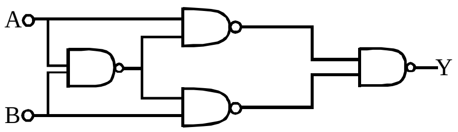
AIEEE – 2012 TEST PAPER WITH ANSWER
(HELD ON SUNDAY 29TH APRIL, 2012)

PART A – PHYSICS

1. Truth table for system of four NAND gates as shown in figure is :-



(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(4)

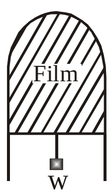
A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

Ans. **(2)**

Sol. By putting A = B = 0 we get Y = 0
and by putting A = B = 1 we get Y = 0
So correct answer is (2)

2. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is :-

(1) 0.025 Nm^{-1}
(2) 0.0125 Nm^{-1}
(3) 0.1 Nm^{-1}
(4) 0.05 Nm^{-1}



Ans. **(1)**

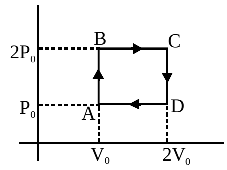
Sol. weight = mg = 1.5×10^{-2} N (given)
length = ℓ = 30 cm (given)
= 0.3 m

$2T\ell = mg$

$$T = \frac{mg}{2\ell} = \frac{1.5\times10^{-2}}{2\times0.3} = 0.025 \text{ N/m.}$$

3. Helium gas goes through a cycle ABCDA (consisting of two isochoric and two isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) :-

(1) 12.5%
(2) 15.4%
(3) 9.1%
(4) 10.5%



Ans. **(2)**

Sol. W = Area bounded by curve = P_0V_0

$$Q_{AB} = nC_V\Delta T = n \times \frac{3}{2} R \times \Delta T = \frac{3}{2}P_0V_0$$

$$Q_{BC} = nC_P \,\Delta T = n \times \frac{5}{2} R \times \Delta T = 5\, P_0V_0$$

$$\text{Total heat supplied} = \frac{3}{2}P_0V_0 + 5P_0V_0 = \frac{13}{2}P_0V_0$$

$$\eta = \frac{W}{Q} \times 100 = \frac{P_0V_0}{13P_0V_0} \times 100 = 15.4\%$$

4. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be :-

(1) 6 (2) 2 (3) 3 (4) 5

Ans. **(1)**

Sol. Number of lines = ${}^nC_2 = {}^4C_2 = \frac{(4)(3)}{(2)} = 6$

5. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :-

(1) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping
(2) Development of air current when the plate is placed
(3) Induction of electrical charge on the plate
(4) Shielding of magnetic lines of force as aluminium is a paramagnetic material

Ans. **(1)**

Sol. Due to conducting nature of Al eddy currents are produced

6. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data :

(1) 59 degree (2) 58.59 degree
(3) 58.77 degree (4) 58.65 degree

Ans. **(4)**

Sol. Least count = MSD – VSD
where MSD = 0.5° & 30VSD = 29MSD

so least count = $0.5° - \left(\frac{29}{30}\right) \times 0.5° = \left(\frac{0.5}{30}\right)^{o}$

Reading = $58.5° + (09) \times \left(\frac{0.5}{30}\right)^{o} = 58.65°$

7. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by :

(1) $\frac{I_m}{9}(1 + 8\cos^2\frac{\phi}{2})$ (2) $\frac{I_m}{9}(4 + 5\cos\phi)$

(3) $\frac{I_m}{3}(1 + 2\cos^2\frac{\phi}{2})$ (4) $\frac{I_m}{5}(1 + 4\cos^2\frac{\phi}{2})$

Ans. **(1)**

Sol. $I_{max} = \left(\sqrt{I} + \sqrt{4I}\right)^2 = 9I = I_m \Rightarrow I = \frac{I_m}{9}$

$I_P = I + 4I + 2\sqrt{(I)(4I)}\cos\phi$

$= 5I + 4I\cos\phi$

$= I + 4I(1 + \cos\phi)$

$= I + 8I\cos^2\frac{\phi}{2}$

$= \frac{I_m}{9}\left(1 + 8\cos^2\frac{\phi}{2}\right)$

8. Proton, Deuteron and alpha particle of the same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relations is correct ?

(1) $r_\alpha = r_d > r_p$ (2) $r_\alpha = r_d = r_p$
(3) $r_\alpha = r_p < r_d$ (4) $r_\alpha > r_d > r_p$

Ans. **(3)**

Sol. $r \propto \frac{\sqrt{2mK}}{qB}$ As K & B are constant

So $r \propto \frac{\sqrt{m}}{q}$

$r_p : r_d : r_\alpha :: \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$

$:: \frac{\sqrt{m_p}}{e} : \frac{\sqrt{2m_p}}{e} : \frac{\sqrt{4m_p}}{2e}$

$:: 1 : \sqrt{2} : 1$

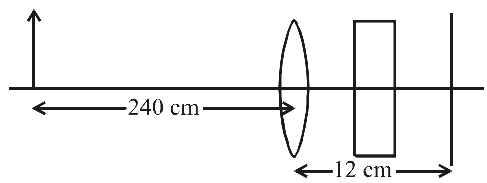
$\Rightarrow r_\alpha = r_p < r_d$

9. An object 2.4 m in front of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object be shifted to be in sharp focus on film ?

(1) 5.6 m (2) 7.2 m (3) 2.4 m (4) 3.2 m

Ans. **(1)**

Sol.



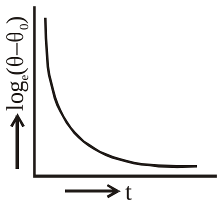
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{(-240)} \Rightarrow f = \frac{240}{21}$

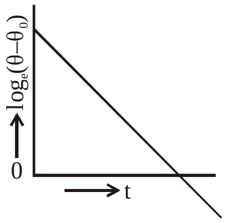
Shift = $h\left(1 - \frac{1}{\mu}\right) = 1\left(1 - \frac{1}{1.5}\right) = \frac{1}{3}$ cm

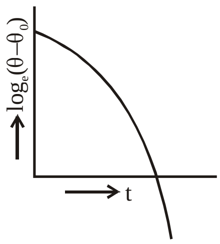
To get image at film, lens should form image at distance = $12 - \frac{1}{3} = \frac{35}{3}$

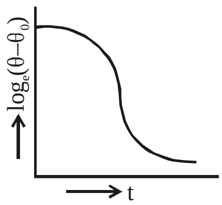
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{21}{240} = \frac{3}{35} - \frac{1}{u} \Rightarrow u = -560$ cm

10. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling the correct graph between $\log_e(\theta - \theta_0)$ and t is :-

(1)


(2)


(3)


(4)


Ans. (2)

Sol. Rate of cooling $= -\dfrac{d\theta}{dt} = k(\theta - \theta_0)$

$$\frac{d\theta}{\theta - \theta_0} = -kdt$$

On integrating $\displaystyle\int \frac{d\theta}{\theta - \theta_0} = -\int kdt$

$\Rightarrow \ell n(\theta - \theta_0) = -k\,t + C$

$\Rightarrow$ correct answer will be (2)

11. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement-1: If stretched by the same amount, work done on S_1, will be more than that on S_2

Statement-2 : $k_1 < k_2$

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

Ans. (2)

Sol. Given same force $F = k_1x_1 = k_2x_2$

$$\Rightarrow \frac{k_1}{k_2} = \frac{x_2}{x_1}$$

$$W_1 = \frac{1}{2}k_1x_1^2 \ \& \ W_2 = \frac{1}{2}k_2x_2^2$$

As $\dfrac{W_1}{W_2} > 1$ so $\dfrac{\frac{1}{2}k_1x_1^2}{\frac{1}{2}k_2x_2^2} > 1$

$$\Rightarrow \frac{Fx_1}{Fx_2} > 1 \Rightarrow \frac{k_2}{k_1} > 1$$

$\therefore$ $k_2 > k_1$ statement 2 is true

OR

if $x_1 = x_2 = x$

$$\frac{W_1}{W_2} = \frac{\frac{1}{2}K_1x^2}{\frac{1}{2}K_2x^2} = \frac{K_1}{K_2}$$

$$\therefore \frac{W_1}{W_2} = \frac{K_1}{K_2} < 1$$

$\therefore W_1 < W_2$

statement 1 is false

12. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1: Davisson-Germer experiment established the wave nature of electrons.

Statement-2 : If electrons have wave nature, they can interfere and show diffraction.

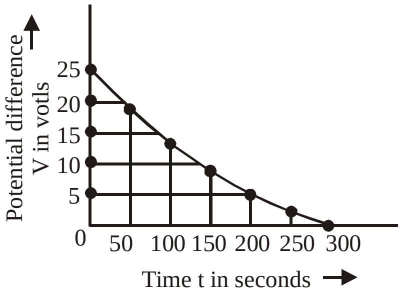
(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

Ans. (4)

13. 

The figure shows an experimental plot for discharging of a capacitor in an R–C circuit. The time constant τ of this circuit lies between:-

(1) 100 sec and 150 sec

(2) 150 sec and 200 sec

(3) 0 and 50 sec

(4) 50 sec and 100 sec

Ans. **(1)**

Sol. τ = 0.37% of V_0

= 0.37 × 25 = 9.25 volt

where is in between 100 and 150 sec.

14. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is :-

(1) 3% (2) 6% (3) zero (4) 1%

Ans. **(2)**

Sol. $R = \dfrac{V}{I} \Rightarrow \dfrac{\Delta R}{R} = \pm\left(\dfrac{\Delta V}{V} + \dfrac{\Delta I}{I}\right)$

$= \pm(3 + 3)\% = \pm 6\%$

15. A Carnot engine, whose efficiency is 40% takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :-

(1) 600 K

(2) efficiency of Carnot engine cannot be made larger than 50%

(3) 1200 K

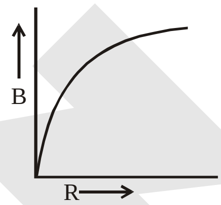
(4) 750 K

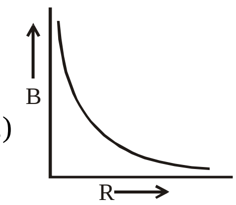
Ans. **(4)**

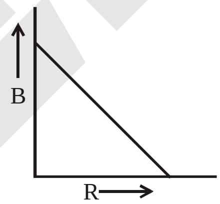
Sol. $\eta = \left(1 - \dfrac{T_2}{T_1}\right) \times 100 \Rightarrow \dfrac{40}{100} = 1 - \dfrac{T_2}{500} \Rightarrow T_2 = 300\text{ K}$

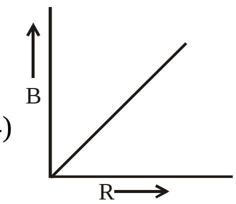
Again $\dfrac{60}{100} = 1 - \dfrac{300}{T_1} \Rightarrow T_1 = 750\text{ K}$

16. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure :-

(1) 

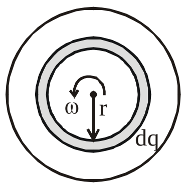
(2) 

(3) 

(4) 

Ans. **(2)**

Sol. Magnetic field at the centre due to element ring

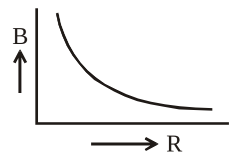


$$dB = \frac{\mu_0 di}{2r} = \frac{\mu_0}{2r}\left(\frac{dq}{T}\right) = \frac{\mu_0 \sigma ds}{2rT}$$

$$= \frac{\mu_0}{2r}\left(\frac{Q}{\pi R^2}\right)\frac{(2\pi r dr)}{2\pi/\omega} = \frac{\mu_0 Q dr \omega}{\pi R^2}$$

$$B = \frac{\mu_0 Q \omega}{\pi R^2}\int_0^R dr = \frac{\mu_0 Q \omega}{\pi R^2} R = \frac{\mu_0 Q \omega}{\pi R}$$

$\boxed{B \propto \dfrac{1}{R}}$



17. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth = 6.4×10^6 m) is :-

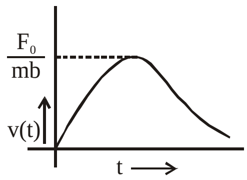
(1) 64 km (2) 80 km
(3) 16 km (4) 40 km

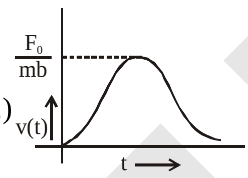
Ans. (2)

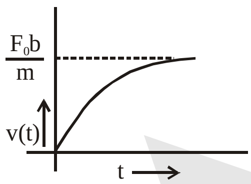
Sol. As $d = \sqrt{2Rh}$

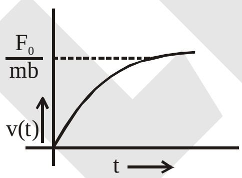
$$= \sqrt{2 \times 6.4 \times 10^6 \times 500} = 80 \text{ km}$$

18. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves ?

(1) 

(2) 

(3) 

(4) 

Ans. (4)

Sol. $F(t) = F_0e^{-bt}$

$$m\frac{dV}{dt} = F_0e^{-bt}$$

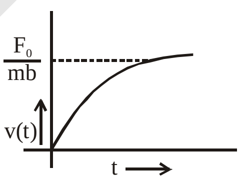
$$\int m dV = \int F_0 e^{-bt} dt$$

$$mV = -\frac{F_0}{b}e^{-bt} + C$$

at t = 0, V = 0

$$\therefore \quad V = -\frac{F_0}{mb}e^{-bt} + \frac{F_0}{mb}$$

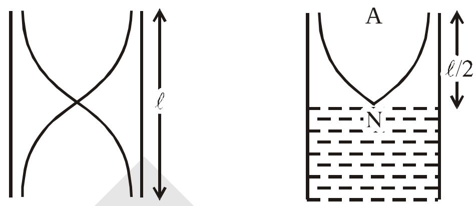
$$V = \frac{F_0}{mb}(1 - e^{-bt})$$



19. A cylindrical tube, open at both ends, has a fundamental frequency, f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now :-

(1) 2f (2) f (3) f/2 (4) 3f/4

Ans. (2)

Sol. 

$$n_0 = \frac{v}{2\ell} \qquad n_c = \frac{v}{4(\ell/2)} = \frac{v}{2\ell}$$

20. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of 'g' and 'R' (radius of earth) are 10 m/s^2 and 6400 km respectively. The required energy for this work will be :-

(1) 6.4×10^{10} Joules (2) 6.4×10^{11} Joules
(3) 6.4×10^{8} Joules (4) 6.4×10^{9} Joules

Ans. (1)

Sol. $PE_i + KE_i = PE_f + KE_f$
$-mgR + KE_i = 0 + 0$
$KE_i = +mgR = 1000 \times 10 \times 6.4 \times 10^6$
work done $= 6.4 \times 10^{10}$ J

21. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be :-

(1) 20 m (2) $20\sqrt{2}$ m (3) 10 m (4) $10\sqrt{2}$ m

Ans. (1)

Sol. $H_{max} = \frac{u^2}{2g} = 10$ m

and

$R_{max} = \frac{u^2}{g} = 20$ m

22. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0 s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :-

(1) $\frac{2}{b}$ (2) $\frac{0.693}{b}$ (3) b (4) $\frac{1}{b}$

Ans. (1)

Sol. Equation of damped simple pendulum

$$\frac{d^2x}{dt^2} = -bv + g\sin\theta$$

$$\Rightarrow \frac{d^2x}{dt^2} = -bv + \frac{g}{\ell}x = 0$$

By solving above equation

$$x = A_0 e^{-\frac{b}{2}t}\sin(\omega t + \phi)$$

At $t = \tau$, $A = \frac{A_0}{2}$ so $\tau = \frac{2}{b}$

23. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarizaiton is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then :-

(1) $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{B}\times\vec{E}$

(2) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{B}\times\vec{E}$

(3) $\vec{X} \| \vec{E}$ and $\vec{k} \| \vec{E}\times\vec{B}$

(4) $\vec{X} \| \vec{B}$ and $\vec{k} \| \vec{E}\times\vec{B}$

Ans. (3)

Sol. The direction of polarization is represented by direction of $\vec{E}$ and wave propagates ⊥ to the $\vec{E}$ and $\vec{B}$ such that $\vec{k} \| \vec{E}\times\vec{B}$

24. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

An insulating solid sphere of radius R has a uniformaly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphre and also at a point out side the sphere. The electric potential at infinity is zero.

Statement-1: When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\epsilon_0}$

Statement-2 : The electric field at a distance r (r < R) from the centre of the sphere is $\frac{\rho r}{3\epsilon_0}$

(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.

(2) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1.

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (4)

25. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by : (n is an integer)

(1) $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$ (2) $\frac{(m_1+m_2)^2n^2h^2}{2m_1^2m_2^2r^2}$

(3) $\frac{n^2h^2}{2(m_1+m_2)r^2}$ (4) $\frac{2n^2h^2}{(m_1+m_2)r^2}$

Ans. (1)

Sol. Energy $= \frac{L^2}{2I} = \frac{(n\hbar)^2}{2(\mu r^2)}$

where μ = reduced mass $= \frac{m_1m_2}{m_1+m_2}$

so energy $= \frac{n^2\hbar^2(m_1+m_2)}{2m_1m_2r^2}$

26. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is :-

(1) 1 : 1 (2) $m_1 r_1 : m_2 r_2$

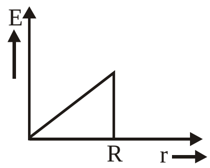
(3) $m_1 : m_2$ (4) $r_1 : r_2$

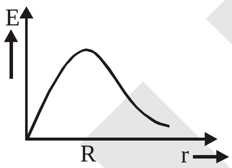
Ans. **(4)**

Sol. As $T_1 = T_2 \Rightarrow \omega_1 = \omega_2$

$$a_c = \omega^2 r \quad \Rightarrow \quad \frac{a_{c_1}}{a_{c_2}} = \frac{r_1}{r_2}$$

27. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be :-

(1)


(2)


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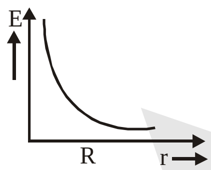
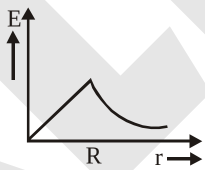


(4)


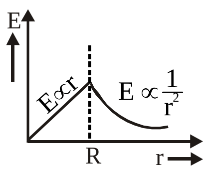
Ans. **(4)**

Sol. For uniformly charged sphere

$$E = \frac{Kqr}{R^3} \quad (r < R)$$

$$E = \frac{Kq}{R^2} \quad (r = R)$$

$$E = \frac{Kq}{r^2} \quad (r > R)$$

so


28. Assume that a neutron breaks into a proton and an electron. The energy released during this process is :

(Mass of neutron = 1.6725×10^{-27} kg

Mass of proton = 1.6725×10^{-27} kg

Mass of electron = 9×10^{-31} kg)

(1) 5.4 MeV (2) 0.73 MeV

(3) 7.10 MeV (4) 6.30 MeV

Ans. **(Bonus)**

Sol. **If** mass of neutron = 1.6747×10^{-27} kg then answer will be (2)

Released energy = $[1.6747 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31}] \times (3 \times 10^8)^2$ J = 0.73 MeV

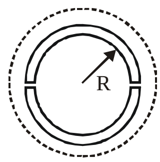
29. A wooden wheel of radius R is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross sectional area S and Length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is α, and its Young's modulus is Y, the force that one part of the wheel applies on the other part is :

(1) $2SY\alpha\Delta T$

(2) $2\pi SY\alpha\Delta T$

(3) $SY\alpha\Delta T$

(4) $\pi SY\alpha\Delta T$



Ans. **(1)**

Sol. $Y = \dfrac{\text{stress}}{\text{strain}} \Rightarrow \text{stress} = Y \times \text{strain}$

$$\frac{F}{S} = Y(\alpha\Delta T) \quad \Rightarrow \quad F = YS\alpha\Delta T$$

Therefore force by one part on other part $= 2F = 2SY\alpha\Delta T$

30. Two electric bulbs marked 25W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse ?

(1) Neither (2) Both

(3) 100 W (4) 25 W

Ans. **(4)**

Sol. Due to greater heating as $H = I^2R$

25W get fused.

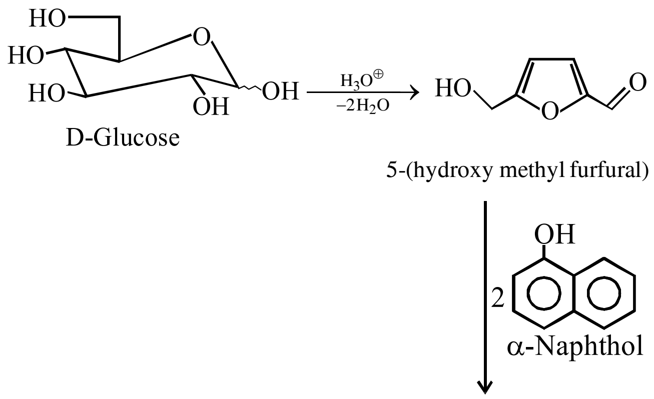
PART C – CHEMISTRY

61. Which of the following compounds can be detected by Molisch's test ?

(1) Primary alcohols (2) Nitro compounds
(3) Sugars (4) Amines

Ans (3)

Sol. Molisch's test is used to detect carbohydrates eg. sugars.



62. The increasing order of the ionic radii of the given isoelectronic species is :-

(1) $K^+, S^{2-}, Ca^{2+}, Cl^-$ (2) $Cl^-, Ca^{2+}, K^+, S^{2-}$
(3) $S^{2-}, Cl^-, Ca^{2+}, K^+$ (4) $Ca^{2+}, K^+, Cl^-, S^{2-}$

Ans (4)

Sol. In isoelectronic ions

$\left(\frac{Z}{e}\right) \propto \frac{1}{\text{size}}$

	Ca^{+2}	K^+	Cl^-	S^{-2}
$\frac{Z}{e} =$	$\frac{20}{18}$	$\frac{19}{18}$	$\frac{17}{18}$	$\frac{16}{18}$

Size $Ca^{+2} < K^+ < Cl^- < S^{-2}$

63. Which one of the following statements is correct ?

(1) All amino acids except glutamic acid are optically active
(2) All amino acids except lysine are optically active
(3) All amino acids are optically active
(4) All amino acids except glycine are optically active

Ans (4)

Sol. Glycine $NH_2–CH_2–COOH$ [1st member of Amino acid] is optically inactive while.

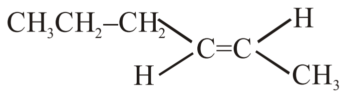
other Amino acids $\left[\begin{array}{c} NH_2–\overset{*}{C}H–COOH \\ \quad | \\ \quad R \end{array}\right]$ are optically active.

64. 2–Hexyne gives trans –2–Hexene on treatment with :-

(1) $Li\ AlH_4$ (2) Pt/H_2
(3) Li/NH_3 (4) $Pd/BaSO_4$

Ans (3)

Sol. $CH_3–CH_2–CH_2–C\equiv C–CH_3 \xrightarrow{Li / Liq.\ NH_3}$



trans-2-hexene

65. The species which can best serve as an initiator for the cationic polymerization is :-

(1) BuLi (2) $LiAlH_4$ (3) HNO_3 (4) $AlCl_3$

Ans (4)

Sol. Lewis acid $AlCl_3$ is best initiater because it is non oxidising agent where are HNO_3 is oxidising agent.

66. The standard reduction potentials for Zn^{2+} / Zn, Ni^{2+} / Ni and Fe^{2+} / Fe are – 0.76, – 0.23 and – 0.44 V respectively. The reaction $X + Y^{+2} \rightarrow X^{2+} + Y$ will be spontaneous when

(1) X = Zn, Y = Ni
(2) X = Ni, Y = Fe
(3) X = Ni, Y = Zn
(4) X = Fe, Y = Zn

Ans (1)

Sol. $E^o_{cell} = E^o_{X/X^{+2}} + E^o_{Y^{+2}/Y}$

X = Zn Y = Ni

$E^o_{cell} = E^o_{Zn/Zn^{+2}} + E^o_{Ni^{+2}/Ni}$

$= 0.76 - 0.23 = 0.53$ V

$E^o_{cell} = +ve$

$\Delta G^\circ = -ve$ so spontaneous process

67. Lithium forms body centred cubic structure. The length of the side of its unit cell is 351 pm. Atomic radius of the lithium will be :-

(1) 152 pm (2) 75 pm
(3) 300 pm (4) 240 pm

Ans (1)

Sol. For BCC $r = \frac{\sqrt{3}a}{4}$ a = 351 pmr

$= \frac{\sqrt{3} \times 351}{4}$

r = 151.80 pm

r ≈ 152 pm (approx)

68. The electrons identified by quantum numbers n and ℓ :-
(a) $n = 4$, $\ell = 1$ (b) $n = 4$, $\ell = 0$
(c) $n = 3$, $\ell = 2$ (d) $n = 3$, $\ell = 1$
Can be placed in order of increasing energy as
(1) (a) < (c) < (b) < (d)
(2) (c) < (d) < (b) < (a)
(3) (d) < (b) < (c) < (a)
(4) (b) < (d) < (a) < (c)

Ans **(3)**

Sol. Energy $\propto (n + \ell)$ a {from $(n + \ell)$ rule}. If $(n + \ell)$ is same than energy will be higher for higher value of 'n'.
(a) $n + \ell = 4 + 1 = 5$
(b) $4 + 0 = 4$
(c) $3 + 2 = 5$
(d) $3 + 1 = 4$
In (a) & (c) $n + \ell = 5$, n is more in (a) So (a) > (c) and in (b) & (d) $n + \ell = 4$, n is more in (b) so - (b) > (d) so order of energy (a) > (c) > (b) > (d)

69. According to Freundlich adsorption isotherm, which of the following is correct ?
(1) $\frac{x}{m} \propto p^0$ (2) $\frac{x}{m} \propto p^1$
(3) $\frac{x}{m} \propto p^{1/n}$
(4) All the above are correct for different ranges of pressure

Ans **(4)**

Sol. According to Freundlich adsorption isotherm
$\frac{x}{m} = Kp^{1/n}$
At very low pressure $\frac{1}{n} = 1$
$$\frac{x}{m} = Kp$$
At very high pressure $\frac{1}{n} = 0$
$$\frac{x}{m} = K(P)^\circ$$
At moderate pressure
$$\frac{x}{m} = KP^{1/n}$$

70. The density of a solution prepared by dissolving 120 g of urea (mol. mass = 60 u) in 1000 g of water is 1.15 g/mL. The molarity of this solution is :-
(1) 2.05 M (2) 0.50 M
(3) 1.78 M (4) 1.02 M

Ans **(1)**

Sol. Molarity = $\frac{\text{wt. of solute}}{\text{mol. wt.}} \times \frac{1}{\text{vol. of sol}^n.(\text{Lt.})}$
Vol. = $\frac{M}{d} = \frac{\text{mass of sol}^n.}{\text{density of sol}^n.} = \frac{120+1000}{1.15}$
M = $\frac{120}{60} \times \frac{1.15+1000}{1120}$
M = 2.05

71. The pH of a 0.1 molar solution of the acid HQ is 3. The value of the ionization constant, Ka of this acid is :-
(1) 1×10^{-7} (2) 3×10^{-7}
(3) 1×10^{-3} (4) 1×10^{-5}

Ans **(4)**

Sol. $pH = -\log[H^+]$ $pH = 3$
$[H^+] = 10^{-pH} = 10^{-3}$
For weak acid
$[H^+] = \sqrt{K_aC}$
$[H^+]^2 = K_a \times C$
$(10^{-3})^2 = K_a \times 0.1$
$K_a = 10^{-5}$

72. The incorrect expression among the following is :-
(1) $K = e^{-\Delta G^\circ/RT}$
(2) $\frac{\Delta G_{system}}{\Delta S_{total}} = -T$
(3) In isothermal process,
$$W_{reversible} = -nRT \ln \frac{V_f}{V_i}$$
(4) $\ln K = \frac{\Delta H^\circ - T\Delta S^\circ}{RT}$

Ans **(4)**

Sol. (1) $\Delta G^o = -RT\ell nK$

$K = e^{-\frac{\Delta G^o}{RT}}$

(2) $\frac{\Delta G_{system}}{\Delta S_{total}} = -T$

(3) In isothermal process

$W_{rev.} = -nRT\ell n\,\frac{V_f}{V_i}$

(4) $\Delta G^o = \Delta H^o - T\Delta S^o$

$-RT\ell nK = \Delta H^o - T\Delta S^o$

$\ell nK = \frac{\Delta H^o - T\Delta S^o}{-RT}$

Hence option (4) given in the question is incorrect.

73. Iodoform can be prepared from all except :-

(1) Isobutyl alcohol

(2) Ethyl methyl ketone

(3) Isopropyl alcohol

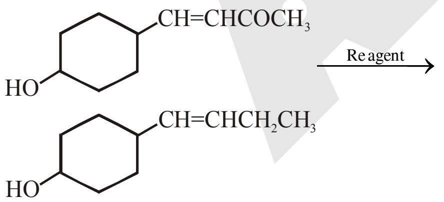
(4) 3-Methyl–2–butanone

Ans **(1)**

Sol. $CH_3-\underset{\underset{O}{||}}{C}-$ / $CH_3-\underset{\underset{OH}{|}}{CH}-$ groups containing compounds give iodoform test

$CH_3-\underset{\underset{CH_3}{|}}{CH}-CH_2-OH$ (isobutyl alcohol) does not give iodoform test

74. In the given transformation, which of the following is the most appropriate reagent ?

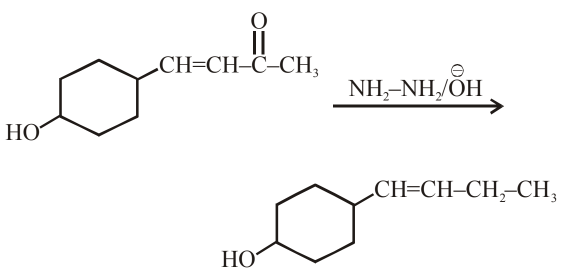


(1) $NaBH_4$ (2) $NH_2\,NH_2, \overset{\ominus}{O}H$

(3) Zn – Hg / HCl (4) Na, Liq.NH_3

Ans **(2)**

Sol.



$NH_2-NH_2/\overset{\ominus}{O}H$ [Wolf kischner reduction] reduce only carbonyl group.

Zn-Hg / HCl affects alcohol, carbonyl compounds and double bond.

75. Very pure hydrogen (99.9%) can be made by which of the following processes ?

(1) Reaction of salt like hydrides with water

(2) Reaction of methane with steam

(3) Mixing natural hydrocarbons of high molecular weight

(4) Electrolysis of water

Ans **(4)**

Sol. There is no impurity present in hydrogen obtained by the electrolysis of pure water.

$2H_2O \xrightarrow{\text{Electrolysis}} 2H_2 + O_2$

76. Which among the following will be named as dibromidobis (ethylene diamine) chromium (III) bromide ?

(1) $[Cr(en)Br_2]Br$

(2) $[Cr(en)_3]Br_3$

(3) $[Cr(en)_2Br_2]Br$

(4) $[Cr(en)Br_4]^-$

Ans **(3)**

77. Ortho-Nitrophenol is less soluble in water than p– and m– Nitrophenols because :-

(1) Melting point of o–Nitrophenol is lower than those of m– and p– isomers

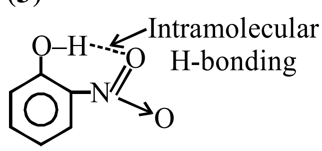
(2) o–Nitrophenol is more volatile in steam than those of m– and p– isomers

(3) o–Nitrophenol shows Intramolecular H–bonding

(4) o–Nitrophenol shows Intermolecular H–bonding

Ans **(3)**

Sol.



due to presence of intramolecular H-bonding its interaction with water molecule will be poor, resulting into less solubility of the compound. While para & meta derivative forms intermolecular H-bonding with water.

78. How many chiral compounds are possible on monochlorination of 2–methyl butane ?

(1) 6 (2) 8 (3) 2 (4) 4

Ans **(4)**

Sol. $CH_3-CH(CH_3)-CH_2-CH_3 \longrightarrow CH_3-CH(CH_3)-\overset{*}{C}H(Cl)-CH_3$;

d + ℓ form = 2

$CH_2(Cl)-\overset{*}{C}H(CH_3)-CH_2-CH_3$;

d + ℓ form = 2

$CH_3-C(Cl)(CH_3)-CH_2-CH_3$;

$CH_3-CH(CH_3)-CH_2-CH_2Cl$

No.s of chiral compound = 4

79. Iron exhibits +2 and +3 oxidation states. Which of the following statements about iron is **incorrect** ?

(1) Ferrous compounds are more easily hydrolysed than the corresponding ferric compounds.

(2) Ferrous oxide is more basic in nature than the ferric oxide.

(3) Ferrous compounds are relatively more ionic than the corresponding ferric compounds.

(4) Ferrous compounds are less volatile than the corresponding ferric compounds.

Ans **(1)**

Sol. Ferrous salt $\rightarrow Fe^{+2}$

Ferric salt $\rightarrow Fe^{+3}$

ϕ (Ionic potential) = $\dfrac{\text{charge}}{\text{radius}}$

Fe^{+2} has low value of ϕ compared to the Fe^{+3}, therefore

(1) Ferrous compounds will be less hydrolysed in comparison of Ferric.

(2) Ferrous oxide will be more basic than Ferric oxide.

(3) Ferrous compounds are relatively more ionic than Ferric compounds.

(4) Ferrous compounds have higher ionic character i.e. less volatile.

80. What is DDT among the following :

(1) Non-biodegradable pollutant

(2) Greenhouse gas

(3) A fertilizer

(4) Biodegradable pollutant

Ans **(1)**

Sol. DDT is a non-biodegradable pollutant.

81. K_f for water is 1.86 K kg mol^{-1}. If your automobile radiator holds 1.0 kg of water, how many grams of ethylene glycol ($C_2H_6O_2$) must you add to get the freezing point of the solution lowered to –2.8°C ?

(1) 27 g (2) 72 g

(3) 93 g (4) 39 g

Ans **(3)**

Sol. $\because \Delta T_f = \dfrac{1000 \times W_{solute} \times K_f \times i}{(\text{Mol.wt.})_{solute} \times W_{solvent}(\text{in gm})}$

$2.8 = \dfrac{1000 \times W_{solute} \times 1.86 \times 1}{62 \times 1000}$

$W_{solute} = 93.3 \text{ gm} \simeq 93 \text{ gm}$

82. Which method of purification is represented by the following equation :

$Ti(s) + 2I_2(g) \xrightarrow{523K} TiI_4(g) \xrightarrow{1700K} Ti(s) + 2I_2(g)$

(1) Van Arkel (2) Zone refining

(3) Cupellation (4) Poling

Ans **(1)**

Sol. Vapour phase refining of the metals like Ti, Zr & Hf through volatile iodide formation is called Van-Arkel process.

83. Which branched chain isomer of the hydrocarbon with molecular mass 72 u gives only one isomer of mono substituted alkyl halide ?

(1) Neohexane

(2) Tertiary butyl chloride

(3) Neopentane

(4) Isohexane

Ans **(3)**

Sol. $CH_3-C(CH_3)_2-CH_3 \xrightarrow{X_2,hv} CH_3-C(CH_3)_2-CH_2-X$

Neopentane [C_5H_{12} = 72 u] — Only one mono substituted product

84. The equilibrium constant (K_C) for the reaction $N_2(g) + O_2(g) \longrightarrow 2NO(g)$ at temperature T is 4×10^{-4}. The value of K_C for the reaction. $NO(g) \longrightarrow \frac{1}{2}N_2(g) + \frac{1}{2}O_2(g)$ at the same temperature is :-

(1) 50.0 (2) 0.02

(3) 2.5×10^2 (4) 4×10^{-4}

Ans **(1)**

Sol. $N_2 + O_2 \rightarrow 2NO \qquad K_C = 4 \times 10^{-4}$

$$2NO \rightarrow N_2 + O_2 \qquad K_C' = \frac{1}{K_C} = \frac{1}{4\times10^{-4}}$$

$$NO \rightarrow \frac{1}{2}N_2 + \frac{1}{2}O_2 \qquad K_C'' = \sqrt{K_C'}$$

$$= \sqrt{\frac{1}{4\times10^{-4}}}$$

$$= \frac{10^2}{2} = 50$$

85. For a first order reaction, (A) $\rightarrow$ products, the concentration of A changes from 0.1 M to 0.025 M in 40 minutes. The rate of reaction when the concentration of A is 0.01 M is :-

(1) 1.73×10^{-4} M/min

(2) 1.73×10^{-5} M/min

(3) 3.47×10^{-4} M/min

(4) 3.47×10^{-5} M/min

Ans **(3)**

Sol. For first order reaction

$$\because\ K = \frac{2.303}{t}\log_{10}\frac{[A]_0}{[A]_t}$$

Given $[A]_0 = 0.1$ M, $[A]_t = 0.025$ M

$$\therefore\ K = \frac{2.303}{40}\log_{10}\frac{0.1}{0.025} = \frac{0.693}{20}\text{min}^{-1}$$

Rate = K $[A]^1$

$$\text{Rate} = \frac{0.693}{20} \times 0.01 = 0.0003465 \text{ M/min}$$

$= 3.47 \times 10^{-4}$ M/min

86. Aspirin is known as :-

(1) Methyl salicylic acid

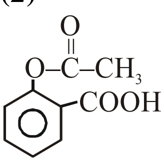
(2) Acetyl salicylic acid

(3) Phenyl salicylate

(4) Acetyl salicylate

Ans **(2)**

Sol.



Aspirin [Acetyl salicylic acid]

87. The molecule having smallest bond angle is :-

(1) PCl_3 (2) NCl_3

(3) $AsCl_3$ (4) $SbCl_3$

Ans **(4)**

Sol. Bond angle $\propto$ Electronegativity of central atom [When hybridisation & no. of lp & bp are same] since Sb is least electronegative among them that is why it's bond angle would be least

OR

In the similar compounds the requirement of hybridisation will be least for largest size central atom therefore extent of hybridisation is minimum in Sb resulting into lower bond angle.

88. The compressibility factor for a real gas at high pressure is :-

(1) $1-\frac{pb}{RT}$ (2) $1+\frac{RT}{pb}$

(3) 1 (4) $1+\frac{pb}{RT}$

Ans **(4)**

Sol. $\because$ For real gas, according to Vander wall's equation

$$\left(P+\frac{a}{V^2}\right)(V-b) = RT \qquad \text{(For 1 mole gas)}$$

At high pressure

$$\left(P+\frac{a}{V^2}\right) \simeq P$$

$\therefore$ $P(V - b) = RT$

$PV = RT + Pb$

$$\frac{PV}{RT} = 1 + \frac{Pb}{RT}$$

$\because$ Compressibility factor for a real gas

$$Z = 1 + \frac{Pb}{RT}$$

89. Which of the following on thermal-decomposition yields a basic as well as an acidic oxide ?

(1) NH_4NO_3 (2) $NaNO_3$

(3) $KClO_3$ (4) $CaCO_3$

Ans **(4)**

Sol. $$NH_4NO_3 \xrightarrow{\Delta} \underset{\text{(neutral)}}{N_2O} + \underset{\text{(neutral)}}{2H_2O}$$

$$NaNO_3 \xrightarrow{\Delta} \underset{\text{(salt)}}{NaNO_2} + O_2$$

$$KClO_3 \xrightarrow{\Delta} \underset{\text{salt}}{KCl} + \frac{3}{2}O_2$$

$$CaCO_3 \xrightarrow{\Delta} \underset{\text{basic}}{CaO} + \underset{\text{acidic}}{CO_2}$$

90. In which of the following pairs the two species are not isostructural ?

(1) AlF_6^{3-} and SF_6 (2) CO_3^{2-} and NO_3^-

(3) PCl_4^+ and $SiCl_4$ (4) PF_5 and BrF_5

Ans **(4)**

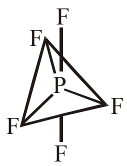
Sol. (1) AlF_6^{3-} and $SF_6 \rightarrow$ both are octahedral.

(2) CO_3^{2-} and $NO_3^- \rightarrow$ both are triangular planar

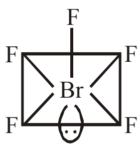
(3) PCl_4^+ and $SiCl_4 \rightarrow$ both are tetrahedral

(4) PF_5 and $BrF_5 \rightarrow PF_5$ has 5bp, 0 ℓp (sp^3d)

Structure is trigonal bipyramidal



BrF_5 has 5bp & 1 ℓp (sp^3d^2) and structure is square pyramidal



PART B – MATHEMATICS

31. **Statement–1 :** The sum of the series $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400)$ is 8000.

Statement–2 : $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

Ans. (3)

Sol. **Statement–1 :**
$(1^3–0^3) + (2^3–1^3) + (3^3–2^3) + ... + (20^3–19^3)$
$= 20^3 = 8000$
Statement–1 is true.
Statement–2 :

$$\sum_{k=1}^{n} k^3 - (k-1)^3 = (1^3 - 0^3) + (2^3–1^3) + (3^3–2^3) + ... n^3 + (n-1)^3 = n^3.$$

Statement–2 is true and Statement–2 is a correct explanation of Statement–1.

32. An ellipse is drawn by taking a diameter of the circle $(x - 1)^2 + y^2 = 1$ as its semi-minor axis and a diameter of the circle $x^2 + (y - 2)^2 = 4$ as its semi-major axis. If the centre of the ellipse is at the origin and its axes are the coordinate axes, then the equation of the ellipse is :

(1) $x^2 + 4y^2 = 16$ (2) $4x^2 + y^2 = 4$
(3) $x^2 + 4y^2 = 8$ (4) $4x^2 + y^2 = 8$

Ans. (1)

Sol. Let the equation of ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$$

from the given conditions
$a = 4$ and $b = 2$
$\therefore$ Eq of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

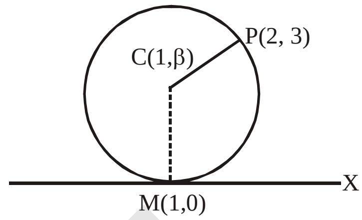
or $x^2 + 4y^2 = 16$

33. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is :

(1) 5/3 (2) 10/3
(3) 3/5 (4) 6/5

Ans. (2)

Sol.



Let center of the circle be $C(1, \beta)$
$\beta^2 = (2 - 1)^2 + (3 - \beta)^2$
$\Rightarrow \beta^2 = - 6 \beta + 10 + \beta^2$
$\Rightarrow \beta = \frac{5}{3}$
$\therefore r = \frac{5}{3}$
diameter $= \frac{10}{3}$

34. Let P and Q be 3 × 3 matrices with $P \neq Q$. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

(1) –1 (2) –2 (3) 1 (4) 0

Ans. (4)

Sol. $(P^2 + Q^2) P = P^3 + Q^2P$ ... (1)
$(P^2 + Q^2) Q = P^2Q + Q^3$ ... (2)
Equation (1) – Equation (2)
$(P^2 + Q^2) (P - Q) = P^3 - Q^3 + Q^2P - P^2Q$
$(P^2 + Q^2) (P - Q) = 0$ $\because (P \neq Q)$
$P^2 + Q^2 = 0$
so $|P^2 + Q^2| = 0$

35. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is :

(1) a rational number other than positive integers
(2) an irrational number
(3) an odd positive integer
(4) an even positive integer

Ans. (2)

Sol. $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$
$= 2[T_2 + T_2 + T_6 + ... + T_{2n}]$
$= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + ...]$
= An Irrational Number

36. **Statement–1 :** An equation of a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$ is $y = 2x + 2\sqrt{3}$.

Statement–2 : If the line $y = mx + \frac{4\sqrt{3}}{m}$, $(m \neq 0)$ is a common tangent to the parabola $y^2 = 16\sqrt{3}x$ and the ellipse $2x^2 + y^2 = 4$, then m satisfies $m^4 + 2m^2 = 24$.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

Ans. (3)

Sol. Let equation of any tangent to $y^2 = 16\sqrt{3}x$

be $y = mx + \frac{4\sqrt{3}}{m}$ ......(i)

and equation of any tangent to $2x^2 + y^2 = 4$

be $y = mx + \sqrt{2m^2+4}$ ....(ii)

but (i) and (ii) are same lines

$\therefore \frac{4\sqrt{3}}{m} = \sqrt{2m^2+4}$

$\Rightarrow m^4 + 2m^2 - 24 = 0$

$\Rightarrow m^2 = -6, 4$

$\therefore m = \pm 2$

37. Three numbers are chosen at random without replacement from {1, 2, 3, ... , 8}. The probability that their minimum is 3, given that their maximum is 6, is :

(1) $\frac{2}{5}$ (2) $\frac{3}{8}$ (3) $\frac{1}{5}$ (4) $\frac{1}{4}$

Ans. (3)

Sol. Let Events A denotes the getting min No. is 3 & B denotes the max. no. is 6

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{{}^2C_1}{{}^5C_2} = \frac{2}{10} = \frac{1}{5}$$

Alt.

$$P\left(\frac{A}{B}\right) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{{}^4C_3-(2)}{{}^8C_3}}{\frac{{}^6C_3-{}^5C_3}{{}^8C_3}} = \frac{2}{10} = \frac{1}{5}$$

38. If $g(x) = \int_0^x \cos 4t\,dt$, then $g(x+\pi)$ equals :

(1) $g(x) \,.\, g(\pi)$ (2) $\frac{g(x)}{g(\pi)}$
(3) $g(x) + g(\pi)$ (4) $g(x) - g(\pi)$

Ans. (3 & 4)

Sol. $g(x) = \int_0^x \cos 4t\,dt$

$$g(x+\pi) = \int_0^{x+\pi} \cos 4t\,dt = \int_0^x \cos 4t\,dt + \int_x^{x+\pi} \cos 4t\,dt$$

$$= \int_0^x \cos 4t\,dt + \int_0^{\pi} \cos 4t\,dt$$

$= g(x) + g(\pi)$

Because $g(\pi) = 0$ so $g(x) - g(\pi)$ is also correct Ans.

39. Assuming the balls to be identical except for difference in colours, the number of ways in which one or more balls can be selected from 10 white, 9 green and 7 black balls is :-

(1) 879 (2) 880 (3) 629 (4) 630

Ans. (1)

Sol. W^{10}, G^9, B^7

selection of one or more balls

$= (10 + 1)(9 + 1)(7 + 1) - 1$

$= 11 \times 10 \times 8 - 1$

$= 879$

40. If 100 times the 100^{th} term of an A.P. with non-zero common difference equals the 50 times its 50^{th} term, then the 150^{th} term of this A.P. is :

(1) zero (2) –150
(3) 150 times its 50^{th} term (4) 150

Ans. (1)

Sol. $100T_{100} = 50T_{50}$

$100(a + 99d) = 50(a + 4d)$

$a + 149d = 0$

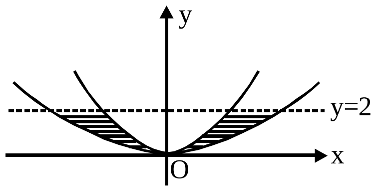
$T_{150} = a + 149d = 0$

41. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$, and the straight line $y = 2$ is :

(1) $10\sqrt{2}$ (2) $20\sqrt{2}$ (3) $\frac{10\sqrt{2}}{3}$ (4) $\frac{20\sqrt{2}}{3}$

Ans. (4)

Sol.



$$\text{Area} = 2\int_0^2 (x_2 - x_1)dy$$

$$= 2\int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2}\right)dy$$

$$= 2\left[\frac{1}{2}(y^{3/2})_0^2 - \frac{1}{3}(y^{3/2})_0^2\right] = \frac{20\sqrt{2}}{3}$$

42. An equation of a plane parallel to the plane $x - 2y + 2z - 5 = 0$ and at a unit distance from the origin is :

(1) $x - 2y + 2z + 5 = 0$
(2) $x - 2y + 2z - 3 = 0$
(3) $x - 2y + 2z + 1 = 0$
(4) $x - 2y + 2z - 1 = 0$

Ans. (2)

Sol. Equation of plane parallel to

$x - 2y + 2z - 5 = 0$

is $x - 2y + 2z = k$

or $\frac{x}{3} - \frac{2}{3}y + \frac{2}{3}z = \frac{K}{3}$

$\left|\frac{K}{3}\right| = 1$

$\Rightarrow K = \pm 3$

$\therefore$ Equation of required plane is

$x - 2y + 2z \pm 3 = 0$

43. The equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ has :

(1) exactly four real roots.
(2) infinite number of real roots.
(3) no real roots.
(4) exactly one real root.

Ans. (3)

Sol. Given $e^{\sin x} - e^{-\sin x} = 4$

let $e^{\sin x} = y$

$$y - \frac{1}{y} = 4 \quad \Rightarrow \quad y^2 - 4y - 1 = 0$$

$y = 2 \pm \sqrt{5}$

$e^{\sin x} = 2 + \sqrt{5}$ $\qquad$ $e^{\sin x} = 2 - \sqrt{5}$

but we know that

$e^{-1} \le e^{\sin x} \le e^1$

so $e^{\sin x} \ne 2 + \sqrt{5}$ and $2 - \sqrt{5}$

so No real solution of given equation.

44. The negation of the statement "If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school.
(2) I will become a teacher and I will not open a school.
(3) Either I will not become a teacher or I will not open a school.
(4) Neither I will become a teacher nor I will open a school.

Ans. (2)

Sol. Let $\quad$ p : I become a teacher

q : I will open a school

Therefore given statement will be $p \to q$

Negation of given statement

$\sim(p \to q) \equiv p \wedge \sim q$

therefore I will become a teacher and I will not open a school.

45. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\, p(t) - 450$. If $p(0) = 850$, then the time at which the population becomes zero is:

(1) ln18 $\quad$ (2) 2 ln18 $\quad$ (3) ln9 $\quad$ (4) $\frac{1}{2}$ ln18

Ans. (2)

Sol. $\frac{dP(t)}{dt} = \frac{1}{2}P(t) - 450$

integrate

$$\int \frac{dP}{P - 900} = \int \frac{1}{2}dt$$

$\ell n|(P - 900)| = \frac{1}{2}t + C \quad ....(1)$

given $\quad t = 0 \to P = 850$

$\therefore\ C = \ell n\ 50$

from (1)

$\ln|(P - 900)| = \frac{1}{2}t + \ell n 50$

$$\frac{1}{2}t = \ell n\left|\left(\frac{P - 900}{50}\right)\right|$$

$$t = 2\,\ell n\left|\left(\frac{P - 900}{50}\right)\right|$$

at $P = 0$

$t = 2\ell n \frac{900}{50}$

$t = 2\ell n 18$

46. If the integral

$$\int \frac{5\tan x}{\tan x - 2}dx = x + a\ln|\sin x - 2\cos x| + k \text{ then}$$

a is equal to :

(1) 2 (2) –1 (3) –2 (4) 1

Ans. (1)

Sol. $\int \frac{5\tan x}{\tan x - 2}dx = \int \frac{5\sin x}{\sin x - 2\cos x}dx$

$5\sin x = A(\sin x - 2\cos x) + B\frac{d}{dx}(\sin x - 2\cos x)$

$= A(\sin x - 2\cos x) + B(\cos x + 2\sin x)$

$\left.\begin{array}{l} A + 2B = 5 \\ -2A + B = 0 \end{array}\right\}$ A = 1, B = 2

Now $\int \frac{5\sin x}{\sin x - 2\cos x}dx$

$= \int\left(\frac{(\sin x - 2\cos x)}{\sin x - 2\cos x} + \frac{2(\cos x + 2\sin x)}{\sin x - 2\cos x}\right)dx$

$= x + 2\log|\sin x - 2\cos x| + K$

a = 2

47. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. (4)

Sol. $\bar{c}.\bar{d} = 0$

$\Rightarrow (\hat{a} + 2\hat{b}).(5\hat{a} - 4\hat{b}) = 0$

$\Rightarrow 5 - 8 + 6\ \hat{a}.\hat{b} = 0$

$\Rightarrow \hat{a}.\hat{b} = 1/2$

$\Rightarrow \cos\theta = 1/2$

$\Rightarrow \theta = \frac{\pi}{3}$

48. A line is drawn through the point (1, 2) to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) –4 (4) –2

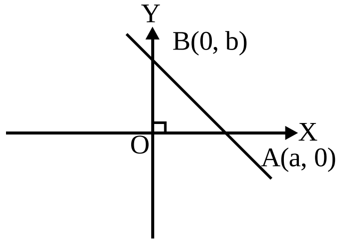
Ans. (4)

Sol. Let the equationof line be $\frac{x}{a} + \frac{y}{b} = 1$

It passes through (1, 2)

$\therefore \frac{1}{a} + \frac{2}{b} = 1$

$\Rightarrow b = \frac{2a}{a-1}$



Area of $\Delta = \frac{1}{2}ab$

$\Rightarrow \Delta = \frac{a^2}{a-1}$

$\frac{d\Delta}{da} = \frac{a^2 - 2a}{(a-1)^2} = 0$

$\Rightarrow \frac{a(a-2)}{(a-1)^2} = 0 \Rightarrow a = 0, 2$

but at a = 0, Δ not possible

$\therefore$ a = 2

slope of line $-\frac{b}{2} = 2$

49. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is :

(1) 5^3 (2) 5^2 (3) 3^5 (4) 2^5

Ans. (3)

Sol. y and z

case 1 set y contain no elemets and set Z contain either ϕ, or 1,2,3,4,5 elements.

$1 \times \{^5C_0 + {^5C_1} + ... + {^5C_5}\} = 32$

case 2 Set y contain 1 elements then

$^5C_1 \times (^4C_0 + {^4C_1} + {^4C_2} + {^4C_3} + {^4C_4}) = 80$

case 3 set y contain 2 elements, then

$^5C_2 \times (^3C_0 + {^3C_1} + {^3C_2} + {^3C_4}) = 80$

case 4 set y contains 3 elements the

$^5C_3 \times (^2C_0 + {^2C_1} + {^2C_2}) = 40$

case 5 set y contains 4 elements then

$^5C_4 \times (^1C_0 + {^1C_1}) = 10$

case 6 set y contains 5 elements, then $^5C_5 \times 1 = 1$

$32 + 80 + 80 + 40 + 10 + 1 = 243 = 3^5$

Opt. (3)

50. Let ABCD be a parallelogram such that $\overrightarrow{AB} = \vec{q}$, $\overrightarrow{AD} = \vec{p}$ and $\angle BAD$ be an acute angle. If $\vec{r}$ is the vector that coincides with the altitude directed from the vertex B to the side AD, then $\vec{r}$ is given by :

(1) $\vec{r} = -3\vec{q} + \dfrac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

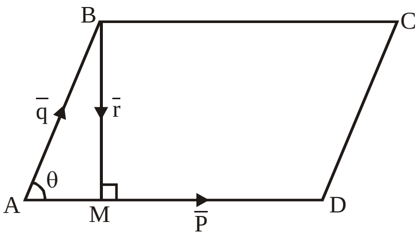
(2) $\vec{r} = 3\vec{q} - \dfrac{3(\vec{p}\cdot\vec{q})}{(\vec{p}\cdot\vec{p})}\vec{p}$

(3) $\vec{r} = -\vec{q} + \left(\dfrac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

(4) $\vec{r} = \vec{q} - \left(\dfrac{\vec{p}\cdot\vec{q}}{\vec{p}\cdot\vec{p}}\right)\vec{p}$

Ans. (3)

Sol.



$\bar{q} + \bar{r} = \overline{AM}$

$\Rightarrow \bar{r} = -\bar{q} + \overline{AM}$

$\Rightarrow \bar{r} = -\bar{q} + \dfrac{\bar{p}.\bar{q}}{|\bar{p}|^2}\bar{p}$

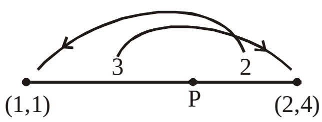
$\Rightarrow \bar{r} = -\bar{q} + \left(\dfrac{\bar{p}.\bar{q}}{\bar{p}.\bar{p}}\right)\bar{p}$

51. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals :

(1) $\dfrac{11}{5}$ (2) $\dfrac{29}{5}$ (3) 5 (4) 6

Ans. (4)

Sol. Let P be the point dividing (1, 1) and (2, 4) in the ratio 3 : 2.



$\therefore$ P is $\left(\dfrac{8}{5}, \dfrac{14}{5}\right)$.

Put $x = \dfrac{8}{5}$ and $y = \dfrac{14}{5}$ in 2x + y = k

$2 \times \dfrac{8}{5} + \dfrac{14}{5} = k$

$\Rightarrow$ k = 6

52. In a ΔPQR, if 3 sinP + 4 cosQ = 6 and 4 sinQ + 3 cos P = 1, then the angle R is equal to:

(1) $\dfrac{3\pi}{4}$ (2) $\dfrac{5\pi}{6}$ (3) $\dfrac{\pi}{6}$ (4) $\dfrac{\pi}{4}$

Ans. (3)

Sol. 3sinP + 4cosQ = 6 ..... (1)

4sinQ + 3cosP = 1 ..... (2)

Squaring (1) and (2). Then adding

9 + 16 + 24sin(P + Q) = 37

$\Rightarrow$ 24sin (P + Q) = 12

$\Rightarrow \sin(P + Q) = \dfrac{1}{2} \Rightarrow \sin R = \dfrac{1}{2}$

$\therefore R = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6}$

But $R = \dfrac{5\pi}{6}$ does't satisfy the given equation so $R = \dfrac{\pi}{6}$

53. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to :

(1) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (4) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$

Ans. (1)

Sol. $A^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}^T = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}$

and $A^{-1} A U_1 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ ...(1)

$A^{-1} A U_2 = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$ ...(2)

Eq. (1) + (2)

$U_1 + U_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

54. If f : R $\rightarrow$ R is a function defined by

$f(x) = [x] \cos\left(\frac{2x-1}{2}\right)\pi$, where [x] denotes the greatest integer function, then f is :

(1) continuous only at x = 0.
(2) continuous for every real x.
(3) discontinuous only at x = 0.
(4) discontinuous only at non-zero integral values of x.

Ans. (2)

Sol. [x] is contincous at R – I

$\therefore$ f(x) is continuous at R – I

Now At x = I

$$\text{LHL} = \lim_{h\to 0} [I - h] \cos \frac{(2(I-h)-1)}{2} \pi$$

$$\lim_{h\to 0} (I-1) \cos [2I-2h-1]\frac{\pi}{2}$$

$$= (I-1) \cos (2I-1)\frac{\pi}{2} = 0$$

similarly,

RHL = 0

and f(I) = 0

$\therefore$ Function is continous everywhere

55. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is :

(1) 9/2 (2) 9/7 (3) 7/9 (4) 2/9

Ans. (4)

Sol. $V = \frac{4}{3}\pi r^3$

Initially $r = 4500\pi$, $r = r_0$

$$4500\pi = \frac{4}{3}\pi r_0^3 \Rightarrow \boxed{r_0 = 15m}$$

$$\text{Now } \frac{dV}{dt} = \frac{4}{3}\pi (3r^2)\frac{dr}{dt}$$

$$-72\pi = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{-18}{r^2} \text{......(i)}$$

$$\int r^2 dr = -\int 18\, dt \Rightarrow \frac{r^3}{3} = -18t + C$$

At t = 0, r = 15 m

$$\text{So, } \frac{(15)^3}{3} = -18(0) + C \Rightarrow C = 1125$$

$\Rightarrow r^3 = -54t + 3375$ ......(ii)

At time t = 49 min   r = 9 m

from eq. (i)

$$\left(\frac{dr}{dt}\right)_{t=49} = \frac{-18}{(9)^2} = -2/9$$

(Negative sign shows decrement in radii)

56. Let a, b $\in$ R be such that the function f given by $f(x) = \ln|x| + bx^2 + ax$, $x \neq 0$ has extreme values at x = – 1 and x = 2.

Statement–1 : f has local maximum at x = –1 and at x = 2.

Statement–2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.

(1) Statement–1 is true, Statement–2 is false.
(2) Statement–1 is false, Statement–2 is true.
(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

Ans. (3)

Sol. $f'(x) = \frac{1}{x} + 2bx + a$

$f'(-1) = -1 - 2b + a = 0$ ......(1)

$f'(2) = \frac{1}{2} + 4b + a = 0$ ...... (2)

solve (1) & (2) $\Rightarrow a = \frac{1}{2},\ b = -\frac{1}{4}$

$\therefore$ st : 2 is true

$$f''(x) = -\frac{1}{x^2} - \frac{1}{2} = -\left(\frac{1}{x^2} + \frac{1}{2}\right) \quad \text{(always –ive)}$$

$$f''(-1) = -\frac{3}{2} < 0$$

$$f''(2) = -\frac{3}{4} < 0$$

$\therefore$ Local maximum at x = –1 & 2

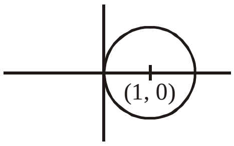
57. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies :

(1) on the imaginary axis.
(2) either on the real axis or on a circle passing through the origin.
(3) on a circle with centre at the origin.
(4) either on the real axis or on a circle not passing through the origin.

Ans. (2)

Sol. $\dfrac{z^2}{z-1}$ is purely real where $(Z \neq 1)$

so $\dfrac{\bar{z}^2}{\bar{z}-1} = \dfrac{z^2}{z-1}$



$z\bar{z}^2 - \bar{z}^2 = \bar{z}z^2 - z^2$

$z\bar{z}(z-\bar{z}) = z^2 - \bar{z}^2$

$z\bar{z}(z-\bar{z}) = (z+\bar{z})\,(z-\bar{z})$

$\Rightarrow \bar{z} - z = 0$ or $z + \bar{z} = z\bar{z}$

$\Rightarrow \bar{z} = z$ or $x^2 + y^2 - 2x = 0$

$(x-1)^2 + y^2 = 1$

so either lie on z real axis or on a circle passing through the origin.

58. Consider the function,

$f(x) = |x-2| + |x-5|,\ x \in R.$

Statement–1 : $f'(4) = 0$.

Statement–2 : f is continuous in [2, 5], differentiable in (2, 5) and $f(2) = f(5)$.

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

Ans. (4)

sol. $f(x) = |x-2| + |x-5|\ ;\ x \in R$

f(x) is continuous in [2, 5] and differentiable is (2, 5) and $f(2) = f(5) = 3$.

$\therefore$ By Rolle's theorem $f'(x) = 0$ for at least one $x \in (2, 5)$.

$$f'(x) = \frac{|x-2|}{x-2} + \frac{|x-5|}{x-5}$$

$f'(4) = 0$ but $f'(x) = 0\ \forall\ x \in (2, 5)$

59. Let $x_1, x_2, \ldots, x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance.

Statement–1 : Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.

Statement–2 : Arithmetic mean of $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$.

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.

Ans. (1)

Sol. Given $\bar{X} = \dfrac{\sum_{i=1}^{n} x_i}{n}$ ..... (1)

and $\sigma^2 = \dfrac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$ ....(2)

let $y_i = 2x_i$; where i = 1, 2, ... n

$\Rightarrow \bar{Y} = \dfrac{\sum_{i=1}^{n} y_i}{n} = \dfrac{2\sum_{i=1}^{n} x_i}{n} = 2\bar{X}$ [using (1)]

Therefore st–2 is false.

again $y_i - \bar{Y} = 2(x_i - \bar{X})$

$\sigma_y^2 = \dfrac{\sum_{i=1}^{n} (y_i - \bar{Y})^2}{n}$

$\sigma_y^2 = \dfrac{4\sum_{i=1}^{n} (x_i - \bar{X})^2}{n} = 4\sigma^2$ [using (2)]

therefore st–1 is true

60. If the lines $\dfrac{x-1}{2} = \dfrac{y+1}{3} = \dfrac{z-1}{4}$ and $\dfrac{x-3}{1} = \dfrac{y-k}{2} = \dfrac{z}{1}$ intersect, then k is equal to:

(1) 0 (2) – 1 (3) $\dfrac{2}{9}$ (4) $\dfrac{9}{2}$

Ans. (4)

Sol. $\begin{vmatrix} 3-1 & K+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} 2 & K+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow 2K - 9 = 0$

$\Rightarrow K = \dfrac{9}{2}$