Joules (4) 6.4×10^{11} Joules

Ans. **[3]**

Sol. Energy required

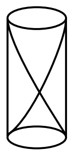
$$= \frac{GMm}{R} = \frac{GM}{R^2}.mR = 10 \times 1000 \times 6400000 \text{ J} = 6.4 \times 10^{10} \text{ J}$$

Q.22 A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now -

(1) $\frac{f}{2}$ (2) $\frac{3f}{4}$ (3) 2 f (4) f

Ans. **[4]**

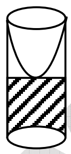
Sol.



$V = n\lambda$

$n = \frac{v}{\lambda}$ $\frac{\lambda}{2} = \ell$

$n_1 = \frac{v}{2\ell} = f$



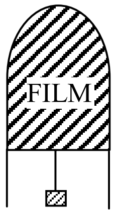
$\frac{\lambda}{4} = \ell$

$n'' = \frac{v}{\lambda} = \frac{v}{4\ell'}$

$n'' = \frac{V}{4 \times \frac{\ell}{2}} = \frac{v}{2\ell} = f$

$f' = f$

Q.23 A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is-



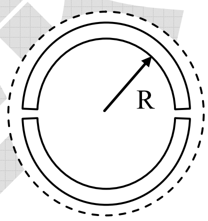
(1) 0.1 Nm^{-1} (2) 0.05 Nm^{-1} (3) 0.025 Nm^{-1} (4) 0.0125 Nm^{-1}

Ans. **[3]**

Sol. $2T\ell = W$

$$T = \frac{W}{2\ell} = \frac{1.5\times10^{-2}}{2\times0.3} = 0.025\ Nm^{-1}$$

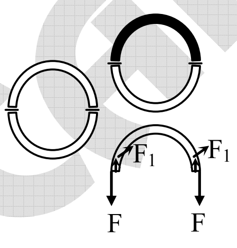
Q.24 A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Youngs' modulus is Y, the force that one part of the wheel applies on the other part is-



(1) $SY\alpha\Delta T$ (2) $\pi SY\alpha\Delta T$ (3) $2SY\alpha\Delta T$ (4) $2\pi SY\alpha\Delta T$

Ans. **[3]**

Sol.

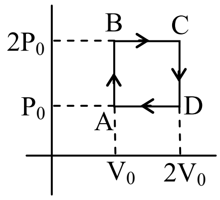


Thermal stress = $Y\alpha\Delta T$

$F = YS\alpha\Delta T$

$2F_1 = 2F$

Q.25 Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly : (Assume the gas to close to ideal gas)



(1) 9.1 % (2) 10.5 % (3) 12.5 % (4) 15.4 %

Ans. **[4]**

Sol. Efficiency = $\dfrac{\text{Work done}}{\text{Heat absorbed}}$

$W_{AB} = 0$; $W_{BC} = 2P_0V_0$; $W_{CD} = 0$; $W_{DA} = -P_0V_0$

So total work done = P_0V_0

$$\Delta Q_{AB} = (\Delta U)_{AB} = n\left(\frac{3}{2}R\right)\left(\frac{P_0V_0}{nR}\right) = \frac{3}{2}P_0V_0$$

$$\Delta Q_{BC} = n\left(\frac{3}{2}R\right)\left(\frac{2P_0V_0}{nR}\right) + 2P_0V_0 = 5P_0V_0$$

$$\Delta Q_{CD} = n\left(\frac{3}{2}R\right)\left(\frac{-2P_0V_0}{nR}\right) = -3P_0V_0$$

$$\Delta Q_{DA} = n\left(\frac{3}{2}R\right)\left(\frac{-P_0V_0}{nR}\right) + (-P_0V_0) = \frac{-5}{2}P_0V_0$$

So total heat absorbed = $\dfrac{13}{2}P_0V_0$

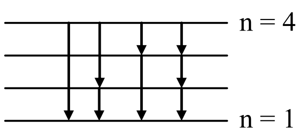
$$\eta = \frac{2P_0V_0}{13(P_0V_0)} \times 100 = \frac{200}{13} = 15.4\%$$

Q.26 Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be-

(1) 3 (2) 5 (3) 6 (4) 2

Ans. **[3]**

Sol. $\dfrac{N(N-1)}{2}$



Q.27 Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct ?

(1) $r_\alpha = r_p < r_d$ (2) $r_\alpha > r_d > r_p$ (3) $r_\alpha = r_d < r_p$ (4) $r_\alpha = r_p = r_d$

Ans. **[1]**

Sol. $\frac{mv^2}{r}\ qvB$

$mv = qBr$

$v = \frac{qBr}{m}$

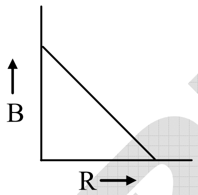
$\frac{1}{2}mv^2 = \frac{1}{2}m \cdot \frac{q^2B^2r^2}{m^2}$

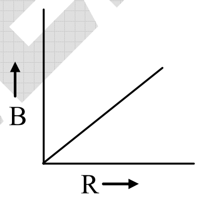
$K = \frac{q^2B^2r^2}{2m}$

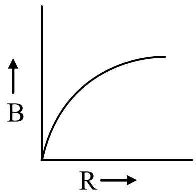
$r^2 = \frac{2mK}{q^2B^2}$

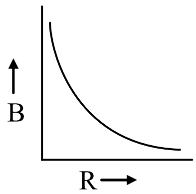
$r_\alpha = r_p < r_d$

Q.28 A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure :

(1) 

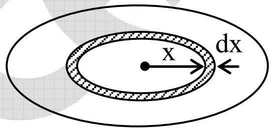
(2) 

(3) 

(4) 

Ans. **[4]**

Sol.



$q = \sigma \times 2\pi x\, dx$

$d\,i = \frac{q\omega}{2\pi}$

$dB = \frac{\mu_0 di}{2x}$

$$dB = \frac{\mu_0 \omega \sigma}{4\pi x} \times 2\pi x\, dx$$

$$B = \frac{\mu_0 \omega \sigma}{2}[R]$$

$$B = \frac{\mu_0 \omega}{2} \times \frac{Q}{\pi R^2} \times R$$

$$B = \frac{\mu_0 \omega \times Q}{2\pi R}$$

$$B \propto \frac{1}{R}$$

Q.29 An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then-

(1) $\vec{X} || \vec{E}$ and $\vec{k} || \vec{E} \times \vec{B}$

(2) $\vec{X} || \vec{B}$ and $\vec{k} || \vec{E} \times \vec{B}$

(3) $\vec{X} || \vec{E}$ and $\vec{k} || \vec{B} \times \vec{E}$

(4) $\vec{X} || \vec{B}$ and $\vec{k} || \vec{B} \times \vec{E}$

Ans. **[1]**

Sol. Direction of electromagnetic wave is along $\vec{E} \times \vec{B}$ and direction of polarization is along electric field.

Q.30 A Carnot engine, whose efficiency is 40 %, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60 %. Then, the intake temperature for the same exhaust (sink) temperature must be-

(1) 1200 K

(2) 750 K

(3) 600 K

(4) efficiency of Carnot engine cannot be made larger than 50 %

Ans. **[2]**

Sol. Let T_1 = Source temperature

T_2 = Sink's temperature

$$\frac{40}{100} = \frac{T_1 - T_2}{T_1} = \frac{500 - T_2}{500}$$

$$\Rightarrow 2000 = 5000 - 10T_2 \Rightarrow 10T_2 = 3000$$

$$T_2 = 300$$

Now, $\frac{60}{100} = \frac{T_1 - 300}{T_1}$

$$\Rightarrow 6T_1 = 10T_1 - 3000 \Rightarrow 4T_1 = 3000 \Rightarrow T_1 = 750 \text{ K}$$

Part B - Chemistry

Q.31 2-Hexyne gives trans -2- Hexene on treatment with -

(1) Li/NH_3 (2) Pd/$BaSO_4$

(3) $LiAlH_4$ (4) Pt/H_2

Ans. **[1]**

Sol.

$$CH_3\text{–}CH_2\text{–}CH_2\text{–}C\equiv C\text{–}CH_3 \xrightarrow{Li/NH_3} \begin{array}{ccc} CH_3\text{–}CH_2\text{–}CH_2 & & H \\ & C=C & \\ H & & CH_3 \end{array}$$

2-hexyne Trans-2-hexene

Q.32 Which of the following on thermal-decomposition yields a basic as well as an acidic oxide ?

(1) $KClO_3$ (2) $CaCO_3$

(3) NH_4NO_3 (4) $NaNO_3$

Ans. **[2]**

Sol. $CaCO_3 \xrightarrow{\Delta} \underset{\text{basic oxide}}{CaO} + \underset{\text{acidic oxide}}{CO_2}$

Q.33 Which one of the following statements is correct ?

(1) All amino acids are optically active

(2) All amino acids except glycine are optically active

(3) All amino acids except glutamic acid are optically active

(4) All amino acids except lysine are optically active

Ans. **[2]**

Sol.

$$\begin{array}{c} H \\ | \\ H_2N\text{–}C\text{–}COOH \\ | \\ H \end{array}$$

Glycine (simplest amino acid)

(Due to absence of chiral carbon it is optically inactive)

Q.34 The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000g of water is 1.15 g/mL. The molarity of this solution is :

(1) 1.78 M (2) 1.02 M

(3) 2.05 M (4) 0.50 M

Ans. **[3]**

Sol. $M = \dfrac{120}{60} \times \dfrac{1000}{\dfrac{1120}{1.15}}$

$= 2 \times \dfrac{100}{112} \times 1.15$

$= 2.05$ M

Q.35 The incorrect expression among the following is -

(1) In isothermal process, $w_{reversible} = -nRT \ln \dfrac{V_f}{V_i}$

(2) $\ln K = \dfrac{\Delta H^\circ - T\Delta S^\circ}{RT}$

(3) $K = e^{-\Delta G^\circ/RT}$

(4) $\dfrac{\Delta G_{system}}{\Delta S_{total}} = -T$

Ans. **[2]**

Sol. $\Delta G^o = -RT \ln K = \Delta H^o - T\Delta S^o$

$\Delta G^o = -RT \ln K$

$-\left(\dfrac{\Delta H^o - T\Delta S^o}{RT}\right) = \ln K$

Q.36 Which branched chain isomer of the hydrocarbon with molecular mass 72u gives only one isomer of mono substituted alkyl halide ?

(1) Neopentane (2) Isohexane

(3) Neohexane (4) Tertiary butyl chloride

Ans. **[1]**

Sol. $$\underset{(C_5H_{12})}{CH_3-\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-CH_3} \xrightarrow{X_2/h\nu} CH_3-\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-CH_2-X$$

Neo pentane

M.W. = 5 × 12 + 12 × 1 = 72

(It contain only one type of 1ºH)

Q.37 According to Freundlich adsorption isotherm, which of the following is correct ?

(1) $\frac{x}{m} \propto P^1$ (2) $\frac{x}{m} \propto P^{1/n}$

(3) $\frac{x}{m} \propto P^0$ (4) All the above are correct for different ranges of pressure

Ans. **[4]**

Sol. $\frac{x}{m} \propto P$ (At low P)

$\frac{x}{m} \propto P^{1/n}$ (At medium P)

$\frac{x}{m} \propto P^o$ (At high P)

= All are correct.

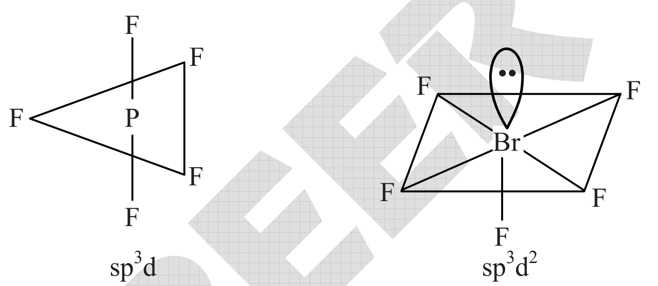
Q.38 In which of the following pairs the two species are not isostructural ?

(1) PCl_4^+ and $SiCl_4$ (2) PF_5 and BrF_5

(3) AlF_6^{3-} and SF_6 (4) CO_3^{2-} and NO_3^-

Ans. **[2]**

Sol.



$PF_5 \rightarrow$ Trigonal bipyramidal $BrF_5 \rightarrow$ Square pyramidal

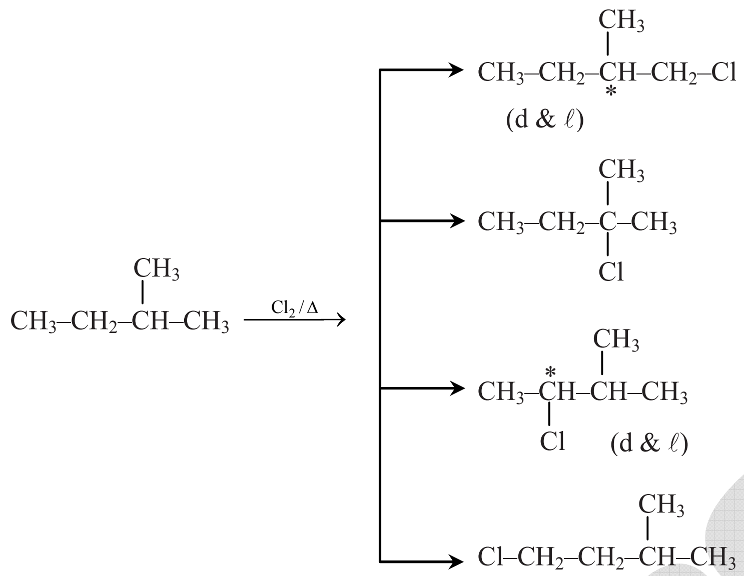
Q.39 How many chiral compounds are possible on monochlorination of 2-methyl butane ?

(1) 2 (2) 4

(3) 6 (4) 8

Ans. **[2]**

Sol.



Total chiral compound (4)

Q.40 The increasing order of the ionic radii of the given isoelectronic species is :

(1) S^{2-}, Cl^{-}, Ca^{2+}, K^{+} (2) Ca^{2+}, K^{+}, Cl^{-}, S^{2-}

(3) K^{+}, S^{2-}, Ca^{2+}, Cl^{-} (4) Cl^{-}, Ca^{2+}, K^{+}, S^{2-}

Ans. **[2]**

Sol. Ionic radii depend upon $\frac{Z}{e}$ ratio

As $\frac{Z}{e}\uparrow$ size $\downarrow$

Correct order of radius

$Ca^{+2} < K^{+} < Cl^{-} < S^{-2}$

Q.41 The compressibility factor for a real gas at high pressure is -

(1) 1 (2) 1 + pb/RT

(3) 1 – pb/RT (4) 1 + RT/pb

Ans. **[2]**

Sol. $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ for 1 mole

At high pressure, $P >> \frac{a}{V^2}$

$P(V - b) = RT \Rightarrow PV = RT + Pb$

$\frac{PV}{RT} = 1 + \frac{Pb}{RT} \Rightarrow \boxed{Z = 1 + \frac{Pb}{RT}}$

Q.42 Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide ?

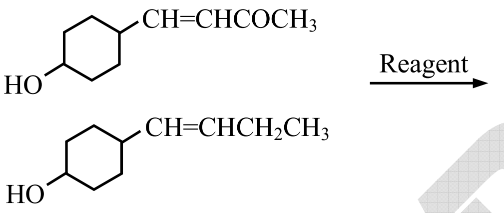
(1) $[Cr(en)_2Br_2]$ Br (2) $[Cr(en)_2Br_2]Br$

(3) $[Cr (en) Br_2]Br$ (4) $[Cr (en)_3]Br_3$

Ans. **[2]**

Sol. $[Cr(en)_2Br_2]Br$

Q.43 In the given transformation, which of the following is the most appropriate reagent ?

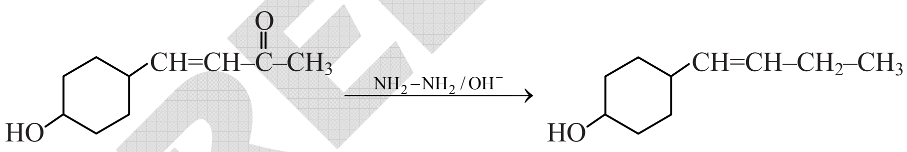


(1) Zn – Hg/HCl (2) Na, Liq. NH_3

(3) $NaBH_4$ (4) $NH_2 NH_2, \overset{\ominus}{O}H$

Ans. **[4]**

Sol.



In order to protect –OH group, reduction of >C=O group takes place in alkaline media.

Q.44 Lithium forms body centered cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be -

(1) 300 pm (2) 240 pm

(3) 152 pm (4) 75 pm

Ans. **[3]**

Sol. For B.C.C.

$\sqrt{3}\, a = 4r$

$r = \frac{\sqrt{3}}{4} \times 351 = 152$ pm

Q.45 K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to – 2.8°C ?

(1) 93 g (2) 39 g
(3) 27 g (4) 72 g

Ans. **[1]**

Sol. $\Delta T_f = K_f m$

$$2.8 = 1.86 \times \frac{W_B}{M_B} \times \frac{1000}{W_A}$$

$$= 1.86 \times \frac{W_B}{62} \times \frac{1000}{1000}$$

$= 93.3$ g

Q.46 The molecule having smallest bond angle is -

(1) $AsCl_3$ (2) $SbCl_3$
(3) PCl_3 (4) NCl_3

Ans. **[2]**

Sol. Order of bond angle :

$NCl_3 > PCl_3 > AsCl_3 > SbCl_3$

Q.47 What is DDT among the following -

(1) A fertilizer (2) Biodegradable pollutant
(3) Non – biodegradable pollutant (4) Greenhouse gas

Ans. **[3]**

Sol. Decomposition of DDT is not possible by micro organisms.

Q.48 The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, Ka of this acid is -

(1) 1×10^{-3} (2) 1×10^{-5}
(3) 1×10^{-7} (4) 3×10^{-1}

Ans. **[2]**

Sol. $HQ \rightleftharpoons H^+ + Q^-$

$pH = -\log[H^+] = 3$; $[H^+] = 10^{-3}$

$[H^+] = \sqrt{K_a \times C}$

$10^{-3} = \sqrt{K_a \times 0.1}$

$$K_a = \frac{(10^{-3})^2}{0.1} = 1 \times 10^{-5}$$

Q.49 Very pure hydrogen (99.9%) can be made by which of the following processes ?

(1) Mixing natural hydrocarbons of high molecular weight

(2) Electrolysis of water

(3) Reaction of salt like hydrides with water

(4) Reaction of methane with steam

Ans. **[3]**

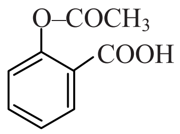
Sol. $\underset{\text{(Metal hydride)}}{MH} \xrightarrow{H_2O} \underset{\text{(metal hydroxide)}}{MOH} + H_2\uparrow$

Q.50 Aspirin is known as -

(1) Phenyl salicylate (2) Acetyl salicylate

(3) Methyl salicylic acid (4) Acetyl salicylic acid

Ans. **[4]**

Sol. Aspirin is  acetyl salicylic acid.

Q.51 Which of the following compounds can be detected by Molisch's test ?

(1) Sugars (2) Amines

(3) Primary alcohols (4) Nitro compounds

Ans. **[1]**

Sol. This is characteristic Test of carbohydrates (sugars)

Q.52 The standard reduction potentials for Zn^{2+}/Zn, Ni^{2+}/Ni, and Fe^{2+}/Fe are – 0.76, – 0.23 and – 0.44 V respectively. The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ will be spontaneous when -

(1) X = Ni, Y= Zn (2) X = Fe, Y = Zn

(3) X = Zn, Y = Ni (4) X = Ni, Y = Fe

Ans. **[3]**

Sol. Order of elements in E.C.S is Zn, Fe, Ni

Zn can reduce Fe or Ni

$\therefore\ Zn(s) + Ni^{+2} \longrightarrow Zn^{+2} + Ni$

$\therefore\ X = Zn,\quad Y = Ni$

Q.53 Ortho-Nitrophenol is less soluble in water than p-and m-Nitrophenols because -

(1) o-Nitrophenol shows Intramolecular H-bonding

(2) o-Nitrophenol shows Intermolecular H-bonding

(3) Melting point of o-Nitrophenol is lower than those of m- and p –isomers.

(4) o-Nitrophenol is more volatile in steam than those of m- and p-isomers

Ans. **[1]**

Sol. o-Nitrophenol shows intramolecular H-bonding

Q.54 Iodoform can be prepared from all except -

(1) Isopropyl alcohol (2) 3-Methyl-2-butanone

(3) Isobutyl alcohol (4) Ethyl methyl ketone

Ans. **[3]**

Sol. $CH_3–\underset{\underset{OH}{|}}{CH}–CH_3 \longrightarrow$ + ve test

$CH_3–\underset{\underset{O}{\|}}{C}–\overset{\overset{CH_3}{|}}{CH}–CH_3 \longrightarrow$ + ve test

$CH_3–\overset{\overset{CH_3}{|}}{CH}–CH_2–OH \longrightarrow$ – ve test

$CH_3–\underset{\underset{O}{\|}}{C}–CH_2–CH_3 \longrightarrow$ + ve test

Q.55 The species which can best serve as an initiator for the cationic polymerization is

(1) HNO_3 (2) $AlCl_3$

(3) BuLi (4) $LiAlH_4$

Ans. **[2]**

Sol. In cationic polymerization lewis acids like $AlCl_3$, BF_3 etc. are act as good initiator.

Q.56 The equilibrium constant (K_c) for the reaction $N_2(g) + O_2(g) \rightarrow 2NO(g)$ at temperature T is 4×10^{-4}. The value of K_c for the reaction, $NO(g) \rightarrow \frac{1}{2}N_2(g) + \frac{1}{2}O_2(g)$ at the same temperature is -

(1) 2.5×10^2 (2) 4×10^{-4}

(3) 50.0 (4) 0.02

Ans. **[3]**

Sol. (2) = (1) reversed × $\frac{1}{2}$

$$\therefore K'_C = \frac{1}{\sqrt{K_C}} = \frac{1}{\sqrt{4\times10^{-4}}} = \frac{1}{2\times10^{-2}} = 50$$

Q.57 For a first order reaction, (A) → products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M, is :

(1) 3.47×10^{-4} M/min (2) 3.47×10^{-5} M/min

(3) 1.73×10^{-4} M/min (4) 1.73×10^{-5} M/min

Ans. **[1]**

Sol. $0.1 \xrightarrow{20} 0.05 \xrightarrow{20} 0.025$

$t_{1/2} = 20$ min

$K = \frac{0.693}{20}$

$r = K[A] = \frac{0.693}{20} \times 0.01$

$= 3.465 \times 10^{-4}$ M/min

Q.58 Which method of purification is represented by the following equation :

$Ti(s) + 2I_2(g) \xrightarrow{523\ K} Ti\ I_4(g) \xrightarrow{1700\ K} Ti(s) + 2I_2(g)$

(1) Cupellation (2) Poling

(3) Van Arkel (4) Zone refining

Ans. **[3]**

Sol. Van-Arkel method

Q.59 Iron exhibits + 2 and + 3 oxidation states. Which of the following statements about iron is incorrect ?

(1) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.

(2) Ferrous compounds are less volatile than the corresponding ferric compounds

(3) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds

(4) Ferrous oxide is more basic in nature than the ferric oxide

Ans. **[3]**

Sol. Fe^{+3} compounds are more easily hydrolysed than the corresponding ferrous compounds.

Q.60 The electrons identified by quantum numbers n and ℓ

(a) $n = 4, \ell = 1$ (b) $n = 4, \ell = 0$

(c) $n = 3, , \ell = 2$ (d) $n = 3, \ell = 1$

can be placed in order of increasing energy as -

(1) (d) < (b) < (c) < (a) (2) (b) < (d) < (a) < (c)

(3) (a) < (c) < (b) < (d) (4) (c) < (d) < (b) < (a)

Ans. **[1]**

Sol. d < b < c < a

3p 4s 3d 4p

Part C - Mathematics

Q.61 Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is:

(1) 3^5 (2) 2^5

(3) 5^3 (4) 5^2

Ans. **[1]**

Sol. For any element x_i present in X, 4 cases arises while making subsets Y and Z.

Case-1 : $x_i \in Y, x_i \in Z \Rightarrow Y \cap Z \neq \phi$

Case-2 : $x_i \in Y, x_i \notin Z \Rightarrow Y \cap Z = \phi$

Case-3 : $x_i \notin Y, x_i \in Z \Rightarrow Y \cap Z = \phi$

Case-4: $x_i \notin Y, x_i \notin Z \Rightarrow Y \cap Z = \phi$

$\therefore$ for every element, number of ways = 3 for which $Y \cap Z = \phi$

$\Rightarrow$ Total ways = $3 \times 3 \times 3 \times 3 \times 3$ [$\because$ no. of elements in set X = 5]

$= (3)^5$

Q.62 The population p(t) at time t of a certain mouse species satisfies the differential equation

$\frac{dp(t)}{dt} = 0.5\, p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is :

(1) ln 9 (2) $\frac{1}{2}$ ln 18

(3) ln 18 (4) 2 ln 18

Ans. **[4]**

Sol. $\frac{dp}{dt} + \left(-\frac{1}{2}\right)p = (-450)$

I.F. = $e^{-\frac{1}{2}t}$

$\left(p\, e^{-\frac{1}{2}t}\right) = \int (-450)\, e^{-t/2}\, dt$

$= (-450)\, \frac{e^{-t/2}}{-\frac{1}{2}} + \lambda$

$p = 900 + \lambda e^{t/2}$

$p(0) = 850 \Rightarrow \lambda = -50$

$900 = 50\, e^{t/2}$

$18 = e^{t/2}$

$t = 2 \ln 18$

Q.63 If f: R → R is a function defined by $f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is :

(1) discontinuous only at x = 0
(2) discontinuous only at non-zero integral values of x
(3) continuous only at x = 0
(4) continuous for every real x

Ans. **[4]**

Sol. f(x) = g(x) . h(x)

g(x) = [x] discontinuous at integer points where h(x) is zero.

So f(x) is continuous everywhere.

Q.64 Let P and Q be 3 × 3 matrices with P ≠ Q. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to:

(1) 1
(2) 0
(3) –1
(4) –2

Ans. **[2]**

Sol. $P^3 = Q^3$ … (i)

$P^2Q = Q^2P$ … (ii)

(i) – (ii)

$P^3 - P^2Q = Q^3 - Q^2P$

$P^2 (P - Q) = Q^2 (Q - P)$

$(P - Q) (P^2 + Q^2) = 0$

But P ≠ Q

$\therefore P^2 + Q^2 = 0$

$\Rightarrow (P^2 + Q^2) = 0$

Q.65 If the integral $\int \frac{5\tan x}{\tan x - 2} dx = x + a \ln |\sin x - 2 \cos x| + k$ then a is equal to :

(1) –2
(2) 1
(3) 2
(4) –1

Ans. **[3]**

Sol. RHS

$$1 + \frac{a(\cos x + 2\sin x)}{\sin x - 2\cos x} = \frac{5\tan x}{\tan x - 2}$$

$(2a + 1) \sin x + (a - 2) \cos x = 5 \sin x$

$\Rightarrow a = 2$

Q.66 If $g(x) = \int_0^x \cos 4t \, dt$, then $g(x + \pi)$ equals:

(1) $g(x) + g(\pi)$ (2) $g(x) - g(\pi)$

(3) $g(x) . g(\pi)$ (4) $\dfrac{g(x)}{g(\pi)}$

Ans. **[1]**

Sol. $g(x + \pi) = \int\limits_0^{x+\pi} \cos 4t \, dt$

$= \int\limits_0^{x} \cos 4t \, dt + \int\limits_x^{x+\pi} \cos 4t \, dt$

$= g(x) + g(\pi)$

Q.67 An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

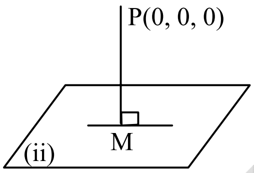
(1) $x - 2y + 2z + 1 = 0$ (2) $x - 2y + 2z - 1 = 0$

(3) $x - 2y + 2z + 5 = 0$ (4) $x - 2y + 2z - 3 = 0$

Ans. **[4]**

Sol. $x - 2y + 2z - 5 = 0$ …(i)

$x - 2y + 2z + \lambda = 0$ … (ii)



$PM = \left|\dfrac{0-0+0+\lambda}{3}\right| = 1$

$\lambda = \pm 3$

Put in (ii)

$x - 2y + 2z \pm 3 = 0$

Q.68 A spherical balloon is filled with 4500 π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72 π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :

(1) 7/9 (2) 2/9

(3) 9/2 (4) 9/7

Ans. [2]

Sol. $V_0 = 4500\ \pi$ cubic meters

So $r_0 = 15$ meters

Given $\frac{dV}{dt} = 72\ \pi$ cubic meters/min.

We know

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$ …(i)

Also at time t = 49 min, let radius is 'r' then

$\frac{4}{3}\pi\,[r_0^3 - r^3] = 72\pi \times 49$

$\Rightarrow r = 9$ meters

So from (i)

$72\pi = 4\pi\,(9)^2.\left(\frac{dr}{dt}\right)$

$\Rightarrow \frac{dr}{dt} = \left(\frac{2}{9}\right)$

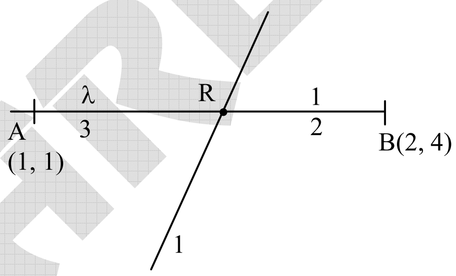
Q.69 If the line $2x + y = k$ passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :

(1) 5 (2) 6

(3) 11/5 (4) 29/5

Ans. [2]

Sol. $2x + y = k$ …..(i)



$\lambda = \frac{-[2(1) + 1 - k]}{2(2) + 4 - k} = \frac{3}{2}$

$\Rightarrow \frac{-3 + k}{8 - k} = \frac{3}{2}$

$\Rightarrow -6 + 2k = 24 - 3k$

$\Rightarrow 5k = 30$

$k = 6$

Q.70 Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

Ans. **[2]**

Sol. $\vec{c} \perp \vec{d}$

$\Rightarrow \vec{c}\,.\,\vec{d} = 0$

$\Rightarrow (\hat{a} + 2\hat{b})\,.\,(5\hat{a} - 4\hat{b}) = 0$

$\Rightarrow 5 + (-8) - 4(\hat{a}\,.\,\hat{b}) + 10(\hat{b}\,.\,\hat{a}) = 0$

$\Rightarrow -3 + 6\,|\hat{a}|\,|\hat{b}|\cos\theta = 0$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Q.71 **Statement 1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement 2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}\,x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is false.

(4) Statement 1 is false, Statement 2 is true.

Ans. **[1]**

Sol. $\begin{cases} y^2 = 16\sqrt{3}x & \text{....(i)} \\ y^2 = 4ax \end{cases}$ & $\begin{cases} \frac{x^2}{2} + \frac{y^2}{4} = 1 & \text{....(ii)} \\ c^2 = a_1^2 m^2 + b_1^2 \\ \quad = 2m^2 + 4 \end{cases}$

$4a = 16\sqrt{3}$

$a = 4\sqrt{3}$

$y = mx + c$ (iii)

for (i) $c = \frac{a}{m} = \frac{4\sqrt{3}}{m}$

$\Rightarrow c^2 = \frac{16\times 3}{m^2} = 2m^2 + 4$

$\Rightarrow m^2 = t \Rightarrow 2t^2 + 4t - 48 = 0$

$\Rightarrow t^2 + 2t - 24 = 0$

$\Rightarrow t^2 + 6t - 4t - 24 = 0$

$(t + 6)(t - 4) = 0$

$t = -6, 4 = m^2 \Rightarrow m = \pm 2$

$\Rightarrow c = \frac{4\sqrt{3}}{\pm 2} = \pm 2\sqrt{3}$

$y = mx + c$

$y = 2x + 2\sqrt{3}$

$y = -2x - 2\sqrt{3}$

Q.72 Three numbers are chosen at random without replacement from {1, 2, 3, ..., 8}. The probability that their minimum is 3, given that their maximum is 6, is :

(1) $\frac{1}{5}$ (2) $\frac{1}{4}$

(3) $\frac{2}{5}$ (4) $\frac{3}{8}$

Ans. **[1]**

Sol. $P\left[\frac{\text{min}.3}{\text{max}.6}\right] = \frac{P[\text{min}\,3 \cap \text{max}\,6]}{P(\text{max}\,6)}$

$= \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = 1/5$

Q.73 A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is :

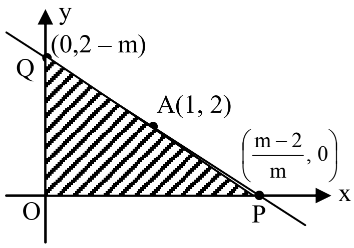
(1) –4 (2) – 2

(3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

Ans. **[2]**

Sol. A(1, 2)

$y - 2 = m(x - 1)$ ...(1)



$y = 0 \Rightarrow x = \frac{m-2}{m}$

$x = 0 \Rightarrow y = 2 - m$

$\Delta_{OPQ} = \left|\frac{1}{2}\left(\frac{m-2}{m}\right)(2-m)\right|$

$= \frac{(m-2)^2}{2m} = -\frac{(m^2+4-4m)}{2m} = -\frac{1}{2}\left[m + \frac{4}{m} - 4\right]$

$\frac{d\Delta}{dm} = -\frac{1}{2}\left[1 - \frac{4}{m^2}\right] = 0$

$m^2 = 4 \Rightarrow m = \pm 2$

$\frac{d^2\Delta}{dm^2} = \frac{d}{dm}\left\{-\frac{1}{2}(1 - 4m^{-2})\right\}$

$= 0 + 4(-2)m^{-3} = -\frac{8}{m^3}$

If $m = -2$

$\frac{d^2\Delta}{dm^2} > 0$

Q.74 Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :

(1) 629 (2) 630

(3) 879 (4) 880

Ans. **[3]**

Sol. Number of ways of selection at least one ball = [(10 + 1) (9 + 1) (8 + 1)] –1 = 880 –1 = 879

Q.75 **Statement 1 :** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement 2 : $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

(1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is false.

(4) Statement 1 is false, Statement 2 is true.

Ans. **[1]**

Sol. Statement 2 : $(1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + \ldots\ldots + [n^3 - (n-1)^3] = n^3$

$\therefore$ Statement 2 is correct.

Statement – 1 : $\sum_{k=1}^{20}[k^3 - (k-1)^3] = 20^3$

$\Rightarrow \sum_{k=1}^{20}[k^2 + (k-1)^2 + k(k-1)] = 20^3$

$\Rightarrow [1 + 0 + 0] + [4 + 1 + 2] + [9 + 4 + 6] + [16 + 9 + 12] \ldots\ldots + [400 + 361 + 380] = 8000$

$\therefore$ True

& correct explanation.

Q.76 Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

(1) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$

(2) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

(3) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

(4) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

Ans. **[3]**

Sol. Let $u_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$, $u_2 = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

$Au_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow a = 1,\ 2a + b = 0 \ \Rightarrow b = -2$

$3a + 2b + c = 0$

$\Rightarrow 3 - 4 + c = 0 \Rightarrow c = 1$

$\therefore\ u_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$$Au_2 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\Rightarrow p = 0,\ 2p + q = 1 \ \Rightarrow q = 1$

$3p + 2q + r = 0$

$r = -2$

$$u_2 = \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}$$

$$u_1 + u_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

Q.77 The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is :

(1) $\frac{10\sqrt{2}}{3}$

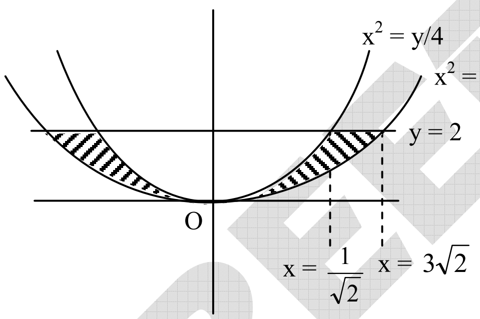
(2) $\frac{20\sqrt{2}}{3}$

(3) $10\sqrt{2}$

(4) $20\sqrt{2}$

Ans. **[2]**

Sol.



$$= 2\left(\int_0^2 3\sqrt{y}\,dy - \int_0^2 \frac{\sqrt{y}}{2}dy\right)$$

$$= 2\left(3\left(\frac{y^{3/2}}{3/2}\right)_0^2 - \left(\frac{y^{3/2}}{2\times 3/2}\right)_0^2\right)$$

$$= 2\left(2(2^{3/2} - 0) - \left(\frac{2^{3/2}}{3} - 0\right)\right)$$

$$= 2\left\{2\times 2\sqrt{2} - \frac{2\sqrt{2}}{3}\right\}$$

$$= 2.\ \frac{10\sqrt{2}}{3} = \frac{20\sqrt{2}}{3}$$

Q.78 Let $x_1, x_2, ..., x_n$ be n observations, and let $\overline{x}$ be their arithmetic mean and σ^2 be their variance.

Statement 1 : Variance of $2x_1, 2x_2, ... 2x_n$ is $4\sigma^2$.

Statement 2 : Arithmetic mean of $2x_1, 2x_2, ..., 2x_n$ is $4\overline{x}$.

(1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is false.

(4) Statement 1 is false, Statement 2 is true.

Ans. **[3]**

Sol. Variance of $x_1, x_2, ... x_n$ is σ^2 then variance of $2x_1, 2x_2, 2x_3$ ...... $2x_n$ is $4\sigma^2$

[If each no. is multiply by λ then variance is λ^2 times the old variance]

mean of $2x_1, 2x_2, 2x_3$ ...... $2x_n$ is $2\,\overline{x}$

so statement 1 is true and statement 2 is not correct.

Q.79 If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is :

(1) an odd positive integer (2) an even positive integer

(3) a rational number other than positive integers (4) an irrational number

Ans. **[4]**

Sol. $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$

$= 2[T_2 + T_4 + T_6 + T_8 + ... + T_{2n}]$

$= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + ... + {}^{2n}C_{2n-1}(\sqrt{3})]$

$= 2\sqrt{3}\,[{}^{2n}C_1(\sqrt{3})^{2n-2} + {}^{2n}C_3(\sqrt{3})^{2n-4} .... + {}^{2n}C_{2n-1}]$

$= \sqrt{3}.\lambda\ \ [\lambda \in I]$

an ir-rational number.

Q.80 If 100 times the 100^{th} term of an AP with non zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this AP is :

(1) 150 times its 50^{th} term (2) 150

(3) zero (4) – 150

Ans. **[3]**

Sol. $100.T_{100} = 50.T_{50}$

$2T_{100} = T_{50}$

$2a + 198\,d = a + 49d$

$a + 149d = 0$

$T_{150} = 0$

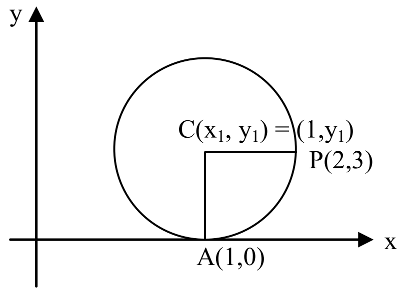
Q.81 The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(1) 3/5 (2) 6/5

(3) 5/3 (4) 10/3

Ans. **[4]**

Sol. $CA = r = |y_1|$



equation of circle

$\Rightarrow (x - x_1)^2 + (y - y_1)^2 = |y_1|^2$

$\Rightarrow (x - 1)^2 + (y - y_1)^2 = y_1^2$

$\Rightarrow (2 - 1)^2 + (3 - y_1)^2 = y_1^2$

$\Rightarrow 1 + 9 + y_1^2 - 6y_1 = y_1^2 \quad \Rightarrow y_1 = \frac{10}{6} = \frac{5}{3}$

$r = |y_1| = \frac{5}{3} \Rightarrow \text{diameter} = 2r = \frac{10}{3}$

Q.82 Let a, b $\in$ R be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$.

Statement 1 : f has local maximum at $x = -1$ and at $x = 2$.

Statement 2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is false.

(4) Statement 1 is false, Statement 2 is true.

Ans. **[1]**

Sol. $f(x) = \frac{1}{x} + 2bx + a$

At $x = -1, 2 \quad \Rightarrow f(x) = 0$

$-1 - 2b + a = 0 \quad \ldots(i)$

$\frac{1}{2} + 4b + a = 0 \quad \ldots(ii)$

By solving $a = \frac{1}{2}, b = -\frac{1}{4}$

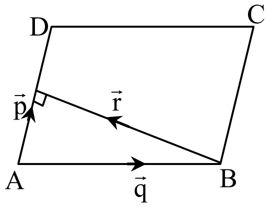
Q.83 Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by :

(1) $\vec{r} = -\vec{q} + \left(\dfrac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$ (2) $\vec{r} = \vec{q} - \left(\dfrac{\vec{p}.\vec{q}}{\vec{p}.\vec{p}}\right)\vec{p}$

(3) $\vec{r} = -3\vec{q} + \dfrac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$ (4) $\vec{r} = 3\vec{q} - \dfrac{3(\vec{p}.\vec{q})}{(\vec{p}.\vec{p})}\vec{p}$

Ans. **[1]**

Sol.



$$-\vec{r} = \vec{q} - \left(\frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\right)\vec{p}$$

$$\vec{r} = -\vec{q} + \left(\frac{\vec{p}.\vec{q}}{|\vec{p}|^2}\right)\vec{p}$$

Q.84 If the lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$ intersect, then k is equal to :

(1) $\dfrac{2}{9}$ (2) $\dfrac{9}{2}$

(3) 0 (4) –1

Ans. **[2]**

Sol.
$$\begin{vmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Solve the determinant, then

$k = \dfrac{9}{2}$

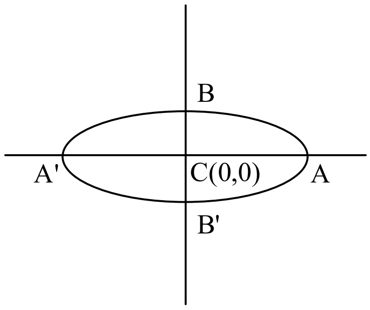
Q.85 An ellipse is drawn by taking a diameter of the circle $(x-1)^2 + y^2 = 1$ as its semi minor axis and a diameter of the circle $x^2 + (y-2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

(1) $x^2 + 4y^2 = 8$ (2) $4x^2 + y^2 = 8$

(3) $x^2 + 4y^2 = 16$ (4) $4x^2 + y^2 = 4$

Ans. **[3]**

Sol.



$(x-1)^2 + y^2 = 1$ ...(1) $\Rightarrow r_1 = 1$

$x^2 + (y-2)^2 = 4$ ...(2) $\Rightarrow r_2 = 2$

CB $\leftarrow$ (Semi minor axis) $2r_1 = 2(1) = 2 = b$

CA $\leftarrow$ (Semi major axis) $2r_2 = 2(2) = 4 = a$

Equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1$

Q.86 The negation of the statement "If I become a teacher, then I will open a school", is :

(1) Either I will not become a teacher or I will not open a school

(2) Neither I will become a teacher nor I will open a school

(3) I will not become a teacher or I will open a school

(4) I will become a teacher and I will not open a school

Ans. **[4]**

Sol. P : I become a teacher

q : I will open a school

"If I become a teacher, then I will open a school."

is $p \to q$ so negation of $\sim(p \to q) = p \wedge \sim q$

which is I will become a teacher and I will not open a school.

Q.87 Consider the function, $f(x) = |x-2| + |x-5|$, $x \in R$.

Statement 1 : $f'(4) = 0$

Statement 2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

(1) Statement 1 is true, Statement 2 is true, Statement 2 is a correct explanation for Statement 1.

(2) Statement 1 is true, Statement 2 is true, Statement 2 is not a correct explanation for Statement 1.

(3) Statement 1 is true, Statement 2 is false.

(4) Statement 1 is false, Statement 2 is true.

Ans. **[2]**

Sol. $f(x) = |x-2| + |x-5|$, $x \in R$

$2 < x < 5$ $f(x) = x - 2 + 5 - x$

$f(x) = 3$

Now statement-I : $f'(x) = 0$ at $2 < x < 5$

Statement 2 : f is continuous in [2, 5]

Differentiable in (2, 5) and f (2) = f (5) (Rolle's theorem)

Q.88 If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

(1) on a circle with centre at the origin.

(2) either on the real axis or on a circle not passing through the origin.

(3) on the imaginary axis.

(4) either on the real axis or on a circle passing through the origin.

Ans. **[4]**

Sol. $\frac{z^2}{z-1} = \frac{z^2(\bar{z}-1)}{|z-1|^2}$ is purely real

$\Rightarrow z^2.\ \bar{z} - z^2$

$= z.\ |z|^2 - z^2$

$= (x^2 + y^2)(x + iy) - (x^2 - y^2 + 2ixy)$ is purely real

$\Rightarrow y(x^2 + y^2) - 2xy = 0$

$\Rightarrow y[x^2 + y^2 - 2x] = 0$

$y = 0$ or $x^2 + y^2 - 2x = 0$

Imaginary axis or circle passing through origin.

Q.89 The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

(1) no real roots.

(2) exactly one real root.

(3) exactly four real roots.

(4) infinite number of real roots.

Ans. **[1]**

Sol. $e^{\sin x} = t$ (let)

$t - \frac{1}{t} = 4$

$t^2 - 4t - 1 = 0$

$t = \frac{4 \pm \sqrt{20}}{2} = 2 \pm \sqrt{5}$

$e^{\sin x} = 2 + \sqrt{5} > e$ (not possible)

$e^{\sin x} = 2 - \sqrt{5} = -$ ve number (not possible)

No real root.

Q.90 In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$

(3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{6}$

Ans. **[1]**

Sol. $P + Q + R = \pi$

$9 \sin^2 P + 16 \cos^2 Q + 24 \sin P \cos Q = 36$

$16 \sin^2 Q + 9 \cos^2 P + 24 \sin Q \cos P = 1$

$25 + 24 \sin (P + Q) = 37$

$\sin (P + Q) = \frac{12}{24} = \frac{1}{2}$

$P + Q = \frac{\pi}{6}, \frac{5\pi}{6}$

$P + Q = \frac{\pi}{6}$ does not satisfy

$P + Q = \frac{5\pi}{6}$

$\therefore R = \frac{\pi}{6}$