Physics

1. A particle of mass M and charge Q moving with velocity $\vec{v}$ describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is :

(a) $\left(\frac{Mv^2}{R}\right) 2\pi R$ (b) zero

(c) $BQ\ 2\pi R$ (d) $BQv\ 2\pi R$

2. A particle of charge -16×10^{-18} C moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is :

(a) $10^3\ Wb/m^2$ (b) $10^5\ Wb/m^2$

(c) $10^{16}\ Wb/m^2$ (d) $10^{-3}\ Wb/m^2$

3. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio T'/T is :

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$

(c) 2 (d) $\frac{1}{4}$

4. A magnetic needle lying parallel to a magnetic field requires W unit of work to turn it through 60°. The torque needed to maintain the needle in this position will be :

(a) $\sqrt{3}W$ (b) W

(c) $(\sqrt{3}/2)W$ (d) $2W$

5. The magnetic lines of force inside a bar magnet :

(a) are from north-pole to south-pole of the magnet

(b) do not exist

(c) depend upon the area of cross-section of the bar magnet

(d) are from south-pole to north-pole of the magnet

6. Curie temperature is the temperature above which :

(a) a ferromagnetic material becomes para magnetic

(b) a paramagnetic material becomes diamagnetic

(c) a ferromagnetic material becomes diamagnetic

(d) a paramagnetic material becomes ferromagnetic

7. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be :

(a) 24 N (b) 74 N

(c) 15 N (d) 49 N

8. The length of a wire of a potentiometer is 100 cm, and the emf of its stand and cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at $l = 30$ cm from the positive end, the emf of the battery is :

(a) $\frac{30E}{100.5}$

(b) $\frac{30E}{100 - 0.5}$

(c) $\frac{30(E - 0.5i)}{100}$, where i is the current in the potentiometer wire

(d) $\frac{30E}{100}$

9. A strip of copper and another of germanium are cooled from room temperature to 80 K. The resistance of :

(a) each of these decreases
(b) copper strip increases and that of germanium decreases
(c) copper strip decreases and that of germanium increases
(d) each of the above increases

10. Consider telecommunication through optical fibres. Which of the following statements is not true ?
(a) Optical fibres can be of graded refractive index
(b) Optical fibres are subjected to electro-magnetic interference from outside
(c) Optical fibres have extremely low transmission loss
(d) Optical fibres may have homogeneous core with a suitable cladding

11. The thermo-emf of a thermocouple is 25 $\mu V/^\circ C$ at room temperature. A galvanometer of 40Ω resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is :
(a) 16°C (b) 12°C
(c) 8°C (d) 20°C

12. The negative Zn pole of Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 minutes. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is :
(a) 0.180 g (b) 0.141 g
(c) 0.126 g (d) 0.242 g

13. Dimensions of $\frac{1}{\mu_0 \varepsilon_0}$, where symbols have their usual meaning, are :
(a) $[L^{-1}T]$ (b) $[L^2T^2]$
(c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

14. A circular disc X of radius R is made from an iron plate of thickness t, and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is :
(a) $I_Y = 32 I_X$ (b) $I_Y = 16 I_X$
(c) $I_Y = I_X$ (d) $I_Y = 64 I_X$

15. The time period of a satellite of earth is 5 hours. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become :
(a) 10 h (b) 80 h
(c) 40 h (d) 20 h

16. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is :
(a) $\frac{L}{4}$ (b) $2L$
(c) $4L$ (d) $\frac{L}{2}$

17. Which of the following radiations has the least wavelength ?
(a) γ-rays (b) β-rays
(c) α-rays (d) X-rays

18. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is :
(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$
(c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$

19. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is :
(a) $2.5R$ (b) $4.5R$
(c) $7.5R$ (d) $1.5R$

20. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the :
(a) crystal structure
(b) variation of the number of charge carriers with temperature
(c) type of bonding
(d) variation of scattering mechanism with temperature

21. A car moving with a speed of 50 km/h, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/h, the minimum stopping distance is :
(a) 12 m (b) 18 m
(c) 24 m (d) 6 m

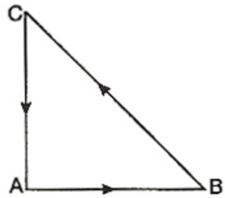
22. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ? [$g = 10$ m/s^2, $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$]
(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m

23. An ammeter reads upto 1A. Its internal resistance is 0.81 Ω. To increase the range to 10 A, the value of the required shunt is :

(a) 0.03 Ω (b) 0.3 Ω
(c) 0.9 Ω (d) 0.09 Ω

24. The physical quantities not having same dimensions are :
(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0 \varepsilon_0)^{-1/2}$

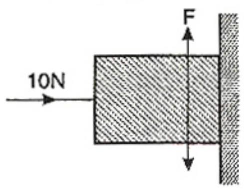
25. Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle *ABC* (as shown). The particle will now move with velocity :



(a) less than $\vec{v}$
(b) greater than $\vec{v}$
(c) $|v|$ in the direction of largest force *BC*
(d) $\vec{v}$, remaining unchanged

26. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be :
(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $(\phi_1 + \phi_2)/\varepsilon_0$
(c) $(\phi_2 - \phi_1)/\varepsilon_0$ (d) $(\phi_1 + \phi_2)\varepsilon_0$

27. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is :



(a) 20 N (b) 50 N
(c) 100 N (d) 2 N

28. A marble block of mass 2 kg lying on ice when given a velocity of 6 m/s is stopped by friction in 10 s. Then the coefficient of friction is :
(a) 0.02 (b) 0.03
(c) 0.06 (d) 0.01

29. Consider the following two statements :
A. Linear momentum of a system of particles is zero.
B. Kinetic energy of a system of particles is zero.
Then :
(a) *A* does not imply *B* and *B* does not imply *A*
(b) *A* implies *B* but *B* does not imply *A*
(c) *A* does not imply *B* but *B* implies *A*
(d) *A* implies *B* and *B* implies *A*

30. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon :
(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils

31. A block of mass *M* is pulled along a horizontal frictionless surface by a rope of mass *m*. If a force *P* is applied at the free end of the rope, the force exerted by the rope on the block is :
(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$
(c) *P* (d) $\frac{PM}{M+m}$

32. A light spring balance hangs from the hook of the other light spring balance and a block of mass *M* kg hangs from the former one. Then the true statement about the scale reading is :
(a) both the scales read *M* kg each
(b) the scale of the lower one reads *M* kg and of the upper one zero
(c) the reading of the two scales can be anything but the sum of the readings will be *M* kg
(d) both the scales read *M*/2 kg

33. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then the elastic energy stored in the wire is :
(a) 0.2 J (b) 10 J
(c) 20 J (d) 0.1 J

34. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, the escape velocity will be :
(a) $11\sqrt{2}$ km/s (b) 22 km/s
(c) 11 km/s (d) $11/\sqrt{2}$ m/s

35. A mass *M* is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period *T*. If the mass is increased by *m*, the time period becomes 5*T*/3, then the ratio of $\frac{m}{M}$ is :
(a) $\frac{3}{5}$ (b) $\frac{25}{9}$
(c) $\frac{16}{9}$ (d) $\frac{5}{3}$

36. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of :

(a) second law of thermodynamics
(b) conservation of momentum
(c) conservation of mass
(d) first law of thermodynamics

37. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is :

(a) $\sqrt{k_1/k_2}$ (b) k_1/k_2
(c) $\sqrt{k_2/k_1}$ (d) k_2/k_1

38. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is :

(a) 11% (b) 21%
(c) 42% (d) 10.5%

39. The displacement y of a wave travelling in the x-direction is given by

$$y = 10^{-4} \sin\left(600t - 2x + \frac{\pi}{3}\right) \text{ metre,}$$

where, x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is :

(a) 300 (b) 600
(c) 1200 (d) 200

40. When the current changes from + 2A to − 2A in 0.05 s, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is :

(a) 0.2 H (b) 0.4 H
(c) 0.8 H (d) 0.1 H

41. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is :

(a) $Q/2$ (b) $Q/\sqrt{3}$
(c) $Q/\sqrt{2}$ (d) Q

42. The core of any transformer is laminated so as to :

(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

43. Let $\vec{F}$ be the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin. Then :

(a) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$
(b) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} = 0$
(c) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$
(d) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} = 0$

44. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is :

(a) 0.4 ln 2 (b) 0.2 ln 2
(c) 0.1 ln 2 (d) 0.8 ln 2

45. A nucleus with $Z = 92$ emits the following in a sequence : $\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha; \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is :

(a) 76 (b) 78
(c) 82 (d) 74

46. Two identical, photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then :

(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

(b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$

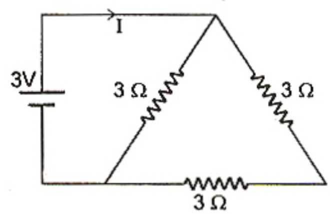
(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$

(d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

47. Which of the following cannot be emitted by radioactive substances during their decay ?

(a) Protons (b) Neutrinos
(c) Helium nuclei (d) Electrons

48. A 3V battery with negligible internal resistance is connected in a circuit as shown in the figure. The current I, in the circuit will be :



(a) 1 A (b) 1.5 A
(c) 2 A (d) $\frac{1}{3}$ A

49. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor :

(a) decreases (b) remains unchanged
(c) becomes infinite (d) increases

50. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is :
(a) -4 (b) 4
(c) $4\sqrt{2}$ (d) 8

51. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is :
(a) $\frac{2Q}{4\pi\varepsilon_0 R}$ (b) $\frac{2Q}{4\pi\varepsilon_0 R} - \frac{2q}{4\pi\varepsilon_0 R}$
(c) $\frac{2Q}{4\pi\varepsilon_0 R} + \frac{q}{4\pi\varepsilon_0 R}$ (d) $\frac{(q+Q)}{4\pi\varepsilon_0}\frac{2}{R}$

52. The work done in placing a charge of 8×10^{-18} C on a condenser of capacity 100 μ F is :
(a) 16×10^{-32} J (b) 3.1×10^{-26} J
(c) 4×10^{-10} J (d) 32×10^{-32} J

53. The co-ordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by :
(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$
(c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

54. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_P/C_V for the gas is :
(a) 4/3 (b) 2
(c) 5/3 (d) 3/2

55. Which of the following parameters does not characterise the thermodynamic state of matter ?
(a) Temperature (b) Pressure
(c) Work (d) Volume

56. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is :
(a) 4.2×10^6 J (b) 8.4×10^6 J
(c) 16.8×10^6 J (d) zero

57. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is :
(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

58. A metal wire of linear mass density of 9.8 g/m is stretched with a tension of 10 kg-wt between two rigid supports 1 m apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is :
(a) 50 Hz (b) 100 Hz
(c) 200 Hz (d) 25 Hz

59. A tuning fork of known frequency 256 Hz makes 5 beats/s with the vibrating string of a piano. The beat frequency decreases to 2 beats/s when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was :
(a) (256 + 2) Hz (b) (256 − 2) Hz
(c) (256 − 5) Hz (d) (256 + 5) Hz

60. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as function of displacement x. Which of the following statement is true ?
(a) KE is maximum when $x = 0$
(b) TE is zero when $x = 0$
(c) KE is maximum when x is maximum
(d) PE is maximum when $x = 0$

61. In the nuclear fusion reaction,
$$^2_1H + ^3_1H \rightarrow ^4_2He + n$$
given that the repulsive potential energy between the two nuclei is 7.7×10^{-14}J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K] :
(a) 10^7 K (b) 10^5 K
(c) 10^3 K (d) 10^9 K

62. Which of the following atoms has the lowest ionization potential ?
(a) $^{14}_7N$ (b) $^{133}_{55}Cs$
(c) $^{40}_{18}Ar$ (d) $^{16}_8O$

63. The wavelengths involved in the spectrum of deuterium (^2_1D) are slightly different from that of hydrogen spectrum, because :
(a) sizes of the two nuclei are different
(b) nuclear forces are different in the two cases
(c) masses of the two nuclei are different
(d) attraction between the electron and the nucleus is different in the two cases

64. In the middle of the depletion layer of reverse biased p-n junction, the :
(a) electric field is zero
(b) potential is maximum
(c) electric field is maximum
(d) potential is zero

65. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is :

(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV

66. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to :

(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$

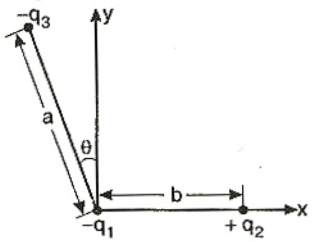
67. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 m/s^2. Then the initial thrust of the blast is :

(a) 3.5×10^5 N (b) 7.0×10^5 N
(c) 14.0×10^5 N (d) 1.75×10^5 N

68. To demonstrate the phenomenon of interference we require two sources which emit radiations of :

(a) nearly the same frequency
(b) the same frequency
(c) different wavelength
(d) the same frequency and having a definite phase relationship

69. Three charges $-q_1$, $+q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to :



(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2} \cos \theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2} \sin \theta$
(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2} \cos \theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2} \sin \theta$

70. A 220 V, 1000 W bulb is connected across a 110 V mains supply. The power consumed will be :

(a) 750 W (b) 500 W
(c) 250 W (d) 1000 W

71. The image formed by an objective of a compound microscope is :

(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

72. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by :

(a) Rayleigh Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law

73. To get three images of a single object, one should have two plane mirrors at an angle of :

(a) 60° (b) 90°
(c) 120° (d) 30°

74. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta \theta)^n$, where $\Delta \theta$ is the difference of the temperature of the body and the surroundings, and n is equal to :

(a) 2 (b) 3
(c) 4 (d) 1

75. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter the change in the resistance of the wire will be :

(a) 200% (b) 100%
(c) 50% (d) 300%

Chemistry

76. In Bohr series of lines of hydrogen spectrum, the third line from the red end corresponds to which one of the following inner-orbit jumps of the electron for Bohr orbits in an atom of hydrogen ?

(a) $3 \rightarrow 2$ (b) $5 \rightarrow 2$
(c) $4 \rightarrow 1$ (d) $2 \rightarrow 5$

77. The de-Broglie wavelength of a tennis ball of mass 60g moving with a velocity of 10 m/s is approximately :

(Planck's constant, $h = 6.63 \times 10^{-34}$ Js)

(a) 10^{-33} m (b) 10^{-31} m
(c) 10^{-16} m (d) 10^{-25} m

78. The orbital angular momentum for an electron revolving in an orbit is given by $\sqrt{l(l+1)} \cdot \frac{h}{2\pi}$. This momentum for an s-electron will be given by :

(a) $+\frac{1}{2} \cdot \frac{h}{2\pi}$ (b) zero
(c) $\frac{h}{2\pi}$ (d) $\sqrt{2} \cdot \frac{h}{2\pi}$

79. How many unit cells are present in a cube shaped ideal crystal of NaCl of mass 1.00 g ?
[Atomic masses : Na = 23, Cl = 35.5]

(a) 2.57×10^{21} (b) 5.14×10^{21}
(c) 1.28×10^{21} (d) 1.71×10^{21}

80. Glass is a :
(a) micro-crystalline solid
(b) super-cooled liquid
(c) gel
(d) polymeric mixture

81. Which one of the following statements is correct ?
(a) Manganese salts give a violet borax- bead test in the reducing flame
(b) From a mixed precipitate of AgCl and AgI, ammonia solution dissolves only AgCl
(c) Ferric ions give a deep green precipitate on adding potassium ferrocyanide solution
(d) On boiling a solution having K^+, Ca^{2+} and HCO_3^- ions we get a precipitate of $K_2Ca(CO_3)_2$

82. According to the periodic law of elements, the variation in properties of elements is related to their :
(a) atomic masses
(b) nuclear masses
(c) atomic numbers
(d) nuclear neutron-proton number ratios

83. Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite :
(a) is a non-crystalline substance
(b) is an allotropic form of diamond
(c) has molecules of variable molecular masses like polymers
(d) has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds

84. The IUPAC name of $CH_3COCH(CH_3)_2$ is :
(a) isopropylmethyl ketone
(b) 2-methyl-3-butanone
(c) 4-methylisopropyl ketone
(d) 3-methyl-2-butanone

85. When $CH_2=CH-COOH$ is reduced with $LiAlH_4$, the compound obtained will be :
(a) CH_3-CH_2-COOH
(b) $CH_2=CH-CH_2OH$
(c) $CH_3-CH_2-CH_2OH$
(d) CH_3-CH_2-CHO

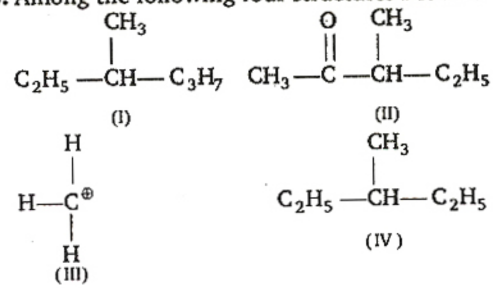
86. According to the kinetic theory of gases, in an ideal gas, between two successive collisions a gas molecule travels :
(a) in a circular path
(b) in a wavy path
(c) in a straight line path
(d) with an accelerated velocity

87. Which of the following group of transition metals is called coinage metals ?
(a) Cu, Ag, Au
(b) Ru, Rh, Pd
(c) Fe, Co, Ni
(d) Os, Ir, Pt

88. The general formula $C_nH_{2n}O_2$ could be for open chain :
(a) diketones
(b) carboxylic acids
(c) diols
(d) dialdehydes

89. An ether is more volatile than an alcohol having the same molecular formula. This is due to :
(a) dipolar character of ethers
(b) alcohols having resonance structures
(c) inter-molecular hydrogen bonding in ethers
(d) inter-molecular hydrogen bonding in alcohols

90. Among the following four structures I to IV :



it is true that :
(a) all four are chiral compounds
(b) only I and II are chiral compounds
(c) only III is a chiral compound
(d) only II and IV are chiral compounds

91. Which one of the following processes will produce hard water ?
(a) Saturation of water with $CaCO_3$
(b) Saturation of water with $MgCO_3$
(c) Saturation of water with $CaSO_4$
(d) Addition of Na_2SO_4 to water

92. Which one of the following compounds has the smallest bond angle in its molecule ?
(a) SO_2 (b) OH_2
(c) SH_2 (d) NH_3

93. Which one of the following pairs of molecules will have permanent dipole moments for both members ?
(a) SiF_4 and NO_2 (b) NO_2 and CO_2
(c) NO_2 and O_3 (d) SiF_4 and CO_2

94. Which one of the following groupings represents a collection of isoelectronic species ? (At. numbers : Cs-55, Br-35)
(a) Na^+, Ca^{2+}, Mg^{2+} (b) N^{3-}, F^-, Na^+
(c) Be, Al^{3+}, Cl^- (d) Ca^{2+}, Cs^+, Br

95. In the anion $HCOO^-$ the two carbon-oxygen bonds are found to be of equal length. What is the reason for it ?

(a) Electronic orbits of carbon atom are hybridised
(b) The $C=O$ bond is weaker than the $C-O$ bond
(c) The anion $HCOO^-$ has two resonating structures
(d) The anion is obtained by removal of a proton from the acid molecule

96. The pair of species having identical shapes for molecules of both species is :

(a) CF_4, SF_4
(b) XeF_2, CO_2
(c) BF_3, PCl_3
(d) PF_5, IF_5

97. The atomic numbers of vanadium (V), chromium (Cr), manganese (Mn) and iron (Fe) are respectively 23, 24, 25 and 26. Which one of these may be expected to have the highest second ionization enthalpy ?

(a) V
(b) Cr
(c) Mn
(d) Fe

98. Consider the reaction equilibrium :

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g); \quad \Delta H^\circ = -198 \text{ kJ}$$

On the basis of Le-Chatelier's principle, the condition favourable for the forward reaction is :

(a) lowering of temperature as well as pressure
(b) increasing temperature as well as pressure
(c) lowering the temperature and increasing the pressure
(d) any value of temperature and pressure

99. What volume of hydrogen gas, at 273K and 1 atm pressure will be consumed in obtaining 21.6g of elemental boron (atomic mass = 10.8) from the reduction of boron trichloride by hydrogen ?

(a) 89.6 L
(b) 67.2 L
(c) 44.8 L
(d) 22.4 L

100. For the reaction equilibrium,

$$N_2O_4(g) \rightleftharpoons 2NO(g)$$

the concentrations of N_2O_4 and NO_2 at equilibrium are 4.8×10^{-2} and 1.2×10^{-2} mol L^{-1} respectively. The value of K_c for the reaction is :

(a) 3.3×10^2 mol L^{-1}
(b) 3×10^{-1} mol L^{-1}
(c) 3×10^{-3} mol L^{-1}
(d) 3×10^3 mol L^{-1}

101. The solubility in water of a sparingly soluble salt AB_2 is 1.0×10^{-5} mol L^{-1}. Its solubility product number will be :

(a) 4×10^{-15}
(b) 4×10^{-10}
(c) 1×10^{-15}
(d) 1×10^{-10}

102. When during electrolysis of a solution of $AgNO_3$, 9650 coulombs of charge pass through the electroplating bath, the mass of silver deposited on the cathode will be :

(a) 1.08 g
(b) 10.8 g
(c) 21.6 g
(d) 108 g

103. For the redox reaction

$$Zn(s) + Cu^{2+}\ (0.1\ M) \rightarrow Zn^{2+}\ (1M) + Cu(s)$$

taking place in a cell, E°_{cell} is 1.10 volt. E_{cell} for the cell will be : $\left(2.303\frac{RT}{F} = 0.0591\right)$

(a) 2.14 V
(b) 1.80 V
(c) 1.07 V
(d) 0.82 V

104. In a 0.2 molal aqueous solution of a weak acid HX, the degree of ionisation is 0.3. Taking K_f for water as 1.85, the freezing point of the solution will be nearest to :

(a) $-0.480°C$
(b) $-0.360°C$
(c) $-0.260°C$
(d) $+0.480°C$

105. The rate law for a reaction between the substances A and B is given by rate $= k[A]^n[B]^m$. On doubling the concentration of A and halving the concentration of B, the ratio of the new rate to the earlier rate of the reaction will be as :

(a) $\frac{1}{2^{m+n}}$
(b) $(m+n)$
(c) $(n-m)$
(d) $2^{(n-m)}$

106. 25mL of a solution of barium hydroxide on titration with 0.1 molar solution of hydrochloric acid gave a titre value of 35 mL. The molarity of barium hydroxide solution was :

(a) 0.07
(b) 0.14
(c) 0.28
(d) 0.35

107. The correct relationship between free energy change in a reaction and the corresponding equilibrium constant K_c is :

(a) $\Delta G = RT \ln K_c$
(b) $-\Delta G = RT \ln K_c$
(c) $\Delta G^\circ = RT \ln K_c$
(d) $-\Delta G^\circ = RT \ln K_c$

108. If at 298K the bond energies of C—H, C—C, C=C and H—H bonds are respectively 414, 347, 615 and 435 kJ mol^{-1}, the value of enthalpy change for the reaction

$$H_2C=CH_2(g) + H_2(g) \longrightarrow H_3C-CH_3(g)$$

at 298K will be :

(a) + 250 kJ
(b) − 250 kJ
(c) + 125 kJ
(d) − 125 kJ

109. The enthalpy change for a reaction does not depend upon the :
(a) physical state of reactants and products
(b) use of different reactants for the same product
(c) nature of intermediate reaction steps
(d) difference in initial or final temperatures of involved substances

110. A pressure cooker reduces cooking time for food because :
(a) heat is more evenly distributed in the cooking space
(b) boiling point of water involved in cooking is increased
(c) the higher pressure inside the cooker crushes the food material
(d) cooking involves chemical changes helped by a rise in temperature

111. If liquids A and B form an ideal solution, the :
(a) enthalpy of mixing is zero
(b) entropy of mixing is zero
(c) free energy of mixing is zero
(d) free energy as well as the entropy of mixing are each zero

112. For the reaction system :

$$2NO(g) + O_2(g) \longrightarrow 2NO_2(g)$$

volume is suddenly reduced to half its value by increasing the pressure on it. If the reaction is of first order with respect to O_2 and second order with respect to NO; the rate of reaction will :
(a) diminish to one-fourth of its initial value
(b) diminish to one-eighth of its initial value
(c) increase to eight times of its initial value
(d) increase to four times of its initial value

113. For a cell reaction involving a two-electron change, the standard emf of the cell is found to be 0.295 V at 25°C. The equilibrium constant of the reaction at 25°C will be :
(a) 1×10^{-10} (b) 29.5×10^{-2}
(c) 10 (d) 1×10^{10}

114. In an irreversible process taking place at constant T and P and in which only pressure-volume work is being done, the change in Gibbs free energy (dG) and change in entropy (dS), satisfy the criteria :
(a) $(dS)_{V,E} < 0, (dG)_{T,P} < 0$
(b) $(dS)_{V,E} > 0, (dG)_{T,P} < 0$
(c) $(dS)_{V,E} = 0, (dG)_{T,P} = 0$
(d) $(dS)_{V,E} = 0, (dG)_{T,P} > 0$

115. Which one of the following characteristics is not correct for physical adsorption ?
(a) Adsorption on solids is reversible
(b) Adsorption increases with increase in temperature
(c) Adsorption is spontaneous
(d) Both enthalpy and entropy of adsorption are negative

116. In the respect of the equation $k = Ae^{-E_a/RT}$ in chemical kinetics, which one of the following statements is correct ?
(a) k is equilibrium constant
(b) A is adsorption factor
(c) E_a is energy of activation
(d) R is Rydberg constant

117. Standard reduction electrode potentials of three metals A, B and C are + 0.5 V, − 3.0 V and −1.2 V respectively. The reducing power of these metals are :
(a) $B > C > A$ (b) $A > B > C$
(c) $C > B > A$ (d) $A > C > B$

118. Which one of the following substances has the highest proton affinity ?
(a) H_2O (b) H_2S
(c) NH_3 (d) PH_3

119. Which one of the following is an amphoteric oxide ?
(a) ZnO (b) Na_2O
(c) SO_2 (d) B_2O_3

120. A red solid is insoluble in water. However it becomes soluble if some KI is added to water. Heating the red solid in a test tube results in liberation of some violet coloured fumes and droplets of a metal appear on the cooler parts of the test tube. The red solid is :
(a) $(NH_4)_2Cr_2O_7$ (b) HgI_2
(c) HgO (d) Pb_3O_4

121. Concentrated hydrochloric acid when kept in open air sometimes produces a cloud of white fumes. The explanation for it is that :
(a) concentrated hydrochloric acid emits strongly smelling HCl gas all the time
(b) oxygen in air reacts wtih the emitted HCl gas to form a cloud of chlorine gas
(c) strong affinity of HCl gas for moisture in air results in forming of droplets of liquid solution which appears like a cloudy smoke
(d) due to strong affinity for water, concentrated hydrochloric acid pulls moisture of air towards itself. This moisture forms droplets of water and hence the cloud

122. The substance used in Holmes signals of the ship is a mixture of :
(a) $CaC_2 + Ca_3P_2$ (b) $Ca_3(PO_4)_2 + Pb_3O_4$
(c) $H_3PO_4 + CaCl_2$ (d) $NH_3 + HOCl$

123. The number of *d*-electrons retained in Fe^{2+} (At. no. Fe = 26) ions is :

(a) 3 (b) 4

(c) 5 (d) 6

124. What would happen when a solution of potassium chromate is treated with an excess of dilute nitric acid ?

(a) Cr^{3+} and $Cr_2O_7^{2-}$ are formed

(b) $Cr_2O_7^{2-}$ and H_2O are formed

(c) CrO_4^{2-} is reduced to + 3 state of Cr

(d) None of the above

125. In the co-ordination compound, $K_4[Ni(CN)_4]$, the oxidation state of nickel is :

(a) – 1 (b) 0

(c) + 1 (d) + 2

126. Ammonia forms the complex ion $[Cu(NH_3)_4]^{2+}$ with copper ions in the alkaline solutions but not in acidic solutions. What is the reason for it ?

(a) In acidic solutions hydration protects copper ions

(b) In acidic solutions protons co-ordinate with ammonia molecules forming NH_4^+ ions and NH_3 molecules are not available

(c) In alkaline solutions insoluble $Cu(OH)_2$ is precipitated which is soluble in excess of any alkali

(d) Copper hydroxide is an amphoteric substance

127. One mole of the complex compound $Co(NH_3)_5Cl_3$, gives 3 moles of ions on dissolution in water. One mole of the same complex reacts with two moles of $AgNO_3$ solution to yield two moles of AgCl(*s*). The structure of the complex is :

(a) $[Co(NH_3)_5Cl]Cl_2$

(b) $[Co(NH_3)_3Cl_2] \cdot 2NH_3$

(c) $[Co(NH_3)_4Cl_2]Cl \cdot NH_3$

(d) $[Co(NH_3)_4Cl]Cl_2 \cdot NH_3$

128. The radius of La^{3+} (Atomic number of La = 57) is 1.06 Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71)?

(a) 1.60 Å (b) 1.40 Å

(c) 1.06 Å (d) 0.85 Å

129. The radionuclide $^{234}_{90}Th$ undergoes two successive β-decays followed by one α-decay. The atomic number and the mass number respectively of the resulting radionuclide are :

(a) 92 and 234 (b) 94 and 230

(c) 90 and 230 (d) 92 and 230

130. The half-life of a radioactive isotope is 3 h. If the initial mass of the isotope were 256 g, the mass of it remaining undecayed after 18 h would be :

(a) 4.0 g (b) 8.0 g

(c) 12.0 g (d) 16.0 g

131. Several blocks of magnesium are fixed to the bottom of a ship to :

(a) keep away the sharks

(b) make the ship lighter

(c) prevent action of water and salt

(d) prevent puncturing by under-sea rocks

132. In curing cement plasters water is sprinkled from time to time. This helps in :

(a) keeping it cool

(b) developing interlocking needle-like crystals of hydrated silicates

(c) hydrating sand and gravel mixed with cement

(d) converting sand into silicic acid

133. Which one of the following statements is not true ?

(a) The conjugate base of $H_2PO_4^-$ is HPO_4^{2-}

(b) pH + pOH = 14 for all aqueous solutions

(c) The pH of 1×10^{-8} M HCl is 8

(d) 96,500 coulombs of electricity when passed through a $CuSO_4$ solution deposit 1g equivalent of copper at the cathode

134. The correct order of increasing basic nature for the bases NH_3, CH_3NH_2 and $(CH_3)_2NH$ is :

(a) $CH_3NH_2 < NH_3 < (CH_3)_2NH$

(b) $(CH_3)_2NH < NH_3 < CH_3NH_2$

(c) $NH_3 < CH_3NH_2 < (CH_3)_2NH$

(d) $CH_3NH_2 < (CH_3)_2NH < NH_3$

135. Butene-1 may be converted to butane by reaction with :

(a) Zn-HCl (b) Sn-HCl

(c) Zn-Hg (d) Pd/H_2

136. The solubilities of carbonates decrease down the magnesium group due to a decrease in :

(a) lattice energies of solids

(b) hydration energies of cations

(c) inter-ionic attraction

(d) entropy of solution formation

137. During dehydration of alcohols to alkenes by heating with concentrated H_2SO_4 the initiation step is :

(a) protonation of alcohol molecule

(b) formation of carbocation

(c) elimination of water

(d) formation of an ester

138. Which one of the following nitrates will leave behind a metal on strong heating ?

(a) Ferric nitrate (b) Copper nitrate

(c) Manganese nitrate (d) Silver nitrate

139. When rain is accompanied by a thunderstorm, the collected rain water will have a pH value :

(a) slightly lower than that of rain water without thunderstorm

(b) slightly higher than that when the thunderstorm is not there

(c) uninfluenced by occurrence of thunderstorm

(d) which depends on the amount of dust in air

140. Complete hydrolysis of cellulose gives :

(a) D-fructose (b) D-ribose

(c) D-glucose (d) L-glucose

141. For making good quality mirrors, plates of float glass are used. These are obtained by floating molten glass over a liquid metal which does not solidify before glass. The metal used can be :

(a) mercury (b) tin

(c) sodium (d) magnesium

142. The substance not likely to contain $CaCO_3$ is :

(a) a marble statue (b) calcined gypsum

(c) sea shells (d) dolomite

143. The reason for double helical structure of DNA is operation of :

(a) van der Waals' forces

(b) dipole-dipole interaction

(c) hydrogen bonding

(d) electrostatic attractions

144. Bottles containing C_6H_5I and $C_6H_5CH_2I$ lost their original labels. They were labelled *A* and *B* for testing. *A* and *B* were separately taken in a test tube and boiled with NaOH solution. The end solution in each tube was made acidic with dilute HNO_3 and then some $AgNO_3$ solution was added. Substance *B* gave a yellow precipitate. Which one of the following statements is true for this experiment ?

(a) *A* was C_6H_5I

(b) *A* was $C_6H_5CH_2I$

(c) *B* was C_6H_5I

(d) Addition of HNO_3 was unnecessary

145. Ethyl isocyanide on hydrolysis in acidic medium generates :

(a) ethylamine salt and methanoic acid

(b) propanoic acid and ammonium salt

(c) ethanoic acid and ammonium salt

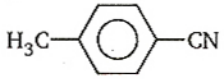
(d) methylamine salt and ethanoic acid

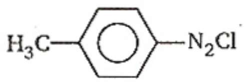
146. The internal energy change when a system goes from state *A* to *B* is 40 k J/mol. If the system goes from *A* to *B* by a reversible path and returns to state *A* by an irreversible path, what would be the net change in internal energy ?

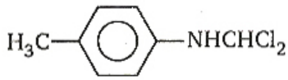
(a) 40 kJ (b) > 40 kJ

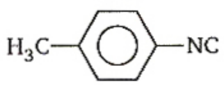
(c) < 40 kJ (d) zero

147. The reaction of chloroform with alcoholic KOH and *p*-toluidine form :

(a) 

(b) 

(c) 

(d) 

148. Nylon threads are made of :

(a) polyvinyl polymer

(b) polyester polymer

(c) polyamide polymer

(d) polyethylene polymer

149. On mixing a certain alkane with chlorine and irradiating it with ultraviolet light, it forms only one monochloroalkane. This alkane could be :

(a) propane (b) pentane

(c) isopentane (d) neopentane

150. Which of the following could act as a propellant for rockets ?

(a) Liquid hydrogen + liquid nitrogen

(b) Liquid oxygen + liquid argon

(c) Liquid hydrogen + liquid oxygen

(d) Liquid nitrogen + liquid oxygen

1. A function f from the set of natural numbers to integers defined by

$$f(n)=\begin{cases}\dfrac{n-1}{2}, \text{ when } n \text{ is odd}\\ -\dfrac{n}{2}, \text{ when } n \text{ is even}\end{cases}$$

is :
(a) one-one but not onto
(b) onto but not one-one
(c) one-one and onto both
(d) neither one-one nor onto

2. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral triangle. Then :
(a) $a^2 = b$ (b) $a^2 = 2b$
(c) $a^2 = 3b$ (d) $a^2 = 4b$

3. If z and ω are two non-zero complex numbers such that $|z\,\omega| = 1$, and $\arg(z) - \arg(\omega) = \dfrac{\pi}{2}$, then $\bar{z}\omega$ is equal to :
(a) 1 (b) -1
(c) i (d) $-i$

4. If $\left(\dfrac{1+i}{1-i}\right)^x = 1$, then :
(a) $x = 4n$, where n is any positive integer
(b) $x = 2n$, where n is any positive integer
(c) $x = 4n + 1$, where n is any positive integer
(d) $x = 2n + 1$, where n is any positive integer

5. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$

and vectors $(1, a, a^2)$, $(1, b, b^2)$ and $(1, c, c^2)$ are non-coplanar, then the product abc equals :
(a) 2 (b) -1
(c) 1 (d) 0

6. If the system of linear equations

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

has a non-zero solution, then a, b, c :
(a) are in AP
(b) are in GP
(c) are in HP
(d) satisfy $a + 2b + 3c = 0$

7. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\dfrac{a}{c}, \dfrac{b}{a}$ and $\dfrac{c}{b}$ are in :
(a) arithmetic progression
(b) geometric progression
(c) harmonic progression
(d) arithmetico-geometric progression

8. The number of the real solutions of the equation $x^2 - 3|x| + 2 = 0$ is :
(a) 2 (b) 4
(c) 1 (d) 3

9. The value of 'a' for which one root of the quadratic equation

$$(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$$

is twice as large as the other, is :
(a) 2/3 (b) −2/3
(c) 1/3 (d) −1/3

10. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then :
(a) $\alpha = a^2 + b^2, \beta = ab$
(b) $\alpha = a^2 + b^2, \beta = 2ab$
(c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(d) $\alpha = 2ab, \beta = a^2 + b^2$

11. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is :
(a) 140 (b) 196
(c) 280 (d) 346

12. The number of ways in which 6 men and 5 women can dine at a round table, if no two women are to sit together, is given by :
(a) $6! \times 5!$ (b) 30
(c) $5! \times 4!$ (d) $7! \times 5!$

13. If $1, \omega, \omega^2$ are the cube roots of unity, then :

$$\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$$

is equal to :
(a) 0 (b) 1
(c) ω (d) ω^2

14. If ${}^nC_r$ denotes the number of combinations of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals :

(a) ${}^{n+2}C_r$ (b) ${}^{n+2}C_{r+1}$

(c) ${}^{n+1}C_r$ (d) ${}^{n+1}C_{r+1}$

15. The number of integral terms in the expansion of $(\sqrt{3} + \sqrt[8]{5})^{256}$ is :

(a) 32 (b) 33

(c) 34 (d) 35

16. If x is positive, the first negative term in the expansion of $(1 + x)^{27/5}$ is :

(a) 7th term (b) 5th term

(c) 8th term (d) 6th term

17. The sum of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots$ upto ∞ is equal to :

(a) $2 \log_e 2$ (b) $\log_e 2 - 1$

(c) $\log_e 2$ (d) $\log_e \left(\frac{4}{e}\right)$

18. Let $f(x)$ be a polynomial function of second degree. If $f(1) = f(-1)$ and a, b, c are in AP, then $f'(a), f'(b)$ and $f'(c)$ are in :

(a) AP

(b) GP

(c) HP

(d) arithmetico-geometric progression

19. If x_1, x_2, x_3 and y_1, y_2, y_3 are both in GP with the same common ratio, then the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) :

(a) lie on a straight line

(b) lie on an ellipse

(c) lie on a circle

(d) are vertices of a triangle

20. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side a, is :

(a) $a \cot \left(\frac{\pi}{n}\right)$ (b) $\frac{a}{2} \cot \left(\frac{\pi}{2n}\right)$

(c) $a \cot \left(\frac{\pi}{2n}\right)$ (d) $\frac{a}{4} \cot \left(\frac{\pi}{2n}\right)$

21. If in a triangle ABC

$$a \cos^2 \left(\frac{C}{2}\right) + c \cos^2 \left(\frac{A}{2}\right) = \frac{3b}{2},$$

then the sides a, b and c :

(a) are in AP (b) are in GP

(c) are in HP (d) satisfy $a + b = c$

22. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the $\Delta\, ABC$ is :

(a) 8/3 (b) 16/3

(c) 32/3 (d) 64/3

23. The trigonometric equation $\sin^{-1} x = 2 \sin^{-1} a$, has a solution for :

(a) $\frac{1}{2} < |a| < \frac{1}{\sqrt{2}}$ (b) all real values of a

(c) $|a| < \frac{1}{2}$ (d) $|a| \geq \frac{1}{\sqrt{2}}$

24. The upper 3/4th portion of a vertical pole subtends an angle $\tan^{-1} 3/5$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. A possible height of the vertical pole is :

(a) 20 m (b) 40 m

(c) 60 m (d) 80 m

25. The real number x when added to its inverse gives the minimum value of the sum at x equals to :

(a) 2 (b) 1

(c) -1 (d) -2

26. If $f : R \to R$ satisfies $f(x + y) = f(x) + f(y)$, for all $x, y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is :

(a) $\frac{7n}{2}$ (b) $\frac{7(n+1)}{2}$

(c) $7n(n+1)$ (d) $\frac{7n(n+1)}{2}$

27. If $f(x) = x^n$, then the value of

$$f(1) - \frac{f'(1)}{1!} + \frac{f''(1)}{2!} - \frac{f'''(1)}{3!} + \ldots + \frac{(-1)^n f^n(1)}{n!}$$ is :

(a) 2^n (b) 2^{n-1}

(c) 0 (d) 1

28. Domain of definition of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is :

(a) $(1, 2)$

(b) $(-1, 0) \cup (1, 2)$

(c) $(1, 2) \cup (2, \infty)$

(d) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

29. $\lim_{x \to \frac{\pi}{2}} \frac{\left[1 - \tan\left(\frac{x}{2}\right)\right][1 - \sin x]}{\left[1 + \tan\left(\frac{x}{2}\right)\right][\pi - 2x]^3}$ is :

(a) $\frac{1}{8}$ (b) 0

(c) $\frac{1}{32}$ (d) ∞

30. If $\lim_{x \to 0} \frac{\log(3 + x) - \log(3 - x)}{x} = k$, the value of k is :

(a) 0 (b) $-1/3$

(c) 2/3 (d) $-2/3$

31. If $f^n(a)$, $g^n(a)$ exist and are not equal for some n. Further if $f(a) = g(a) = k$ and $\lim_{x \to a} \frac{f(a)\,g(x) - f(a) - g(a)\,f(x) + g(a)}{g(x) - f(x)} = 4$, then the value of k is equal to :

(a) 4 (b) 2
(c) 1 (d) 0

32. The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is :

(a) an even function
(b) an odd function
(c) a periodic function
(d) neither an even nor an odd function

33. If $f(x) = \begin{cases} xe^{-\left[\frac{1}{|x|} + \frac{1}{x}\right]} & , x \neq 0 \\ 0 & ; x = 0 \end{cases}$ then $f(x)$ is :

(a) continuous as well as differentiable for all x
(b) continuous for all x but not differentiable at $x = 0$
(c) neither differentiable nor continuous at $x = 0$
(d) discontinuous everywhere

34. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals :

(a) 3 (b) 1
(c) 2 (d) 1/2

35. If $f(y) = e^y$, $g(y) = y; y > 0$ and $F(t) = \int_0^t f(t-y)\,g(y)\,dy$, then :

(a) $F(t) = 1 - e^{-t}(1 + t)$
(b) $F(t) = e^t - (1 + t)$
(c) $F(t) = te^t$
(d) $F(t) = te^{-t}$

36. If $f(a + b - x) = f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to :

(a) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$
(b) $\frac{a+b}{2}\int_a^b f(x)\,dx$
(c) $\frac{b-a}{2}\int_a^b f(x)\,dx$
(d) $\frac{a+b}{2}\int_a^b f(a+b+x)\,dx$

37. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sec^2 t\,dt}{x \sin x}$ is :

(a) 3 (b) 2
(c) 1 (d) –1

38. The value of the integral $I = \int_0^1 x(1-x)^n\,dx$ is :

(a) $\frac{1}{n+1}$ (b) $\frac{1}{n+2}$
(c) $\frac{1}{n+1} - \frac{1}{n+2}$ (d) $\frac{1}{n+1} + \frac{1}{n+2}$

39. $\lim_{n \to \infty} \frac{1 + 2^4 + 3^4 + \ldots + n^4}{n^5} - \lim_{n \to \infty} \frac{1 + 2^3 + 3^3 + \ldots + n^3}{n^5}$ is :

(a) 1/30 (b) 0
(c) 1/4 (d) 1/5

40. Let $\frac{d}{dx}F(x) = \left(\frac{e^{\sin x}}{x}\right)$, $x > 0$.

If $\int_1^4 \frac{3}{x} e^{\sin x^3}\,dx = F(k) - F(1)$,

then one of the possible values of k, is :

(a) 15 (b) 16
(c) 63 (d) 64

41. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is :

(a) 2 sq unit (b) 3 sq unit
(c) 4 sq unit (d) 6 sq unit

42. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be a function that satisfies $f(x) + g(x) = x^2$. Then the value of the integral $\int_0^1 f(x)\,g(x)\,dx$, is :

(a) $e - \frac{e^2}{2} - \frac{5}{2}$ (b) $e + \frac{e^2}{2} - \frac{3}{2}$
(c) $e - \frac{e^2}{2} - \frac{3}{2}$ (d) $e + \frac{e^2}{2} + \frac{5}{2}$

43. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively :

(a) 2, 1 (b) 1, 2
(c) 3, 2 (d) 2, 3

44. The solution of the differential equation

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0, \text{ is :}$$

(a) $(x - 2) = ke^{-\tan^{-1} y}$
(b) $2x\,e^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$
(c) $xe^{\tan^{-1} y} = \tan^{-1} y + k$
(d) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

45. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$, then the value of '$C$' is :

(a) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$
(b) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

(c) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$

(d) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

46. Locus of centroid of the triangle whose vertices are $(a \cos t, a \sin t)$, $(b \sin t, -b \cos t)$ and $(1, 0)$, where t is a parameter, is :

(a) $(3x-1)^2 + (3y)^2 = a^2 - b^2$

(b) $(3x-1)^2 + (3y)^2 = a^2 + b^2$

(c) $(3x+1)^2 + (3y)^2 = a^2 + b^2$

(d) $(3x+1)^2 + (3y)^2 = a^2 - b^2$

47. If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then :

(a) $p = q$ (b) $p = -q$

(c) $pq = 1$ (d) $pq = -1$

48. A square of side a lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left(0 < \alpha < \frac{\pi}{4}\right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is :

(a) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

(b) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

(c) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

(d) $y(\cos\alpha + \sin\alpha) + x(\cos\alpha - \sin\alpha) = a$

49. If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then :

(a) $2 < r < 8$ (b) $r < 2$

(c) $r = 2$ (d) $r > 2$

50. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq unit. Then the equation of the circle is :

(a) $x^2 + y^2 + 2x - 2y = 62$

(b) $x^2 + y^2 + 2x - 2y = 47$

(c) $x^2 + y^2 - 2x + 2y = 47$

(d) $x^2 + y^2 - 2x + 2y = 62$

51. The normal at the point $(bt_1^2, 2bt_1)$ on a parabola meets the parabola again in the point $(bt_2^2, 2bt_2)$, then :

(a) $t_2 = -t_1 - \frac{2}{t_1}$ (b) $t_2 = -t_1 + \frac{2}{t_1}$

(c) $t_2 = t_1 - \frac{2}{t_1}$ (d) $t_2 = t_1 + \frac{2}{t_1}$

52. The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide. Then the value of b^2 is :

(a) 1 (b) 5

(c) 7 (d) 9

53. A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, 2, 1)$, $B(2, 1, 3)$ and $C(-1, 1, 2)$. Then the angle between the faces OAB and ABC will be :

(a) $\cos^{-1}\left(\frac{19}{35}\right)$ (b) $\cos^{-1}\left(\frac{17}{31}\right)$

(c) 30° (d) 90°

54. The radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$ is :

(a) 1 (b) 2

(c) 3 (d) 4

55. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if :

(a) $k = 0$ or -1 (b) $k = 1$ or -1

(c) $k = 0$ or -3 (d) $k = 3$ or -3

56. The two lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ will be perpendicular, if and only if :

(a) $aa' + bb' + cc' + 1 = 0$

(b) $aa' + bb' + cc' = 0$

(c) $(a + a')(b + b') + (c + c') = 0$

(d) $aa' + cc' + 1 = 0$

57. The shortest distance from the plane $12x + 4y + 3z = 327$ to the sphere $x^2 + y^2 + z^2 + 4x - 2y - 6z = 155$ is :

(a) 26 (b) $11\frac{4}{13}$

(c) 13 (d) 39

58. Two systems of rectangular axes have the same origin. If a plane cuts them at distances a, b, c and a', b', c' from the origin, then :

(a) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2} = 0$

(b) $\frac{1}{a^2} + \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} + \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(c) $\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{c^2} + \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

(d) $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$

59. $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 1$, $|\vec{b}| = 2$, $|\vec{c}| = 3$, then $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$ is equal to :

(a) 0 (b) -7
(c) 7 (d) 1

60. If $\vec{u}$, $\vec{v}$ and $\vec{w}$ are three non-coplanar vectors, then $(\vec{u} + \vec{v} - \vec{w})\cdot[(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})]$ equals :

(a) 0 (b) $\vec{u}\cdot\vec{v} \times \vec{w}$
(c) $\vec{u}\cdot\vec{w} \times \vec{v}$ (d) $3\vec{u}\cdot\vec{v} \times \vec{w}$

61. Consider points A, B, C and D with position vectors $7\hat{i} - 4\hat{j} + 7\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$, $-\hat{i} - 3\hat{j} + 4\hat{k}$ and $5\hat{i} - \hat{j} + 5\hat{k}$ respectively. Then $ABCD$ is a :

(a) square
(b) rhombus
(c) rectangle
(d) parallelogram but not a rhombus

62. The vectors $\vec{AB} = 3\hat{i} + 4\hat{k}$, and $\vec{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a triangle ABC. The length of the median through A is :

(a) $\sqrt{18}$ (b) $\sqrt{72}$
(c) $\sqrt{33}$ (d) $\sqrt{288}$

63. A particle acted on by constant forces $4\hat{i} + \hat{j} - 3\hat{k}$ and $3\hat{i} + \hat{j} - \hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to the point $5\hat{i} + 4\hat{j} + \hat{k}$. The total work done by the forces is :

(a) 20 unit (b) 30 unit
(c) 40 unit (d) 50 unit

64. Let $\vec{u} = \hat{i} + \hat{j}$, $\vec{v} = \hat{i} - \hat{j}$ and $\vec{w} = \hat{i} + 2\hat{j} + 3\hat{k}$. If $\hat{n}$ is a unit vector such that $\vec{u}\cdot\hat{n} = 0$ and $\vec{v}\cdot\hat{n} = 0$, then $|\vec{w}\cdot\hat{n}|$ is equal to :

(a) 0 (b) 1
(c) 2 (d) 3

65. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set :

(a) is increased by 2
(b) is decreased by 2
(c) is two times the original median
(d) remains the same as that of the original set

66. In an experiment with 15 observations on x, the following results were available

$$\Sigma x^2 = 2830,\ \Sigma x = 170.$$

One observation that was 20, was found to be wrong and was replaced by the correct value 30. Then the corrected variance is :

(a) 78.00 (b) 188.66
(c) 177.33 (d) 8.33

67. Five horses are in a race. Mr. A selects two of the horses at random and bets on them. The probability that Mr. A selected the winning horse, is :

(a) $\frac{4}{5}$ (b) $\frac{3}{5}$
(c) $\frac{1}{5}$ (d) $\frac{2}{5}$

68. Events A, B, C are mutually exclusive events such that $P(A) = \frac{3x+1}{3}$, $P(B) = \frac{1-x}{4}$ and $P(C) = \frac{1-2x}{2}$. The set of possible values of x are in the interval :

(a) $\left[\frac{1}{3}, \frac{1}{2}\right]$ (b) $\left[\frac{1}{3}, \frac{2}{3}\right]$
(c) $\left[\frac{1}{3}, \frac{13}{3}\right]$ (d) $[0, 1]$

69. The mean and variance of a random variable X having a binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is :

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$
(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

70. The resultant of forces $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, then $\vec{R}$ is doubled. If the direction of $\vec{Q}$ is reversed, then $\vec{R}$ is again doubled, then $P^2 : Q^2 : R^2$ is :

(a) $3:1:1$ (b) $2:3:2$
(c) $1:2:3$ (d) $2:3:1$

71. Let R_1 and R_2 respectively be the maximum ranges up and down an inclined plane and R be the maximum range on the horizontal plane. Then R_1, R, R_2 are in :

(a) arithmetico-geometric progression (AGP)
(b) AP
(c) GP
(d) HP

72. A couple is of moment $\vec{G}$ and the force forming the couple is $\vec{P}$. If $\vec{P}$ is turned through a right angle, the moment of the couple thus formed is $\vec{H}$. If instead, the forces $\vec{P}$ is turned through an angle α, then the moment of couple becomes :

(a) $\vec{\mathbf{G}} \sin\alpha - \vec{\mathbf{H}} \cos\alpha$

(b) $\vec{\mathbf{H}} \cos\alpha + \vec{\mathbf{G}} \sin\alpha$

(c) $\vec{\mathbf{G}} \cos\alpha + \vec{\mathbf{H}} \sin\alpha$

(d) $\vec{\mathbf{H}} \sin\alpha - \vec{\mathbf{G}} \cos\alpha$

73. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity $\vec{\mathbf{u}}$ and the other from rest with uniform acceleration $\vec{\mathbf{f}}$. Let α be the angle between their directions of motion. The relative velocity of the second particle w.r.t. the first is least after a time :

(a) $\dfrac{u \sin\alpha}{f}$ (b) $\dfrac{f \cos\alpha}{u}$

(c) $u \sin\alpha$ (d) $\dfrac{u \cos\alpha}{f}$

74. Two stones are projected from the top of a cliff h metres high, with the same speed u so as to hit the ground at the same spot. If one of the stones is projected horizontally and the other is projected at an angle θ to the horizontal, then $\tan\theta$ equals :

(a) $\sqrt{\dfrac{2u}{gh}}$ (b) $2g\sqrt{\dfrac{u}{h}}$

(c) $2h\sqrt{\dfrac{u}{g}}$ (d) $u\sqrt{\dfrac{2}{gh}}$

75. A body travels a distance s in t seconds. It starts from rest and ends at rest. In the first part of the journey, it moves with constant acceleration f and in the second part with constant retardation r. The value of t is given by :

(a) $2s\left(\dfrac{1}{f} + \dfrac{1}{r}\right)$ (b) $\dfrac{2s}{\dfrac{1}{f} + \dfrac{1}{r}}$

(c) $\sqrt{2s(f + r)}$ (d) $\sqrt{2s\left(\dfrac{1}{f} + \dfrac{1}{r}\right)}$

ANSWERS

PHYSICS AND CHEMISTRY

1.	(b)	2.	(a)	3.	(b)	4.	(a)	5.	(d)	6.	(a)	7.	(a)	8.	(d)
9.	(c)	10.	(b)	11.	(a)	12.	(c)	13.	(c)	14.	(d)	15.	(c)	16.	(a)
17.	(a)	18.	(c)	19.	(c)	20.	(b)	21.	(c)	22.	(d)	23.	(d)	24.	(b)
25.	(d)	26.	(a)	27.	(d)	28.	(c)	29.	(c)	30.	(b)	31.	(d)	32.	(a)
33.	(d)	34.	(c)	35.	(c)	36.	(a)	37.	(c)	38.	(d)	39.	(a)	40.	(d)
41.	(c)	42.	(a)	43.	(d)	44.	(a)	45.	(b)	46.	(a)	47.	(a)	48.	(b)
49.	(b)	50.	(c)	51.	(c)	52.	(d)	53.	(b)	54.	(d)	55.	(c)	56.	(b)
57.	(b)	58.	(a)	59.	(c)	60.	(a)	61.	(d)	62.	(b)	63.	(c)	64.	(a)
65.	(a)	66.	(b)	67.	(a)	68.	(d)	69.	(b)	70.	(c)	71.	(c)	72.	(a)
73.	(b)	74.	(d)	75.	(d)	76.	(b)	77.	(a)	78.	(b)	79.	(a)	80.	(b)
81.	(b)	82.	(c)	83.	(d)	84.	(d)	85.	(b)	86.	(c)	87.	(a)	88.	(b)
89.	(d)	90.	(b)	91.	(c)	92.	(c)	93.	(c)	94.	(b)	95.	(c)	96.	(b)
97.	(b)	98.	(c)	99.	(b)	100.	(c)	101.	(a)	102.	(b)	103.	(c)	104.	(a)
105.	(d)	106.	(a)	107.	(d)	108.	(d)	109.	(c)	110.	(b)	111.	(a)	112.	(c)
113.	(d)	114.	(b)	115.	(b)	116.	(c)	117.	(a)	118.	(c)	119.	(a)	120.	(b)
121.	(b)	122.	(a)	123.	(d)	124.	(b)	125.	(b)	126.	(b)	127.	(a)	128.	(d)
129.	(c)	130.	(a)	131.	(b)	132.	(b)	133.	(c)	134.	(c)	135.	(d)	136.	(b)
137.	(a)	138.	(d)	139.	(a)	140.	(c)	141.	(a)	142.	(b)	143.	(c)	144.	(a)
145.	(a)	146.	(d)	147.	(d)	148.	(c)	149.	(d)	150.	(c)				

MATHEMATICS

1.	(c)	2.	(c)	3.	(d)	4.	(a)	5.	(b)	6.	(c)	7.	(c)	8.	(b)
9.	(a)	10.	(b)	11.	(b)	12.	(a)	13.	(a)	14.	(b)	15.	(b)	16.	(c)
17.	(d)	18.	(a)	19.	(a)	20.	(b)	21.	(a)	22.	(*)	23.	(a)	24.	(b)
25.	(c)	26.	(d)	27.	(c)	28.	(d)	29.	(c)	30.	(c)	31.	(a)	32.	(b)
33.	(b)	34.	(c)	35.	(b)	36.	(b)	37.	(c)	38.	(c)	39.	(d)	40.	(d)
41.	(c)	42.	(c)	43.	(b)	44.	(b)	45.	(a)	46.	(b)	47.	(d)	48.	(d)
49.	(a)	50.	(c)	51.	(a)	52.	(c)	53.	(a)	54.	(c)	55.	(c)	56.	(d)
57.	(c)	58.	(d)	59.	(b)	60.	(b)	61.	(*)	62.	(c)	63.	(c)	64.	(d)
65.	(d)	66.	(a)	67.	(d)	68.	(a)	69.	(a)	70.	(b)	71.	(d)	72.	(c)
73.	(d)	74.	(d)	75.	(d)										

(*) No option is correct in question paper.

HINTS & SOLUTIONS

Physics

1. When particle describes circular path in a magnetic field, its velocity is always perpendicular to the magnetic force.

Power $P = \vec{F} \cdot \vec{v} = Fv \cos\theta$

Here, $\theta = 90°$

$\therefore \quad P = 0$

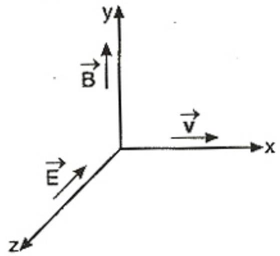
But $P = \dfrac{W}{t}$

$\Rightarrow \quad W = P \cdot t$

Hence, work done

$W = 0$ (everywhere)

2. The force on a particle is



So, $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

or $\vec{F} = \vec{F}_e + \vec{F}_m$

$\therefore \vec{F}_e = q\vec{E} = -16 \times 10^{-18} \times 10^4 (-\hat{k})$

$= 16 \times 10^{-14} \hat{k}$

and $\vec{F}_m = -16 \times 10^{-18} (10\hat{i} \times B\hat{j})$

$= -16 \times 10^{-17} \times B (+\hat{k})$

$= -16 \times 10^{-17} B\hat{k}$

Since, particle will continue to move along $+x$-axis, so resultant force is equal to 0.

$\vec{F}_e + \vec{F}_m = 0$

$\therefore \quad 16 \times 10^{-14} = 16 \times 10^{-17} B$

$\Rightarrow \quad B = \dfrac{16 \times 10^{-14}}{16 \times 10^{-17}} = 10^3$

$B = 10^3 \text{ Wb/m}^2$

3. When magnet is divided into two equal parts, the magnetic dipole moment.

$M' = \text{pole strength} \times \dfrac{l}{2} = \dfrac{M}{2}$

[pole strength remains same]

Also, the mass of magnet becomes half *i. e.*,

$m' = \dfrac{m}{2}$

Moment of inertia of magnet

$I = \dfrac{ml^2}{12}$

New moment of inertia

$I' = \dfrac{1}{12}\left(\dfrac{m}{2}\right)\left(\dfrac{l}{2}\right)^2 = \dfrac{ml^2}{12 \times 8}$

$\therefore \quad I' = \dfrac{I}{8}$

Now, $T = 2\pi\sqrt{\left(\dfrac{I}{MB}\right)}$

$T' = 2\pi\sqrt{\left(\dfrac{I'}{M'B}\right)} = 2\pi\sqrt{\left(\dfrac{I/8}{MB/2}\right)}$

$\therefore \quad T' = \dfrac{T}{2} \Rightarrow \dfrac{T'}{T} = \dfrac{1}{2}$

4. $W = MB(1 - \cos 0)$

$\Rightarrow \quad W = MB(1 - \cos 60°) \quad (\because \theta = 60°)$

$\Rightarrow \quad W = \dfrac{MB}{2}$

$\therefore \quad MB = 2W$

Torque, $\tau = MB \sin 60°$

$= \dfrac{MB\sqrt{3}}{2} = \dfrac{2W\sqrt{3}}{2}$

$= W\sqrt{3}$

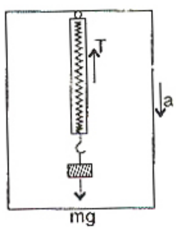
5. Inside bar magnet, lines of force are from south to north.

6. At Curie temperature, a ferromagnetic substance transits into paramagnetic substance.

7. In stationary position, spring balance reading

$= mg = 49$

$m = \dfrac{49}{9.8} = 5 \text{ kg}$



When lift moves downward.

$mg - T = ma$

Reading of balance

$T = mg - ma$

$= 5(9.8 - 5)$

$= 5 \times 4.8$

$= 24.0 \text{ N}$

8. $$V \propto l$$

$\therefore$ $$\frac{V}{E} = \frac{l}{L}$$

where, l = balance point

L = length of potentiometer wire

or $$V = \frac{l}{L}E$$

$$V = \frac{30 \times E}{100} = \frac{30}{100}E$$

9. Germanium is semiconductor, whereas copper is conductor.

For conductors, $R \propto \Delta t$

For semiconductors,

$$R \propto \frac{1}{\Delta t}$$

Hence, when both are cooled, then resistance of copper decreases whereas that of germanium increases.

10. Some of the characteristics of an optical fibre are as follows :

(i) This works on the principle of total internal reflection.

(ii) It consists of core made up of glass/silica/plastic with refractive index n_1, which is surrounded by a glass or plastic cladding with refractive index $n_2 (n_2 > n_1)$. The refractive index of cladding can be either changing abruptly or gradually changing (graded index fibre).

(iii) There is a very little transmission loss through optical fibres.

(iv) There is no interference from stray electric and magnetic fields to the signals through optical fibres.

11. Thermo-emf of thermocouple = $25\,\mu V/°C$.

Let θ be the smallest temperature difference.

Therefore, after connecting the thermocouple with the galvanometer, thermo-emf

$$E = (25\,\mu V/°C) \times \theta (°C)$$

$$= 25\theta \times 10^{-6}\ V$$

Potential drop developed across the galvanometer

$$= iR = 10^{-5} \times 40 = 4 \times 10^{-4}\ V$$

$\therefore$ $$4 \times 10^{-4} = 25\theta \times 10^{-6}$$

$\therefore$ $$\theta = \frac{4}{25} \times 10^2 = 16°C$$

12. According to Faraday's law of electrolysis,

$$m = zit$$

$$\frac{m_{Zn}}{m_{Cu}} = \frac{z_{Zn}}{z_{Cu}}$$

$\therefore$ $$\frac{0.13}{m_{Cu}} = \frac{32.5}{31.5}$$

$$m_{Cu} = 0.126\ g$$

13. As we know that formula of velocity is

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\therefore$ $$v^2 = \frac{1}{\mu_0 \varepsilon_0} = [LT^{-1}]^2$$

$\therefore$ $$\frac{1}{\mu_0 \varepsilon_0} = [L^2T^{-2}]$$

14. Mass of disc (X), $m_X = \pi R^2 t\rho$

where, ρ = density of material of disc

$\therefore$ $$I_X = \frac{1}{2} m_X R^2 = \frac{1}{2} \pi R^2 t\rho R^2$$

$$I_X = \frac{1}{2} \pi \rho t R^4 \quad \text{...(i)}$$

Mass of disc (Y)

$$m_Y = \pi (4R)^2 \frac{t}{4} \rho = 4\pi R^2 t\rho$$

and $$I_Y = \frac{1}{2} m_Y (4R)^2 = \frac{1}{2} 4\pi R^2 t\rho \cdot 16R^2$$

$\Rightarrow$ $$I_Y = 32\pi t\rho R^4 \quad \text{...(ii)}$$

$\therefore$ $$\frac{I_Y}{I_X} = \frac{32\pi t\rho R^4}{\frac{1}{2}\pi \rho t R^4}$$

$$= 64$$

$\therefore$ $$I_Y = 64\, I_X$$

15. According to Kepler's law

$$T^2 \propto r^3$$

or $$5^2 \propto r^3 \quad \text{...(i)}$$

$$(T')^2 \propto (4r)^3 \quad \text{...(ii)}$$

From Eqs. (i) and (ii)

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

$$T' = \sqrt{1600}$$

$$T' = 40\ h$$

16. Angular momentum

$$L = I\omega \quad \text{...(i)}$$

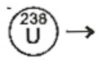
Kinetic energy

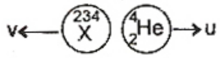
$$K = \frac{1}{2} I\omega^2 = \frac{1}{2} L\omega \quad \text{[from Eq. (i)]}$$

$\therefore$ $$L = \frac{2K}{\omega}$$

Now, $$L' = \frac{2\left(\frac{K}{2}\right)}{2\omega}$$

$\Rightarrow$ $$L' = \frac{L}{4}$$

18.  [Before decay]



[After Decay]

Apply conservation of linear momentum.

$$0 = 4u - 234v$$

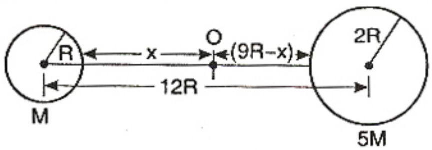
$$\Rightarrow \quad v = \frac{4u}{234}$$

The residual nucleus will recoil with a velocity of $\frac{4u}{234}$ unit.

Recoil speed of residual nucleus is $\frac{4u}{234}$.

Note : If they will ask the recoil velocity, then answer remains same *i.e.*, $\frac{4u}{234}$ and not $-\frac{4u}{234}$ as the word recoil itself is signifying the direction of motion of residual nucleus.

19. Let at 'O' there will be a collision. If smaller sphere moves x distance to reach at O, then bigger sphere will move a distance of $(9R - x)$.



$$F = \frac{GM \times 5M}{(12R - x)^2}$$

$$a_{\text{small}} = \frac{F}{M} = \frac{G \times 5M}{(12R - x)^2}$$

$$a_{\text{big}} = \frac{F}{5M} = \frac{GM}{(12R - x)^2}$$

$$x = \frac{1}{2} a_{\text{small}} t^2 = \frac{1}{2} \frac{G \times 5M}{(12R - x)^2} t^2 \quad \text{...(i)}$$

$$(9R - x) = \frac{1}{2} a_{\text{big}} t^2 = \frac{1}{2} \frac{GM}{(12R - x)^2} t^2 \quad \text{...(ii)}$$

Thus, dividing Eq. (i) by Eq. (ii), we get

$$\therefore \quad \frac{x}{9R - x} = 5$$

$$\Rightarrow \quad x = 45R - 5x$$

$$\Rightarrow \quad 6x = 45R$$

$$\Rightarrow \quad x = 7.5R$$

20. The difference in the variation of resistance with temperature in metal and semiconductor is caused due to difference in the variation of the number of charge carriers with temperature.

21. $v^2 = u^2 + 2as$

$$0 = \left(50 \times \frac{5}{18}\right)^2 + 2a \times 6$$

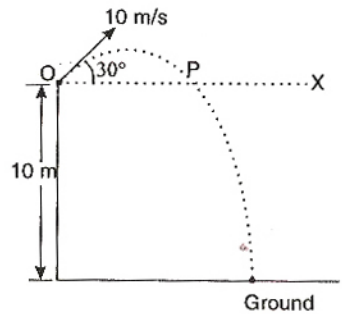
$$a = -16 \text{ m/s}^2 \qquad (a = \text{ retardation})$$

Again $v^2 = u^2 + 2as$

$$0 = \left(100 \times \frac{5}{18}\right)^2 - 16 \times 2 \times s$$

$$s = \frac{(100 \times 5)^2}{18 \times 18 \times 32} = 24.1 \approx 24 \text{ m}$$

22. The ball will be at point P when it is at a height of 10 m from the ground. So, we have to find the distance OP, which can be calculated directly by considering it as a projectile on a levelled plane (OX).

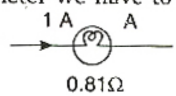


$$OP = R = \frac{u^2 \sin 2\theta}{g} = \frac{10^2 \times \sin(2 \times 30°)}{10}$$

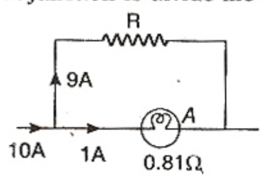
$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3}$$

$$= 8.66 \text{ m}$$

23. To increase the range of ammeter we have to connect a small resistance in parallel (shunt), let its value be R.



Apply KCL at junction to divide the current.



Voltage across R = Voltage across ammeter.

$$\Rightarrow \quad 9R = 0.81 \times 1$$

$$\Rightarrow \quad R = \frac{0.81}{9} = 0.09\,\Omega$$

24. Planck's constant $(h) = \text{J} - \text{s}[\text{ML}^2\text{T}^{-2}]\,[\text{T}]$

$$[\text{ML}^2\text{T}^{-1}]$$

Momentum (p) kg – m – s^{-1}

$$= [\text{M}][\text{L}][\text{T}^{-1}] = [\text{MLT}^{-1}]$$

25. Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order. Hence, by Newton's 2nd law

$\left(\vec{F} = m\dfrac{d\vec{v}}{dt}\right)$, particle velocity ($\vec{v}$) will be same.

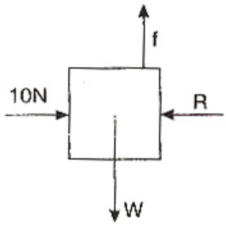
26. [From Gauss law, $\dfrac{\text{Charge enclosed}}{\varepsilon_0}$

$= \text{Flux leaving the surface}]$

$$\frac{q}{\varepsilon_0} = \phi_2 - \phi_1$$

$$\Rightarrow \quad q = (\phi_2 - \phi_1)\,\varepsilon_0$$

27. Let R be the normal contact force by wall on the block.



$R = 10\text{ N}$

$f_L = w$ and $f = \mu R$

$\therefore \quad \mu R = w$

$\Rightarrow \quad w = 0.2 \times 10 = 2\text{ N}$

28. Let coefficient of friction be μ, then retardation will be μg.

From equation of motion, $v = u + at$

$$\Rightarrow \quad 0 = 6 - \mu g \times 10$$

$$\Rightarrow \quad \mu = \frac{6}{100} = 0.06$$

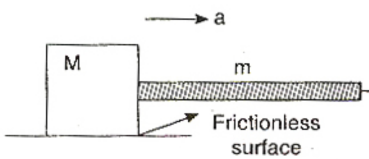
29. Here B is implying A but A is not implying B, as kinetic energy of a system of particles is zero means speed of each and every particle is zero which says that momentum of every particle is zero.

But statement A means linear momentum of a system of particles is zero, which may be true even if particles have equal and opposite momentums and hence, having non-zero KE.

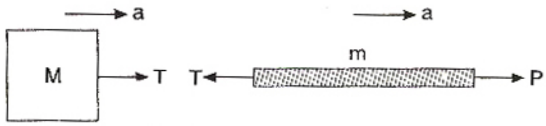
30. Mutual inductance of the pair of coils depends on distance between two coils and geometry of two coils.

31. Let acceleration of system (rope + block) is a along the direction of applied force. Then

$$a = \frac{P}{M + m}$$



Draw the *FBD* of block and rope as shown in figure.

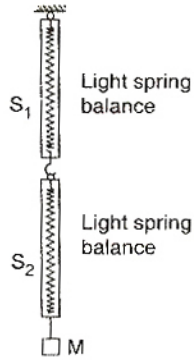


Where, T is the required parameter

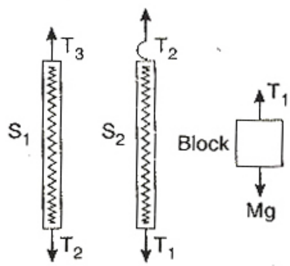
For block $\quad T = Ma$

$$\Rightarrow \quad T = \frac{MP}{M + m}$$

32. The arrangement is shown in figure.



Now, draw the free body diagram of the spring balances and block.



For equilibrium of block, $T_1 = Mg$

where, $T_1 = \text{Reading of } S_2$

For equilibrium of S_2, $T_2 = T_1$

where, $T_2 = \text{Reading of } S_1$

For equilibrium of S_1, $T_2 = T_3$

Hence, $\quad T_1 = T_2 = Mg$

So, both scales read M kg.

33. Elastic energy stored in the wire is

$$U = \frac{1}{2}\text{ stress} \times \text{strain} \times \text{volume}$$

$$= \frac{1}{2}\frac{F}{A} \times \frac{\Delta l}{L} \times AL$$

$$= \frac{1}{2}F\,\Delta l$$

$$= \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1\text{ J}$$

34. The escape velocity is independent of angle of projection, hence, it will remain same *i. e.*, 11 km/s.

35. $$T = 2\pi\sqrt{\frac{M}{k}} \quad \text{...(i)}$$

$$T' = 2\pi\sqrt{\frac{M+m}{k}}$$

$$\Rightarrow \quad \frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}} \quad \text{...(ii)}$$

Dividing Eq. (i) by (ii), we have

$$\therefore \quad \frac{3}{5} = \sqrt{\frac{M}{M+m}}$$

$$\frac{9}{25} = \frac{M}{M+m}$$

$$\Rightarrow \quad 9M + 9m = 25M$$

$$\Rightarrow \quad 16M = 9m$$

$$\frac{m}{M} = \frac{16}{9}$$

36. Heat cannot flow itself from a lower temperature to a body of higher temperature. This corresponds to second law of thermodynamics.

37. $$v_{\max} = a\omega = a\frac{2\pi}{T}$$

$$= \frac{2\pi a}{2\pi\sqrt{\frac{m}{k}}} = a\sqrt{\frac{k}{m}}$$

Hence, $$\frac{v_{\max_1}}{v_{\max_2}} = \frac{a_1}{a_2}\sqrt{\frac{k_1}{k_2}}$$

$$\because \quad v_{\max_1} = v_{\max_2} \quad \text{(given)}$$

$$\frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$$

38. $$\mathrm{T} = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad \frac{\Delta \mathrm{T}}{\mathrm{T}} = \frac{1}{2}\frac{\Delta l}{l}$$

Given, $$\frac{\Delta l}{l} = 21\%$$

$$\therefore \quad \frac{\Delta \mathrm{T}}{\mathrm{T}} = \frac{1}{2} \times 21\% = 10.5\%$$

39. The given equation of wave

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \quad \text{...(i)}$$

Standard equation of wave

$$y = a\sin(\omega t - kx + \phi) \quad \text{...(ii)}$$

Now comparing Eqs. (i) and (ii), we get,

$$\omega = 600 \quad \text{and} \quad k = 2$$

$$\therefore \quad \text{velocity of wave} = \frac{\omega}{k} = \frac{600}{2} = 300 \text{ m/s}$$

40. $$e = -L\frac{di}{dt} = -L\frac{(-2-2)}{0.05}$$

$$8 = L\frac{(4)}{0.05}$$

$$\therefore \quad L = \frac{8 \times 0.05}{4} = 0.1 \text{ H}$$

41. In an LC circuit the energy oscillates between inductor (in the magnetic field) and capacitor (in the electric field).

$U_{E\max}$ [Maximum energy stored in capacitor]

$$= \frac{Q^2}{2C}$$

$U_{B\max}$ [Maximum energy stored in inductor]

$$= \frac{LI_0^2}{2}$$

where, I_0 is the current at this time.

For the given instant $U_E = U_B$

$$i.e., \quad \frac{q^2}{2C} = \frac{Li^2}{2}$$

From energy conservation

$$U_E + U_B = U_{E\max} = U_{B\max}$$

$$\Rightarrow \quad 2\frac{q^2}{2C} = \frac{Q^2}{2C}$$

$$\Rightarrow \quad q = \frac{Q}{\sqrt{2}}$$

42. The core of transformer is laminated to reduce energy loss due to eddy currents.

43. $\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{\tau}$ is perpendicular to both $\vec{r}$ and $\vec{F}$, so $\vec{r} \cdot \vec{\tau}$ as well as $\vec{F} \cdot \vec{\tau}$ has to be zero.

44. Given : $N_0\lambda = 5000$, $N\lambda = 1250$

$$N = N_0 e^{-\lambda t} = N_0 e^{-5\lambda}$$

Where λ is decay constant per min

$$N\lambda = N_0 \lambda e^{-5\lambda}$$

$$1250 = N_0 \lambda e^{-5\lambda}$$

$$\therefore \quad \frac{N_0\lambda}{N_0\lambda e^{-5\lambda}} = \frac{5000}{1250} = 4$$

$$e^{5\lambda} = 4$$

$$5\lambda = 2\log_e 2$$

$$\lambda = 0.4 \ln 2$$

45. Since, 8 α-particles $4\beta^-$-particles and $2\beta^+$-particles are emitted, so new atomic number

$$Z' = Z - 8 \times 2 + 4 \times 1 - 2 \times 1$$

$$= 92 - 16 + 4 - 2$$

$$= 92 - 14$$

$$= 78$$

46. $hf = hf_0 + \frac{1}{2}mv^2$

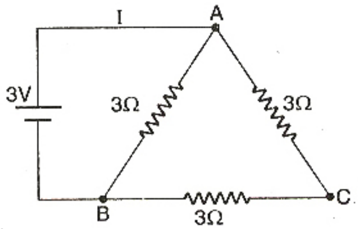
Hence, $$v_1^2 = \frac{2hf_1}{m} - \frac{2hf_0}{m}$$

$$v_2^2 = \frac{2hf_2}{m} - \frac{2hf_0}{m}$$

$\therefore$ $$v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$$

47. Protons cannot be emitted by radioactive substances during their decay.

48. Resistance in the arms AC and BC are in series,



$\therefore$ $$R' = 3 + 3 = 6\,\Omega$$

Now R' and $3\,\Omega$ are in parallel,

$\therefore$ $$R_{eq} = \frac{6 \times 3}{6 + 3} = 2\Omega$$

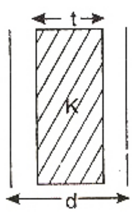
Now, $V = IR$

$\Rightarrow$ $$I = \frac{3}{2} = 1.5 \text{ A}$$

49. Aluminium is a metal, so when we insert an aluminium foil, equal and opposite charges appear on its two surfaces. Since, it is of negligible thickness, it will not effect the capacitance.

Alternative : From the formula

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$



$\Rightarrow$ Here, $K = \infty$ and $t \to 0$

So, $C = \frac{\varepsilon_0 A}{d + 0} = \frac{\varepsilon_0 A}{d} = C_0$

50. $x = 4(\cos \pi t + \sin \pi t)$

$$= \frac{4}{\sqrt{2}} \times \sqrt{2}[\cos \pi t + \sin \pi t]$$

$$x = 4\sqrt{2} \sin\left[\pi t + \frac{\pi}{4}\right]$$

So , amplitude $= 4\sqrt{2}$

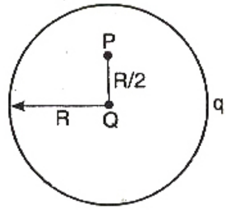
51. At P due to shell, potential

$$V_1 = \frac{q}{4\pi \varepsilon_0 R}$$

at P due to Q, potential

$$V_2 = \frac{Q}{4\pi \varepsilon_0 \frac{R}{2}}$$

$$= \frac{2Q}{4\pi \varepsilon_0 R}$$



$\therefore$ Net potential at P

$$V = V_1 + V_2$$

$$= \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R}$$

52. $W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

$$= \frac{1}{2} \times \frac{(8 \times 10^{-18})^2}{100 \times 10^{-6}}$$

$$= \frac{1}{2} \times \frac{64 \times 10^{-36}}{100 \times 10^{-6}} = 32 \times 10^{-32} \text{ J}$$

53. $x = \alpha t^3$, $y = \beta t^3$

$$v_x = \frac{dx}{dt} = 3\alpha t^2$$

$$v_y = \frac{dy}{dt} = 3\beta t^2$$

Resultant velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$

$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

54. Given : $P \propto T^3$ ...(i)

In an adiabatic process

$$T^{\gamma} P^{1-\gamma} = \text{constant}$$

$$T \propto \frac{1}{P^{(1-\gamma)/\gamma}}$$

$T^{(\gamma/\gamma - 1)} \propto P$ ...(ii)

Comparing Eqs. (i) and (ii), we get

$\therefore$ $$\frac{\gamma}{\gamma - 1} = 3$$

$$3\gamma - 3 = \gamma$$

$$2\gamma = 3$$

$$\frac{C_P}{C_V} = \gamma = \frac{3}{2}$$

55. Work does not characterise the thermodynamic state of matter, it is a path function giving only relationship between two quantities.

56. $T_1 = 627 + 273 = 900$ K

$Q_1 = 3 \times 10^6$ cal

$T_2 = 27 + 273 = 300$ K

$\therefore$ $$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$\Rightarrow \quad Q_2 = \frac{T_2}{T_1} \times Q_1 = \frac{300}{900} \times 3 \times 10^6$

$= 1 \times 10^6$ cal

Work done $= Q_1 - Q_2$

$= 3 \times 10^6 - 1 \times 10^6 = 2 \times 10^6$ cal

$= 2 \times 4.2 \times 10^6 \text{J} = 8.4 \times 10^6 \text{J}$

57. $W_1 = \frac{1}{2} k \times {x_1}^2 = \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2$

$= 6.25$ J

$W_2 = \frac{1}{2} k (x_1 + x_2)^2$

$= \frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2$

$= 25$ J

Net work done $= W_2 - W_1$

$= 25 - 6.25 = 18.75$ J

$= 18.75$ N-m

58. The wire will vibrate with the same frequency as that of source. This can be considered as an example of forced vibration.

$T = 10 \times 9.8\text{N} = 98\text{N},$

$m = 9.8 \times 10^{-3}$ kg/m

Frequency of wire $f = \frac{1}{2L} \sqrt{\left(\frac{T}{m}\right)}$

$= \frac{1}{2 \times 1} \sqrt{\left(\frac{98}{9.8 \times 10^{-3}}\right)}$

$= 50$ Hz

59. $f_1 = 256$ Hz

For tuning fork $f_2 - f_1 = \pm 5,$

f_2 = frequency of piano

$f_2 = (256 + 5)$ Hz, or $(256 - 5)$ Hz

When tension is increased, the beat frequency decreases to 2 beats/s.

If we assume that the frequency of piano string is 261Hz, then on increasing tension, frequency, more than 261Hz. But it is given that beat frequency decreases to 2, therefore 261 is not possible.

Hence, 251 Hz *i.e.*, 256–5 was the frequency of piano string before increasing tension.

60. At $x = 0$, kinetic energy is maximum and potential energy is minimum.

61. $\frac{3}{2} kT = 7.7 \times 10^{-14}$ J

$T = \frac{2 \times 7.7 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}}$

$= 3.7 \times 10^9$ K

62. As $_{55}Cs^{133}$ has larger size among the four atoms given, thus, electrons present in the outermost orbit will be away from the nucleus and the electrostatic force experienced by electrons due to nucleus will be minimum. Therefore, the energy required to liberate electrons from outer orbit will be minimum in case of $_{55}Cs^{133}$.

64. Due to the reverse biasing the width of depletion region increases and current flowing through the diode is almost zero. And in this case electric field is almost zero at the middle of the depletion region.

65. $E = -Z^2 \frac{13.6}{n^2}$ eV

For first excited state

$E_2 = -3^2 \times \frac{13.6}{4}$

$= -30.6$ eV

Ionisation energy for first excited state of Li^{2+} is 30.6 eV.

66. P = constant

$\Rightarrow \quad Fv = P$

$[\because P = \text{force} \times \text{velocity}]$

$\Rightarrow \quad Ma \times v = P \qquad [\because F = Ma]$

$\Rightarrow \quad va = \frac{P}{M}$

$\Rightarrow \quad v \times \left[\frac{v\,dv}{ds}\right] = \frac{P}{M} \qquad \left[\because a = \frac{v\,dv}{ds}\right]$

$\Rightarrow \quad \int_0^v v^2\,dv = \int_0^s \frac{P}{M}\,ds$

[Assuming at $t = 0$ it starts from rest, *i.e.*, from $s = 0$]

$\Rightarrow \quad \frac{v^3}{3} = \frac{P}{M} s$

$\Rightarrow \quad v = \left(\frac{3P}{M}\right)^{1/3} s^{1/3}$

$\Rightarrow \quad \frac{ds}{dt} = k\,s^{1/3} \qquad \left[k = \left(\frac{3P}{M}\right)^{1/3}\right]$

$\Rightarrow \quad \int_0^s \frac{ds}{s^{1/3}} = \int_0^t K\,dt$

$\Rightarrow \quad \frac{s^{2/3}}{2/3} = Kt$

$\Rightarrow \quad s^{2/3} = \frac{2}{3} Kt$

$\Rightarrow \quad s = \left(\frac{2}{3} K\right)^{3/2} \times t^{3/2}$

$\Rightarrow \quad s \propto t^{3/2}$

67. Here, thrust force is responsible to accelerate the rocket, so initial thrust of the blast

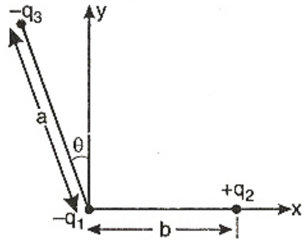
$= ma$

$= 3.5 \times 10^4 \times 10$

$= 3.5 \times 10^5$ N

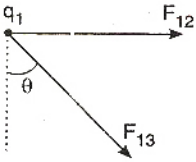
68. Sustained interference is possible with coherent sources only.

69. Force on $-q_1$



$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{b^2}\hat{\mathbf{i}} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{a^2}[\sin\theta\hat{\mathbf{i}} - \cos\theta\hat{\mathbf{j}}]$$

From above, x' component $-q_1$ of force is



$$F_x = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

$$\therefore \quad F_x \propto \left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

70. $R = \frac{V^2}{P} = \frac{(220)^2}{1000}$

where, V and P are denoting rated voltage and power respectively

$$\therefore \ P_{\text{consumed}} = \frac{V^2_{\text{applied}}}{R} = \frac{110 \times 110}{220 \times 220} \times 1000$$

$$= 250 \text{ W}$$

71. Objective of compound microscope is a convex lens. Convex lens forms real and enlarged image when an object is placed between its focus and lens.

73.

$$n = \frac{360°}{\theta} - 1$$

$$3 = \frac{360°}{\theta} - 1$$

$$\theta = 90°$$

74. According to Newton's law of cooling

$$\frac{dQ}{dt} \propto \Delta\theta$$

But $\frac{dQ}{dt} \propto (\Delta\,\theta)^n$ (given)

$\therefore$ $n = 1$

75. Given : $l' = l + 100\%\, l = 2l$

Initial volume = final volume

i.e. $\pi r^2 l = \pi r'^2 l'$

$$\Rightarrow \quad r'^2 = \frac{r^2 l}{l'} = r^2 \times \frac{l}{2l}$$

$$\Rightarrow \quad r'^2 = \frac{r^2}{2}$$

$$\therefore \quad R' = \rho\frac{l'}{A'} = \rho\frac{2l}{\pi r'^2} \qquad \left(\because R = \frac{\rho l}{A}\right)$$

$$= \frac{\rho \cdot 4l}{\pi r^2}$$

$$= 4R$$

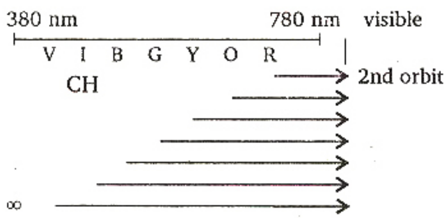
Thus, $\Delta R = R' - R = 4R - R$

$$= 3R$$

$$\therefore \ \%\Delta R = \frac{3R}{R} \times 100\% = 300\%$$

Chemistry

76. Hydrogen spectrum coloured radiation means visible radiation corresponds to Balmer series ($n_1 = 2$, $n_2 = 3, 4 \ldots$.)



3rd line from the red end it means $5 \to 2$

77. $\lambda = \frac{h}{mv} = 1.105 \times 10^{-33}$ m

78. For s-electron, $l = 0$.

79. Mass of one unit-cell (m) = volume × density

$$= a^3 \times d$$

$$= a^3 \times \frac{Mz}{N_0\, a^3} = \frac{Mz}{N_0}$$

$$m = \frac{58.5 \times 4}{6.02 \times 10^{23}} \text{ g}$$

$\therefore$ Number of unit cells in 1 g $= \frac{1}{m}$

$$= \frac{6.02 \times 10^{23}}{58.5 \times 4} = 2.57 \times 10^{21}$$

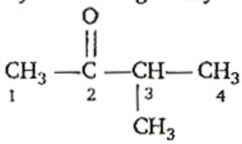
80. Glass is a supercooled liquid, which forms a non-crystalline solid. The common lime-soda glass used for bottles and jars is a supercooled mixture of sodium and calcium silicates.

81. NH_3-an inorganic solvent.

AgI is having much higher covalent characteristics in comparison to AgCl.

82. All physical and chemical properties of elements are periodic function of atomic number- Modern periodic law.

83. C-atoms form covalently bonded plates (layers). Layers are bonded weakly together, that's why one layer can slide over other cause lubricacy. Cannot be melted easily as large number of atoms being bonded strongly in the layer form big entity.

84. 

3-methyl butan-2-one

or 3-methyl 2-butanone

keto ($-\overset{O}{\overset{||}{C}}-$) functional group is given priority.

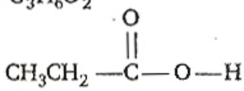
85. $LiAlH_4$ reduces —COOH to —CH_2OH without affecting C $=$C bond.

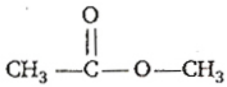
86. In between two successive collision, no force is acting on the gas molecules. Resultantly it travels with uniform velocity during this interval, and hence, it moves along a straight line.

87. Cu, Ag, Au group of elements are called coinage metals as these are used in minting coins.

88. $C_nH_{2n}O_2$ is general formula for open chain acid and ester

$n = 3$ $C_3H_6O_2$

Acid 

Ester 

89. Alcohol has polar H which makes inter-molecular H-bonding possible. Ether is non polar hence no H-bonding. Lack of H-bonding in ether makes it more volatile than alcohol.

90. Chiral compounds which have one chiral centre. All four atoms or groups attached to carbon are different.

91. Alkaline earth metal salts are causing hardness :

Temporary hardness caused by soluble Ca and Mg hydrogen carbonates. calcium and magnesium soluble sulphates and chlorides cause permanent hardness.

92.

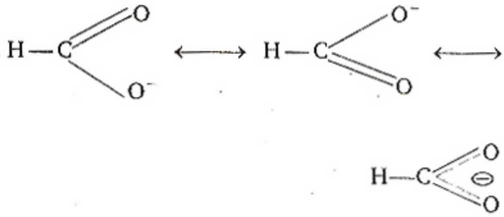
Molecule	Hybridisation	Repulsion	Bond angle
SO_2	sp^2	$lp - bp, bp - bp$	119°
OH_2	sp^3	$lp - lp, bp - lp$ $bp - bp$	104.5°
SH_2	sp^3	—do—	90°
NH_3	sp^3	$lp - bp, bp - bp$	107°

93. NO_2 and O_3 both are having irregular geometry.

94. N^{3-} $7 + 3 = 10$ electrons

F^- $9 + 1 = 10$ electrons

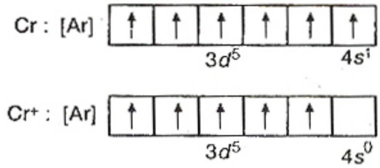
Na^+ $11 - 1 = 10$ electrons

95. 

Resonating (canonical) structures of formate ion

96. In CO_2, C-atom is *sp*-hybridised thus it has linear structure.

In XeF_2, Xe is sp^3d hybridised with three lone-pairs of electrons on equatorial position. This minimises repulsion hence it has also linear structure.



97.

(by first IP)

This is stable EC hence formation of Cr^{2+} by second IP requires maximum enthalpy.

98.

$\Delta n_g = -$ ve	$\Delta H° = -$ ve
Takes place with decrease in number of mole or pressure hence increase in pressure shifts equilibrium in forward side.	Takes place with evolution of heat or increase in temperature hence decrease in temperature shifts this equilibrium in forward side.

99. $2BCl_3 + 3H_2 \longrightarrow 2B + 6HCl$

$\underset{2\text{ mol}}{}\quad \underset{3\text{ mol}}{}\quad \underset{2\text{ mol}}{}$ 21.6 g = 2 mol

$$21.6 \text{ g B} = 2 \text{ mol B} \equiv 3 \text{ mol } H_2$$

$$PV = nRT$$

$$\therefore V = \frac{nRT}{P} = \frac{3 \times 0.0821 \times 273}{1} = 67.2 \text{ L}$$

100. $K_c = \frac{[NO_2]^2}{[N_2O_4]} = \frac{(1.2 \times 10^{-2})^2}{4.8 \times 10^{-2}}$

$$= 3 \times 10^{-3} \text{ mol L}^{-1}$$

101. $AB_2 \rightleftharpoons \underset{S}{A^{2+}} + \underset{2S}{2B^-}$

$$K_{sp} = [A^{2+}][B^-]^2$$
$$= (S)(2S)^2 = 4S^3$$
$$= 4(1 \times 10^{-5})^3$$
$$= 4 \times 10^{-15}$$

102. $Ag^+ + e^- \longrightarrow Ag$

$$9650\text{C} = 0.1\text{F} = 0.1 \text{ equivalent Ag}$$
$$= 0.1 \text{ mol Ag}$$
$$= 10.8 \text{ g Ag}$$

103. $E_{cell} = E^\circ_{cell} - \frac{0.0591}{n} \log Q$

$$\underset{0.1M}{Cu^{2+}} + Zn \longrightarrow \underset{1M}{Zn^{2+}} + Cu$$

$$Q = \frac{[Zn^{2+}]}{[Cu^{2+}]} = \frac{1}{0.1} = 10$$

$$E_{cell} = 1.10 - \frac{0.0591}{2} \log 10$$
$$= 1.10 - 0.0295$$
$$= 1.0705 \text{ V}$$

104. $\Delta T_f = \text{molality} \times K_f \times i$ | $HX \rightleftharpoons H^+ + X^{-1}$

$= 0.2 \times 1.85 \times 1.3$ | $\alpha = 0.3$

$= 0.481°$ | $i = 1 + \alpha = 1.3$

$\therefore \text{fp} = -0.481°\text{C}$

105. Rate becomes x^y times if concentration is made x times of a reactant giving y^{th} order reaction.

$$\text{Rate} = k[A]^n[B]^m$$

Concentration of A is doubled hence $x = 2$, $y = n$ and rate becomes $= 2^n$ times

Concentration of B is halved, hence $x = 1/2$ and $y = m$ and rate becomes $= \left(\frac{1}{2}\right)^m$ times

Net rate becomes $= (2)^n \left(\frac{1}{2}\right)^m$ times

$$= (2)^{n-m} \text{ times}$$

106. Let molarity of $Ba(OH)_2 = M_1$

$\therefore$ Normality $= 2M_1$

Molarity of HCl $= 0.1 \text{ M} = 0.1 \text{ N}$

$$N_1V_1 = N_2V_2$$
$$2M_1 \times 25 = 0.1 \times 35$$
$$M_1 = \mathbf{0.07\ M}$$

107. $\Delta G° = -W_{max.}$ Maximum work done by the system.

108. $CH_2 = CH_2 + H_2 \longrightarrow CH_3 - CH_3$

$$\Delta H = (BE)_{reactants} - (BE)_{products}$$
$$= 4(BE)_{C-H} + (BE)_{C=C} + (BE)_{H-H} - [6(BE)_{C-H} + (BE)_{C-C}]$$
$$= -125 \text{ kJ}$$

109. Enthalpy change is state function and depends only on initial and final condition do not depend on path or intermediates.

110. High pressure inside the cooker increases boiling point, thus heat consistently given is used up by food material instead in boiling and constant boiling evaporation.

111. Ideal solution $\Delta H = 0$

$$\Delta V = 0$$
$$F_{A-A} = F_{B-B} = F_{A-B}$$

112. $\left(\frac{dx}{dt}\right) = k[NO]^2[O_2]$

$$= k\left(\frac{n_{NO}}{V}\right)^2\left(\frac{n_{O_2}}{V}\right)$$

$$\left(\frac{dx}{dt}\right) = \frac{k}{V^3}(n_{NO})^2(n_{O_2})$$

$$\left(\frac{dx}{dt}\right) = \frac{k(n_{NO})^2(n_{O_2})}{\left(\frac{V}{2}\right)^3}$$

$$= 8\left(\frac{dx}{dt}\right)$$

113. $E^\circ_{cell} = \frac{2.303RT}{nF} \log K_{eq}$

$$0.295 = \frac{0.0591}{2} \log K_{eq}$$

$$\therefore \quad \log K_{eq} = 10$$
$$\therefore \quad K_{eq} = 10^{10}$$

114. An irreversible process

$\Rightarrow$ spontaneous process

$\Rightarrow (dS)_{VE}$ (change in entropy) $= +\text{ve} > 0$

$\Rightarrow (dG)_{T,p}$ (change in Gibbs free energy) $-\text{ve} \Rightarrow < 0$

115. As temperature increases desorption increases.

Adsorbent + Adsorbate $\rightleftharpoons$ Adsorbed state + ΔE

Adsorption is exothermic process (forward direction), desorption is endothermic process (Backward direction)

According to Le-Chatelier's principle increase in temperature favours endothermic process.

116. $k = A\, e^{-E_a/RT}$

k = Rate constant
A = Pre-exponential, frequency factor
E_a = Activation energy
R = Gas constant
T = Temperature

117. More – ve value of $E°$ means larger reducing power.

118. Proton affinity it means affinity for proton *i.e.*, basicity

NH_3-Nitrogen has pair of electron to donate as well as higher tendency to donate due to lower electronegativity. P is not suitable as that has larger size.

119. ZnO can react with acid and base both.

$$ZnO + 2HCl \longrightarrow ZnCl_2 + H_2O$$
$$ZnO + 2NaOH \longrightarrow Na_2ZnO_2 + H_2O$$

120. $HgI_2 + 2KI \longrightarrow \underset{\text{soluble}}{K_2HgI_4}$

$$HgI_2 \xrightarrow{\Delta} Hg + I_2$$

121. $2HCl + \frac{1}{2}O_2 \longrightarrow H_2O + Cl_2$

122. In Holmes signals of the ship mixture of CaC_2 and Ca_3P_2 is used.

123. Fe_{26} - [Ar] $3d^6\, 4s^2$

$Fe^{2+}{}_{(24\ \text{electrons})}$ – [Ar] $3d^6\, 4s^0$

124. $2CrO_4^{2-} + 2H^+ \longrightarrow Cr_2O_7^{2-} + H_2O$

125. $K_4[Ni(CN)_4] \longrightarrow 4K^+ + [Ni(CN)_4]^{4-}$

$$x + (4 \times -1) = -4$$
$$x - 4 = -4$$
$$x = 0$$

126. The pair of electron present with nitrogen will not be available to be donated as H^+ will consume that one.

127. $Co(NH_3)_5Cl]\, Cl_2 \rightleftharpoons \underbrace{[Co(NH_3)_5Cl]^{2+} + 2Cl^-}_{\text{three ions}}$

$$\underset{\substack{\text{free}\\ 2\ \text{mol}}}{2Cl^-} + \underset{2\ \text{mol}}{2Ag^+} \longrightarrow \underset{2\ \text{mol}}{2AgCl}$$

128. Due to lanthanide contraction there occurs net decrease in size. Only one 0.85 Å is smaller one.

129. ${}^{234}_{90}Th \xrightarrow{-\beta} \xrightarrow{-\beta} {}^{234}_{92}X \xrightarrow{-\alpha} {}^{230}_{90}Y$

130. $C = C_0 \left(\frac{1}{2}\right)^y$

$$y = \frac{\text{Total time}}{T_{1/2}} = \frac{18}{3} = 6$$
$$C_0 = 256\text{g}$$
$$\therefore \quad C_{(\text{undecayed})} = 256 \left(\frac{1}{2}\right)^6 = \frac{256}{64} = \mathbf{4g}$$

131. Factual

132. $CaSO_4 \cdot \frac{1}{2} H_2O + 1\frac{1}{2} H_2O \xrightarrow{\text{Setting}}$

$$\underset{\substack{\text{Gypsum}\\ \text{(orthorhombic)}}}{CaSO_4 \cdot 2H_2O} \xrightarrow{\text{Hardening}} \underset{\substack{\text{Gypsum}\\ \text{(monoclinic)}}}{CaSO_4 \cdot 2H_2O}$$

133. 1×10^{-8} M HCl solution H_2O is also present there which also undergoes self ionisation

$$H_2O \rightleftharpoons H^+ + OH^-$$
$$10^{-7}\ \text{M at } 25°\text{C}$$

If it is taken simply even without co-... ion effect, higher concentration must be considered which is 10^{-7} M but H^+ from HCl decreases self ionization which further decrease self ionization, hence, $[H^+]$ from H_2O.

Hence, net concentration must be smaller than 10^{-7} M

Hence, acidic

134. NH_3, CH_3NH_2, $(CH_3)_2NH$

$$(CH_3)_2NH + H_2O \overset{K_1}{\rightleftharpoons} (CH_3)_2\overset{+}{N}H_2 + OH^-$$

I^+ effect maximum stabilization more H-bond stability high, hence, most basic

$$CH_3NH_2 + H_2O \overset{K_2}{\rightleftharpoons} CH_3\overset{+}{N}H_3 + OH^-$$
$$NH_3 + H_2O \overset{K_3}{\rightleftharpoons} NH_4^+ + OH^-$$
$$K_1 > K_2 > K_3$$

Hence, basicity order is

$$(CH_3)_2NH > CH_3NH_2 > NH_3$$

135. $\underset{\text{Butene-1}}{CH_2{=}CH{-}CH_2CH_3} \xrightarrow[\Delta,\ \text{pressure}]{Pd/H_2}$

$$\underset{\text{Butane}}{CH_3-CH_2-CH_2-CH_3}$$

other reagents are successful with polar double bonds.

136. Solubilities of carbonates decreases down the group because lattice energy decrease is almost constant while decrease in hydration energy happens sharply. Finally difference of hydration energy and lattice energy decreases thus solubility decreases.

137. Protonation of —OH is first step. Conversion of poor leaving group (—OH) into good leaving group ($-\overset{+}{O}H_2$).

138. Metal oxides lying below Hg in electrochemical series decompose to form metal.

$$ZnO, MgO, CuO \xrightarrow{\Delta} \text{no effect}$$
$$Ag_2O \xrightarrow{\Delta} Ag + O_2$$
$$AgNO_3 \xrightarrow{\Delta} Ag_2O \longrightarrow Ag + O_2$$

139. During thunderstorm, there is formation of NO which changes to NO_2 and ultimately to HNO_3 (acid-rain).

$$N_2 + O_2 \longrightarrow NO \xrightarrow{O_2} NO_2$$
$$\longrightarrow N_2O_5 \xrightarrow{H_2O} HNO_3 \ (pH < 7)$$

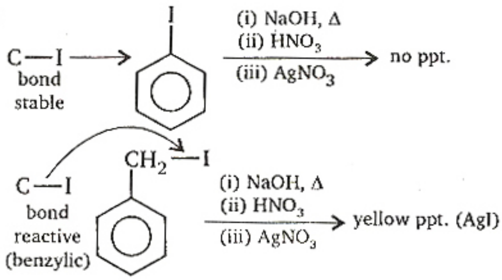
140. Partial hydrolysis of cellulose gives the disaccharide cellobiose ($C_{12}H_{22}O_{11}$). Cellobiose resembles maltose (which on acid catalysed hydrolysis yields two molar equivalents of *D*-Glucose) in every respect except one : the configuration of its glycosidic linkage.

142. Calcined gypsum does not contain $CaCO_3$ (that's sulphate one).

143. Hydrogen bonding is involved molecular force in the DNA molecule.

Watson and Crick observed the purine-pyrimidine type of hydrogen bonding (instead of purine-purine and pyridine-pyridine)

144.

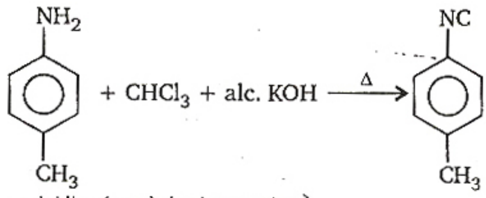


145. $C_2H_5NC + H_2O \xrightarrow{H^+} \underset{\text{Formic acid}}{HCOOH} + C_2H_5NH_2$

$$C_2H_5NH_2 + H^+ \longrightarrow \underset{\text{Salt}}{C_2H_5NH_3^+}$$

146. $\Delta E = 0$, in a cyclic process.

147.



p-toluidine (a carbylamine reaction.)

148. Nylon threads are made up of polyamide some common are

Nylon 6 $H_2N{-}(CH_2)_5{-}\overset{O}{\overset{\|}{C}}{-}OH$ Monomer

$$-HN{-}(CH_2)_5{-}\overset{O}{\overset{\|}{C}}{-}NH{-}(CH_2)_5{-}\overset{O}{\overset{\|}{C}}{-}NH-$$

Nylon 66 $HO{-}\overset{O}{\overset{\|}{C}}{-}(CH_2)_4{-}COOH$

and $H_2N{-}(CH_2)_6{-}NH_2$ Co-polymer

$$-HN{-}\overset{O}{\overset{\|}{C}}{-}(CH_2)_4{-}\overset{O}{\overset{\|}{C}}{-}NH{-}(CH_2)_6{-}$$

$$NH{-}\overset{O}{\overset{\|}{C}}{-}(CH_2)_4{-}\overset{O}{\overset{\|}{C}}-$$

149. In neopentane, all H are equivalents.

$$H_3C{-}\underset{CH_3}{\overset{CH_3}{C}}{-}CH_3 \xrightarrow[Cl_2]{h\nu} H_3C{-}\underset{CH_3}{\overset{CH_3}{C}}{-}CH_2Cl$$

150. Liquid hydrogen and liquid oxygen are good fuel.

Mathematics

1. $f(n) = \begin{cases} \dfrac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\dfrac{n}{2}, & \text{when } n \text{ is even} \end{cases}$

and $f : N \to I$, where N is the set of natural numbers and I is the set of integers.

Let $x, y \in N$ and both are even.

Then $f(x) = f(y)$

$\Rightarrow$ $-\dfrac{x}{2} = -\dfrac{y}{2} \Rightarrow x = y$

Again $x, y \in N$ and both are odd.

Then $f(x) = f(y)$

$\Rightarrow$ $\dfrac{x-1}{2} = \dfrac{y-1}{2} \Rightarrow x = y$

i. e., mapping is one-one.

Since each negative integer is an image of even natural number and positive integer is an image of odd natural number. So mapping is onto. Hence, mapping is one-one onto.

2. Key Idea : *If z_1, z_2 and z_3 are the vertices of an equilateral triangle. Then*

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1 .$$

$\because$ Origin, z_1 and z_2 are the vertices of an equilateral triangle, then

$$z_1^2 + z_2^2 = z_1z_2$$

$$\Rightarrow \quad (z_1 + z_2)^2 = 3z_1z_2 \quad \ldots(i)$$

Again z_1, z_2 are the roots of the equation

$$z^2 + az + b = 0 ,$$

then $\quad z_1 + z_2 = -a$ and $z_1z_2 = b$

Putting these values in Eq. (i), we have

$$(-a)^2 = 3b \quad \Rightarrow a^2 = 3b$$

3. Let $z = r_1\, e^{i\theta}$ and $\omega = r_2\, e^{i\phi}$

$$\Rightarrow \quad \bar{z} = r_1\, e^{-i\theta}$$

Given $\quad |z\omega| = 1 \Rightarrow |r_1e^{i\theta} \cdot r_2e^{i\phi}| = 1$

$$\Rightarrow \quad r_1r_2 = 1 \quad \ldots(i)$$

and $\quad \arg(z) - \arg(\omega) = \dfrac{\pi}{2}$

$$\Rightarrow \quad \theta - \phi = \frac{\pi}{2} \quad \ldots(ii)$$

then $\quad \bar{z}\omega = r_1e^{-i\theta} \cdot r_2e^{i\phi} = r_1r_2\, e^{-i(\theta - \phi)}$

From Eq. (i) and Eq. (ii), we get

$$\bar{z}\omega = 1 \cdot e^{-i\pi/2}$$

$$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$$

$$\bar{z}\omega = -i$$

Note : We know that $r(\cos\theta + i\sin\theta) = r\,e^{i\theta}$

4. $$\left(\frac{1+i}{1-i}\right)^x = \left[\frac{(1+i)(1+i)}{(1-i)(1+i)}\right]^x = \left[\frac{(1+i)^2}{1-i^2}\right]^x = \left[\frac{1-1+2i}{2}\right]^x$$

$$\Rightarrow \quad \left(\frac{1+i}{1-i}\right)^x = (i)^x = 1 \text{ (given)}$$

$$\Rightarrow \quad (i)^x = (i)^{4n},$$

where n is any positive integer.

$$\Rightarrow \quad x = 4n$$

Note : 4 is the least value for which $i^4 = 1$.

5. $$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & b^3 \\ c & c^2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc)\begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = 0 \quad \left[\because \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} \neq 0\right]$$

$$\Rightarrow \quad 1 + abc = 0 \Rightarrow abc = -1$$

Alternative Solution :

The vectors $(1, a, a^2)$, $(1, b, b^2)$, $(1, c, c^2)$ are non coplanar.

$\therefore$ Their scalar triple product $\neq 0$

$$i.e., \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0 \quad \ldots(i)$$

It is given that

$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad abc \neq -1 \text{ (using (i))}$$

6. Key Idea : *The system of linear equations*

$$a_1x + b_1y + c_1z = 0$$

$$a_2x + b_2y + c_2z = 0$$

and $\quad a_3x + b_3y + c_3z = 0$

has a non-zero solution ,

if $$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

The system of linear equations has a non-zero solution, then

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{vmatrix} = 0$$

$$\Rightarrow (3b - 2a)(c - a) - (4c - 2a)(b - a) = 0$$

$$\Rightarrow 3bc - 3ba - 2ac + 2a^2 = 4bc - 2ab - 4ac + 2a^2$$

$$\Rightarrow \quad 4ac - 2ac = 4bc - 2ab - 3bc + 3ab$$

$$\Rightarrow \quad 2ac = bc + ab$$

On dividing by abc, we get

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

i.e., a, b, c are in HP.

7. Key Idea : *If α, β be the roots of the equation $ax^2 + bx + c + 0$, then*

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}.$$

Given equation is

$$ax^2 + bx + c = 0$$

Let α, β are the roots of this equation.

Then $\quad \alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Also $\quad \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$

$$\alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \quad \alpha + \beta = \left(\frac{\alpha + \beta}{\alpha\beta}\right)^2 - \frac{2}{\alpha\beta}$$

$$\Rightarrow \quad \left(-\frac{b}{a}\right) = \left(\frac{-b/a}{c/a}\right)^2 - \frac{2}{c/a}$$

$$\Rightarrow \quad -\frac{b}{a} = \left(\frac{b}{c}\right)^2 - \frac{2a}{c}$$

$$\Rightarrow \quad \frac{2a}{c} = \left(\frac{b}{c}\right)^2 + \frac{b}{a}$$

$$\Rightarrow \quad \frac{2a}{c} = \frac{b}{c}\left[\frac{b}{c} + \frac{c}{a}\right]$$

$$\Rightarrow \quad \frac{2a}{b} = \frac{b}{c} + \frac{c}{a}$$

$\Rightarrow \frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in AP.

$\Rightarrow \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in HP.

8. $x^2 - 3|x| + 2 = 0$

If $x > 0$, then $|x| = x$

therefore $\quad x^2 - 3x + 2 = 0$

$\Rightarrow \quad x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 2) - 1(x - 2) = 0$

$\Rightarrow \quad (x - 1)(x - 2) = 0$

$\Rightarrow \quad x = 1, 2$

If $x < 0$, then $|x| = -x$

therefore $\quad x^2 + 3x + 2 = 0$

$\Rightarrow \quad x^2 + 2x + x + 2 = 0$

$\Rightarrow x(x + 2) + 1(x + 2) = 0$

$\Rightarrow \quad (x + 1)(x + 2) = 0$

$\Rightarrow \quad x = -1, -2$

Hence, four solutions are possible.

9. Key Idea : *If α and 2α are the roots of the equation $ax^2 + bx + c = 0$, then $2b^2 = 9ac$.*

Since one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, then

$$2(3a - 1)^2 = 2 \times 9(a^2 - 5a + 3)$$

$\Rightarrow \quad 9a^2 - 6a + 1 = 9a^2 - 45a + 27$

$\Rightarrow \quad 45a - 6a = 27 - 1$

$\Rightarrow \quad a = \frac{26}{39} = \frac{2}{3}$

Alternate Solution

The given equation is

$$(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$$

Let α and 2α are the roots of this equation, then

$$\alpha + 2\alpha = -\frac{(3a - 1)}{(a^2 - 5a + 3)}$$

$$\Rightarrow \quad 3\alpha = -\frac{(3a - 1)}{(a^2 - 5a + 3)}$$

and $\quad \alpha \cdot 2\alpha = \frac{2}{(a^2 - 5a + 3)}$

$$\Rightarrow \quad 2\alpha^2 = \frac{2}{(a^2 - 5a + 3)}$$

$$\Rightarrow \quad 2\left[\frac{-(3a - 1)}{3(a^2 - 5a + 3)}\right]^2 = \frac{2}{(a^2 - 5a + 3)}$$

$$\Rightarrow \quad \frac{(3a - 1)^2}{9(a^2 - 5a + 3)^2} = \frac{1}{(a^2 - 5a + 3)}$$

$\Rightarrow \quad (3a - 1)^2 = 9(a^2 - 5a + 3)$

$\Rightarrow \quad 9a^2 + 1 - 6a = 9a^2 - 45a + 27$

$\Rightarrow \quad 45a - 6a = 27 - 1$

$\Rightarrow \quad 39a = 26$

$\Rightarrow \quad a = \frac{2}{3}$

10. $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}, A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} a^2 + b^2 & ab + ba \\ ba + ab & b^2 + a^2 \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 2ab \\ 2ab & a^2 + b^2 \end{bmatrix}$$

$\Rightarrow \quad \alpha = a^2 + b^2, \quad \beta = 2ab$

11. The number of choices available to him

$$= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$$

$$= \frac{5!}{4!1!} \times \frac{8!}{6!2!} + \frac{5!}{5!0!} \times \frac{8!}{5!3!}$$

$$= 5 \times \frac{8 \times 7}{2} + 1 \times \frac{8 \times 7 \times 6}{3 \times 2}$$

$$= 5 \times 4 \times 7 + 8 \times 7$$

$$= 140 + 56 = 196$$

12. The number of ways to sit men = 5!

and the number of ways to sit women = ${}^6C_5 \times 5!$

Total number of ways = $5! \times {}^6C_5 \times 5!$

$$= 5! \times 6 \times 5! = 6! \times 5!$$

13. $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$

$= 1\,(\omega^{3n} - 1) - \omega^n(\omega^{2n} - \omega^{2n}) + \omega^{2n}(\omega^n - \omega^{4n})$

$= 1\,(1 - 1) - 0 + \omega^{2n}\,(\omega^n - \omega^n) = 0$

14. Key Idea : ${}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1}$.

${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$

$= {}^nC_{r+1} + {}^nC_r + {}^nC_{r-1} + {}^nC_r$

$= {}^{n+1}C_{r+1} + {}^{n+1}C_r$

$= {}^{n+2}C_{r+1}$

15. $(r+1)$th term of $(\sqrt{3} + \sqrt[8]{5})^{256}$

i.e., $T_{r+1} = {}^{256}C_r\,(3)^{(256-r)/2}\,(5)^{r/8}$

The terms are integral, if $\frac{256 - r}{2}$ and $\frac{r}{8}$ are both positive integer.

$r = 0, 8, 16, 24, 32, \ldots, 256$

Hence total terms are 33.

16. $\because$ $(r+1)$th term in the expansion of $(1+x)^{27/5}$

$$= \frac{\frac{27}{5}\left(\frac{27}{5} - 1\right)\ldots\left(\frac{27}{5} - r + 1\right)}{r!}\,x^r$$

Now this term will be negative, if the last factor in Nr. is the only one negative factor.

$\Rightarrow \quad \frac{27}{5} - r + 1 < 0 \Rightarrow \frac{32}{5} < r$

$\Rightarrow \quad 6.4 < r \Rightarrow$ least value of r is 7.

Thus first negative term will be 8th.

Alternate Solution

$$(1+x)^{27/5} = 1 + \frac{27}{5}x + \frac{27}{5}\left(\frac{27}{5} - 1\right)\frac{x^2}{2!} + \frac{27}{5}\left(\frac{27}{5} - 1\right)\left(\frac{27}{5} - 2\right)\frac{x^3}{3!}$$
$$+ \frac{27}{5}\left(\frac{27}{5} - 1\right)\left(\frac{27}{5} - 2\right)\left(\frac{27}{5} - 3\right)\frac{x^4}{4!}$$
$$+ \frac{27}{5}\left(\frac{27}{5} - 1\right)\left(\frac{27}{5} - 2\right)\left(\frac{27}{5} - 3\right)\left(\frac{27}{5} - 4\right)\frac{x^5}{5!}$$
$$+ \frac{27}{5}\left(\frac{27}{5} - 1\right)\left(\frac{27}{5} - 2\right)\left(\frac{27}{5} - 3\right)\left(\frac{27}{5} - 4\right)\left(\frac{27}{5} - 5\right)\frac{x^6}{6!}$$
$$+ \frac{27}{5}\left(\frac{27}{5} - 1\right)\left(\frac{27}{5} - 2\right)\left(\frac{27}{5} - 3\right)\left(\frac{27}{5} - 4\right)\left(\frac{27}{5} - 5\right) \times \left(\frac{27}{5} - 6\right)\frac{x^7}{7!} + \ldots$$

Here $\frac{27}{5} - 6$ is negative *i.e.*, 8th term is negative in the expansion of $(1+x)^{27/5}$.

17. $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \ldots$

$= \left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) - \ldots$

$= 1 - 2\cdot\frac{1}{2} + 2\cdot\frac{1}{3} - 2\cdot\frac{1}{4} + \ldots$

$= 2\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right] - 1$

$= 2\log(1+1) - 1$

$= 2\log 2 - \log e$

$= \log 4 - \log e$

$= \log\frac{4}{e}$

18. Let $f(x) = Ax^2 + Bx + C$

$\therefore \quad f(1) = A + B + C$

and $\quad f(-1) = A - B + C$

$\because \quad f(1) = f(-1)$

$\Rightarrow \quad A + B + C = A - B + C$

$\Rightarrow \quad 2B = 0$

$\Rightarrow \quad B = 0$

$\therefore \quad f(x) = Ax^2 + C$

Now $f'(x) = 2Ax$

$\therefore \; f'(a) = 2Aa,\; f'(b) = 2Ab,\; f'(c) = 2Ac$

Also a, b, c are in AP.

$\therefore$ $2Aa, 2Ab, 2Ac$ are in AP.

$\Rightarrow f'(a), f'(b), f'(c)$ are in AP.

19. If x_1, x_2, x_3 and y_1, y_2, y_3 are in GP,

then let $\quad x_2 = rx_1, \quad x_3 = r^2x_1$

and $\quad y_2 = ry_1, \quad y_3 = r^2y_1$

with common ratio r, then the points are $(x_1, y_1), (rx_1, ry_1)$ and (r^2x_1, r^2y_1).

Now $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & ry_1 & 1 \\ r^2x_1 & r^2y_1 & 1 \end{vmatrix}$

$= x_1y_1 \begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix}$

$= x_1y_1\,(0) = 0$

(Since two columns are identical)

Thus these points lie on a straight line.

Alternate Solution

Let $x_1 = a \Rightarrow x_2 = ar$ and $x_3 = ar^2$

and $y = b \Rightarrow y_2 = br$ and $y_3 = br^2$

Let the points are $A(a, b), B(ar, br)$ and $C(ar^2, br^2)$.

Now slope of $AB = \frac{b(r-1)}{a(r-1)} = \frac{b}{a}$

and slope of $BC = \frac{b(r^2 - r)}{a(r^2 - r)} = \frac{b}{a}$

$\because \quad$ slope of AB = slope of BC

$\Rightarrow \quad AB \,||\, BC$

But B is a common point.

$\therefore$ A, B and C are collinear.

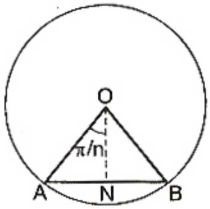
i.e., the points (x_1, y_1), (x_2, y_2) and (x_3, y_3) lie on a straight line.

20. $AB = a$

$ON \perp AB$

and $AN = BN$

In ΔAON,



$$\tan\frac{\pi}{n} = \frac{AN}{ON}$$

$$ON = AN\cot\frac{\pi}{n} = \frac{a}{2}\cot\frac{\pi}{n}$$

and $\sin\frac{\pi}{n} = \frac{AN}{OA}$

$$OA = AN\,\text{cosec}\,\frac{\pi}{n} = \frac{a}{2}\,\text{cosec}\,\frac{\pi}{n}$$

Sum of the radii $= ON + OA$

$$= \frac{a}{2}\cot\frac{\pi}{n} + \frac{a}{2}\,\text{cosec}\,\frac{\pi}{n}$$

$$= \frac{a}{2}\left[\frac{\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}} + \frac{1}{\sin\frac{\pi}{n}}\right]$$

$$= \frac{a}{2}\left[\frac{1+\cos\frac{\pi}{n}}{\sin\frac{\pi}{n}}\right] = \frac{a}{2}\left[\frac{1+2\cos^2\frac{\pi}{2n}-1}{2\sin\frac{\pi}{2n}\cos\frac{\pi}{2n}}\right]$$

$$= \frac{a}{2}\cot\frac{\pi}{2n}$$

21. $a\cos^2\frac{C}{2} + c\cos^2\frac{A}{2} = \frac{3b}{2}$

$$\Rightarrow \quad a\left[\frac{s(s-c)}{ab}\right] + c\left[\frac{s(s-a)}{bc}\right] = \frac{3b}{2}$$

$$\Rightarrow \quad \frac{s[s-c+s-a]}{b} = \frac{3b}{2}$$

$$\Rightarrow \quad 2s[2s-c-a] = 3b^2$$

$$\Rightarrow \quad 2s[a+b+c-c-a] = 3b^2$$

$$\Rightarrow \quad (a+b+c)\,b = 3b^2$$

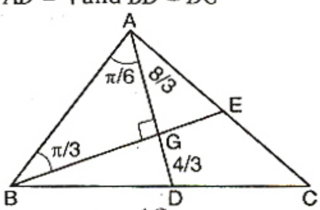
$$\Rightarrow \quad a+b+c = 3b$$

$$\Rightarrow \quad 2b = a+c$$

i.e., a, b, c are in AP.

Note : $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$ and $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$.

22. Given $AD = 4$ and $BD = DC$



In ΔABG, $\tan\frac{\pi}{3} = \frac{AG}{BG}$

$$BG = AG\cot\frac{\pi}{3}$$

$$BG = \frac{8}{3} \times \frac{1}{\sqrt{3}} = \frac{8}{3\sqrt{3}}$$

Area of $\Delta ADB = \frac{1}{2} \times BG \times AD$

$$= \frac{1}{2} \times \frac{8}{3\sqrt{3}} \times 4 = \frac{16}{3\sqrt{3}}$$

Since, median divide a triangle into two triangles of equal area. Therefore

Area of $\Delta ABC = 2 \times$ area of ΔADB

$$= 2 \times \frac{16}{3\sqrt{3}} = \frac{32}{3\sqrt{3}}$$

23. Key Idea : *The range of* $\sin^{-1} x$ *is* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

We have $\quad \sin^{-1} x = 2\sin^{-1} a$

$$\Rightarrow \quad -\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$$

$$\Rightarrow \quad -\frac{\pi}{2} \le 2\sin^{-1} a \le \frac{\pi}{2}$$

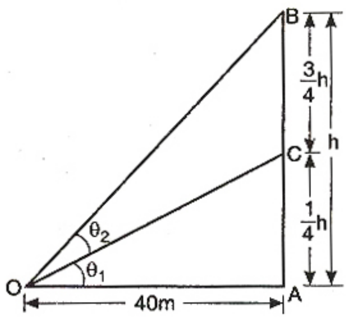
$$\Rightarrow \quad -\frac{\pi}{4} \le \sin^{-1} a \le \frac{\pi}{4}$$

$$\Rightarrow \quad \sin\left(-\frac{\pi}{4}\right) \le a \le \sin\frac{\pi}{4}$$

$$\Rightarrow \quad -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad |a| \le \frac{1}{\sqrt{2}}$$

24. Given $\theta_2 = \tan^{-1}\frac{3}{5}$



$$\tan\theta_2 = \frac{3}{5}$$

In ΔAOC, $\tan\theta_1 = \frac{AC}{OA}$

$$= \frac{\frac{1}{4}h}{40} = \frac{h}{160}$$

In ΔAOB,

$$\tan(\theta_1 + \theta_2) = \frac{AB}{OA} = \frac{h}{40}$$

$$\frac{\tan\theta_1 + \tan\theta_2}{1 - \tan\theta_1\tan\theta_2} = \frac{h}{40}$$

$$\frac{\frac{h}{160}+\frac{3}{5}}{1-\frac{h}{160}\times\frac{3}{5}}=\frac{h}{40}$$

$\Rightarrow \qquad \dfrac{5[h+96]}{800-3h}=\dfrac{h}{40}$

$\Rightarrow \qquad 200[h+96]=800h-3h^2$

$\Rightarrow \qquad 200h+19200=800h-3h^2$

$\Rightarrow \qquad 3h^2-600h+19200=0$

$\Rightarrow \qquad h^2-200h+6400=0$

$\Rightarrow \qquad (h-160)(h-40)=0$

$\Rightarrow \qquad h=160 \text{ or } h=40$

$\therefore$ Height of the vertical pole = 40 m.

25. $f(x)=x+\dfrac{1}{x}$

$$f'(x)=1-\frac{1}{x^2}$$

Put $f'(x)=0$ for maxima and minima

$\Rightarrow \qquad 1-\dfrac{1}{x^2}=0$

$\Rightarrow \qquad x=\pm 1$

$$f''(x)=\frac{2}{x^3}$$

At $x=1$, $f''(x)=+$ ve

$\therefore$ $f(x)$ will be minimum at $x=1$.

Value of $f(x)$ at $x=1$ is 2.

But at $\qquad x=-1$ value of $f(x)=-2$

Thus, $f(x)$ attains minimum value at $x=-1$.

26. $\sum_{r=1}^{n} f(r)=f(1)+f(2)+f(3)+\ldots+f(n)$

$=f(1)+2f(1)+3f(1)+\ldots+nf(1)$

[since $f(x+y)=f(x)+f(y)$]

$=(1+2+3+\ldots+n)\,f(1)$

$=\dfrac{n(n+1)}{2}\cdot 7=\dfrac{7n(n+1)}{2}$

27. $f(x)=x^n \Rightarrow f(1)=1$

$f'(x)=nx^{n-1} \Rightarrow f'(1)=n$

$f''(x)=n(n-1)x^{n-2} \Rightarrow f''(1)=n(n-1)$

....

....

$f^n(x)=n(n-1)(n-2)\ldots 2\cdot 1$

$\Rightarrow \qquad f^n(1)=n(n-1)(n-2)\ldots 2\cdot 1$

We have

$$f(1)-\frac{f'(1)}{1!}+\frac{f''(1)}{2!}-\frac{f'''(1)}{3!}+\ldots+\frac{(-1)^n f^n(1)}{n!}$$

$$=1-\frac{n}{1!}+\frac{n(n-1)}{2!}-\frac{n(n-1)(n-2)}{3!}+\ldots+\frac{(-1)^n n(n-1)(n-2)\ldots 2.1}{n!}$$

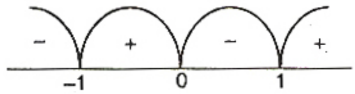
$=(1-1)^n=0$

28. $\because f(x)=\dfrac{3}{4-x^2}+\log_{10}(x^3-x)$

For domain of $f(x)$

$\Rightarrow \qquad x^3-x>0$

$\Rightarrow \qquad x(x-1)(x+1)>0$



Region is $(-1,0)\cup(1,\infty)$

and $\qquad 4-x^2\neq 0$

$\Rightarrow \qquad x\neq \pm 2$

Region is $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$.

$\therefore$ Common region is $(-1,0)\cup(1,2)\cup(2,\infty)$.

29. $\displaystyle\lim_{x\to\frac{\pi}{2}} \frac{\left[1-\tan\frac{x}{2}\right][1-\sin x]}{\left[1+\tan\frac{x}{2}\right][\pi-2x]^3}$

Let $\qquad x=\dfrac{\pi}{2}-h$ as $x\to\dfrac{\pi}{2}$, $h\to 0$

$$\therefore \quad \lim_{h\to 0}\frac{1-\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}{1+\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)}\cdot\frac{(1-\cos h)}{(2h)^3}$$

$$=\lim_{h\to 0}\ \tan\frac{h}{2}\cdot\frac{2\sin^2\frac{h}{2}}{8h^3}$$

$$=\lim_{h\to 0}\frac{1}{4}\cdot\frac{\tan\frac{h}{2}}{\frac{h}{2}\times 2}\times\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{1}{4}$$

$$=\lim_{h\to 0}\frac{1}{32}\cdot\left(\frac{\tan\frac{h}{2}}{\frac{h}{2}}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2=\frac{1}{32}$$

30. $\displaystyle\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$

Applying L' Hospital's rule

$$\Rightarrow \qquad \lim_{x\to 0}\frac{\left[\frac{1}{3+x}+\frac{1}{3-x}\right]}{1}=k$$

$\Rightarrow \qquad \dfrac{1}{3}+\dfrac{1}{3}=k$

$\Rightarrow \qquad k=\dfrac{2}{3}$

Note : L' Hospital's rule can be applied only, if a function is of the form of $\left(\dfrac{0}{0}\right)$ form.

31. $\lim_{x \to a} \frac{f(a)g(x) - f(a) - g(a)f(x) + g(a)}{g(x) - f(x)} = 4$

Applying L' Hospital rule

$$\lim_{x \to a} \frac{f(a)g'(x) - g(a)f'(x)}{g'(x) - f'(x)} = 4$$

$$\Rightarrow \quad \lim_{x \to a} \frac{k\,g'(x) - kf'(x)}{g'(x) - f'(x)} = 4$$

$$\Rightarrow \quad k = 4$$

32. $f(x) = \log(x + \sqrt{x^2 + 1})$

$$\Rightarrow \quad f(-x) = \log(-x + \sqrt{x^2 + 1})$$

$$\Rightarrow \quad f(x) + f(-x) = \log(x + \sqrt{x^2 + 1}) + \log(-x + \sqrt{x^2 + 1})$$

$$= \log(1) = 0$$

Hence $f(x)$ is an odd function.

Note : A function is said to be an even function, if $f(x) = f(-x)$. Otherwise it is called an odd function.

33. Key Idea : *A function is continuous in an interval $[a, b]$, if it is continuous at $x = a$ and $x = b$ and each point of interval (a, b).*

Continuity at $x = 0$

$$f(0 - 0) = \lim_{h \to 0} (0 - h)\, e^{-\left(\frac{1}{|-h|} + \frac{1}{(-h)}\right)}$$

$$= \lim_{h \to 0} (-h)\, e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} = \lim_{h \to 0} (-h) = 0$$

$$f(0 + 0) = \lim_{h \to 0} (0 + h)\, e^{-\left(\frac{1}{|h|} + \frac{1}{h}\right)}$$

$$= \lim_{h \to 0} h e^{-\left(\frac{1}{h} + \frac{1}{h}\right)} = \lim_{h \to 0} \frac{h}{e^{2/h}} = 0$$

$$\Rightarrow \quad f(0 - 0) = f(0) = f(0 + 0)$$

Therefore $f(x)$ is continuous for all x.

Differentiability at $x = 0$

$$Rf'(0) = \lim_{h \to 0} \frac{h e^{-\left(\frac{1}{h} + \frac{1}{h}\right)} - 0}{h - 0}$$

$$= \lim_{h \to 0} \frac{1}{e^{2/h}} = 0$$

$$Lf'(0) = \lim_{h \to 0} \frac{(-h)\, e^{-\left(\frac{1}{h} - \frac{1}{h}\right)} - 0}{(-h) - 0}$$

$$= \lim_{h \to 0} e^0 = 1$$

$$Rf'(0) \neq Lf'(0)$$

$f(x)$ is not differentiable at $x = 0$.

34. $\because f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$$f'(x) = 6x^2 - 18ax + 12a^2$$

Put $f'(x) = 0$

$$6[x^2 - 3ax + 2a^2] = 0$$

$$\Rightarrow \quad x^2 - 3ax + 2a^2 = 0$$

$$\Rightarrow \quad x^2 - 2ax - ax + 2a^2 = 0$$

$$\Rightarrow \quad x(x - 2a) - a(x - 2a) = 0$$

$$\Rightarrow \quad (x - a)(x - 2a) = 0$$

$$\Rightarrow \quad x = a, \quad x = 2a$$

Now $f''(x) = 12x - 18a$

at $x = a$

$$f''(x) = 12a - 18a = -6a$$

$\therefore f(x)$ will be maximum at $x = a$

i.e., $p = a$.

at $x = 2a$

$$f''(x) = 24a - 18a = 6a$$

$\therefore f(x)$ will be minimum at $x = 2a$

i.e., $q = 2a$.

Given $p^2 = q$

$$\Rightarrow \quad a^2 = 2a \Rightarrow a = 2$$

35. $F(t) = \int_0^t f(t - y)\, g(y)\, dy$

$$= \int_0^t e^{t - y} \cdot y\, dy = e^t \cdot \int_0^t e^{-y}\, y\, dy$$

$$= e^t \left[(-ye^{-y})_0^t - \int_0^t 1\,(-e^{-y})\, dy \right]$$

$$= e^t [(-te^{-t} - 0) - (e^{-y})_0^t]$$

$$= e^t [-te^{-t} - (e^{-t} - e^0)]$$

$$= e^t [-te^{-t} - e^{-t} + 1]$$

$$= [1 - e^{-t}(1 + t)]\, e^t$$

$$= e^t - (1 + t)$$

36. Key Idea :

$$\int_a^b x\, f(x)\, dx = \int_a^b (a + b - x)\, f(a + b - x)\, dx$$

Let $\quad I = \int_a^b x\, f(x)\, dx \quad$ ...(i)

$$\Rightarrow \quad I = \int_a^b (a + b - x)\, f(a + b - x)\, dx$$

$$\Rightarrow \quad I = \int_a^b (a + b - x)\, f(x)\, dx$$

[Since given $f(a + b - x) = f(x)$]

$$\Rightarrow \quad I = (a + b)\int_a^b f(x)\, dx - \int_a^b x\, f(x)\, dx$$

$$\Rightarrow \quad I = (a + b)\int_a^b f(x)\, dx - I \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad 2I = (a + b)\int_a^b f(x)\, dx$$

$$\Rightarrow \quad I = \left(\frac{a + b}{2}\right)\int_a^b f(x)\, dx$$

37. $\lim_{x \to 0} \dfrac{\int_0^{x^2} \sec^2 t \, dt}{x \sin x} \left(\dfrac{0}{0} \text{ form}\right)$

Applying L' Hospital's rule

$$= \lim_{x \to 0} \frac{\sec^2 x^2 \cdot 2x}{x \cos x + \sin x}$$

Again applying L'Hospital's rule

$$= \lim_{x \to 0} \frac{2x \cdot 2\sec^2 x^2 \cdot \tan x^2 \cdot 2x + 2\sec^2 x^2}{-x \sin x + \cos x + \cos x}$$

$$= \frac{0 + 2\sec^2 0}{0 + 2\cos 0} = 1$$

38. $I = \int_0^1 x(1-x)^n dx$

Let $1 - x = z \Rightarrow -dx = dz$

$$I = \int_1^0 (1-z) z^n (-dz) = \int_0^1 (1-z) z^n dz$$

$$= \int_0^1 (z^n - z^{n+1}) \, dz = \left[\frac{z^{n+1}}{n+1} - \frac{z^{n+2}}{n+2}\right]_0^1$$

$$= \frac{1}{n+1} - \frac{1}{n+2}$$

39. Key Idea : $\lim_{n \to \infty} \dfrac{1}{n} \sum_{r=1}^{n} \left(\dfrac{r}{n}\right)^4 = \int_0^1 x^4 dx$.

$$\lim_{n \to \infty} \frac{1 + 2^4 + 3^4 + \dots + n^4}{n^5} - \lim_{n \to \infty} \frac{1 + 2^3 + 3^3 + \dots + n^3}{n^5}$$

$$= \lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^4 - \lim_{n \to \infty} \frac{1}{n} \times \lim_{n \to \infty} \frac{1}{n} \sum_{r=1}^{n} \left(\frac{r}{n}\right)^3$$

$$= \int_0^1 x^4 \, dx - \lim_{n \to \infty} \frac{1}{n} \times \int_0^1 x^3 \, dx$$

$$= \left[\frac{x^5}{5}\right]_0^1 - 0 = \frac{1}{5}$$

40. $\dfrac{d}{dx} F(x) = \dfrac{e^{\sin x}}{x}, \quad x > 0$

On integrating both sides, we get

$$F(x) = \int \frac{e^{\sin x}}{x} dx \qquad \dots(i)$$

Also $\int_1^4 \dfrac{3}{x} e^{\sin x^3} \, dx = \int_1^4 \dfrac{3x^2}{x^3} \cdot e^{\sin x^3} dx$

$$= F(k) - F(1)$$

Let $x^3 = z \Rightarrow 3x^2 \, dx = dz$

$$\Rightarrow \int_1^{64} \frac{e^{\sin z}}{z} dz = F(k) - F(1)$$

Using Eq. (i)

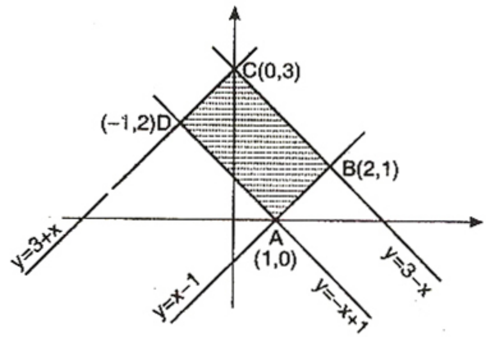
$$[F(z)]_1^{64} = F(k) - F(1)$$

$$\Rightarrow F(64) - F(1) = F(k) - F(1)$$

$$\Rightarrow k = 64$$

41. $y = |x - 1| = \begin{cases} x - 1, & x > 0 \\ -x + 1, & x < 0 \end{cases}$

and $y = 3 - |x| = \begin{cases} 3 + x, & x < 0 \\ 3 - x, & x > 0 \end{cases}$



Solving $y = x - 1$ and $y = 3 - x$

$\Rightarrow \quad x - 1 = 3 - x$

$\Rightarrow \quad 2x = 4$

$\Rightarrow \quad x = 2$

and $\quad y = 3 - 2$

$y = 1$

$AB^2 = (2-1)^2 + (1-0)^2 = 1 + 1 = 2$

$\Rightarrow \quad AB = \sqrt{2}$

and $BC^2 = (0-2)^2 + (3-1)^2 = 4 + 4 = 8$

$BC = 2\sqrt{2}$

Area of rectangle $ABCD = AB \times BC$

$= \sqrt{2} \times 2\sqrt{2}$

$= 4$ sq unit

42. Given $\quad f'(x) = f(x)$ and $f(0) = 1$

Then let $\quad f(x) = e^x$

Also $f(x) + g(x) = x^2$

$g(x) = x^2 - e^x$

Again

$$\int_0^1 f(x) g(x) \, dx = \int_0^1 e^x (x^2 - e^x) \, dx$$

$$= \int_0^1 (x^2 e^x - e^{2x}) \, dx$$

$$= \int_0^1 x^2 e^x \, dx - \int_0^1 e^{2x} dx$$

$$= [x^2 e^x - \int 2x e^x dx]_0^1 - \frac{1}{2}[e^{2x}]_0^1$$

$$= [x^2 e^x - 2(xe^x - e^x)]_0^1 - \frac{1}{2}(e^2 - e^0)$$

$$= [x^2 e^x - 2xe^x + 2e^x]_0^1 - \frac{1}{2}(e^2 - 1)$$

$$= [(x^2 - 2x + 2) e^x]_0^1 - \frac{1}{2} e^2 + \frac{1}{2}$$

$$= [(1-2+2)e^1 - (0-0+2)e^0] - \frac{1}{2}e^2 + \frac{1}{2}$$
$$= [e-2] - \frac{1}{2}e^2 + \frac{1}{2}$$
$$= e - \frac{1}{2}e^2 - \frac{3}{2}$$

43. General equation of parabola whose axis is x-axis, is

$$y^2 = 4a(x+h)$$

On differentiating with respect to x

$$2y\frac{dy}{dx} = 4a \Rightarrow y\frac{dy}{dx} = 2a$$

Again differentiating, we get

$$y\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2} = 0$$

This is a differential equation whose degree and order are 1 and 2 respectively.

44. $(1+y^2) + (x - e^{\tan^{-1}y})\frac{dy}{dx} = 0$

$$\Rightarrow \quad (1+y^2)\frac{dx}{dy} + x = e^{\tan^{-1}y}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{IF} = e^{\int P\,dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

Therefore required solution is

$$x \cdot e^{\tan^{-1}y} = \int e^{\tan^{-1}y} \cdot \frac{e^{\tan^{-1}y}}{1+y^2}dy + k_1$$

$$xe^{\tan^{-1}y} = \int \frac{e^{2\tan^{-1}y}}{1+y^2}dy + k_1$$

$$xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + k_1$$

$$2x\,e^{\tan^{-1}y} = e^{2\tan^{-1}y} + k$$

45. Let $P(\alpha, \beta)$ be the point which is equidistant to $A(a_1, b_1)$ and $B(a_2, b_2)$.

$$\therefore \quad PA = PB$$

$$\Rightarrow (\alpha - a_1)^2 + (\beta - b_1)^2 = (\alpha - a_2)^2 + (\beta - b_2)^2$$

$$\Rightarrow \quad \alpha^2 + a_1^2 + 2a_1\alpha + \beta^2 + b_1^2 - 2\beta b_1 = \alpha^2 + a_2^2 - 2\alpha a_2 + \beta^2 + b_2^2 - 2\beta b_2$$

$$\Rightarrow \quad 2(a_2 - a_1)\alpha + 2(b_2 - b_1)\beta + (a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0$$

$$\Rightarrow \quad (a_2 - a_1)\alpha + (b_2 - b_1)\beta + \frac{1}{2}(a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0$$

Thus the equation of locus is

$$(a_2 - a_1)x + (b_2 - b_1)y + \frac{1}{2}(a_1^2 + b_1^2 - a_2^2 - b_2^2) = 0$$

But the given equation is

$$(a_2 - a_1)x + (b_2 - b_1)y + c = 0$$

$$\therefore \quad c = -\frac{1}{2}(a_1^2 + b_1^2 - a_2^2 - b_2^2)$$
$$= \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$$

Alternate Solution

(a_1, b_1) and (a_2, b_2) satisfy the equation. So that

$$a_1(a_1 - a_2) + b_1(b_1 - b_2) + c = 0 \quad \ldots\text{(i)}$$

and $a_2(a_1 - a_2) + b_2(b_1 - b_2) + c = 0 \quad \ldots$(ii)

On adding Eqs. (i) and (ii), we get

$$(a_1 + a_2)(a_1 - a_2) + (b_1 + b_2)(b_1 - b_2) + 2c = 0$$

$$\Rightarrow \quad 2c = -[a_1^2 - a_2^2 + b_1^2 - b_2^2]$$

$$\Rightarrow \quad c = \frac{1}{2}[a_2^2 + b_2^2 - a_1^2 - b_1^2]$$

46. Co-ordinates of the vertices of a triangle are $(a\cos t, a\sin t)$, $(b\sin t, -b\cos t)$ and $(1, 0)$. centroid is

$$x = \frac{a\cos t + b\sin t + 1}{3}$$

$$\Rightarrow \quad 3x - 1 = a\cos t + b\sin t \quad \ldots\text{(i)}$$

and $$y = \frac{a\sin t - b\cos t + 0}{3}$$

$$\Rightarrow \quad 3y = a\sin t - b\cos t \quad \ldots\text{(ii)}$$

Squaring and adding Eqs. (i) and (ii), we get

$$(3x-1)^2 + (3y)^2 = (a\cos t + b\sin t)^2 + (a\sin t - b\cos t)^2$$

$$(3x-1)^2 + (3y)^2 = a^2 + b^2$$

47. The given equation is $x^2 - 2pxy - y^2 = 0$.

On comparing with

$$ax^2 + 2hxy + by^2 = 0, \text{ we get}$$

$$a = 1, b = -1, 2h = -2p \Rightarrow h = -p$$

Equation of the bisector of angles

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$$\Rightarrow \quad \frac{x^2 - y^2}{1+1} = \frac{xy}{-p}$$

$$\Rightarrow \quad x^2 - y^2 = -\frac{2xy}{p}$$

$$\Rightarrow \quad x^2 + \frac{2xy}{p} - y^2 = 0 \quad \ldots\text{(i)}$$

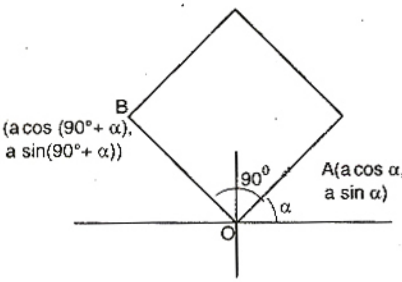
But given equation of the bisector of angles is

$$x^2 - 2qxy - y^2 = 0 \quad \ldots\text{(ii)}$$

On comparing (i) and (ii), we get

$$\frac{2}{p} = -2q \Rightarrow pq = -1$$

48. Line OA makes an angle α with x-axis and $OA = a$, then co-ordinates of A are $(a\cos\alpha, a\sin\alpha)$.



Also $OB \perp OA$, then OB makes an angle $(90° + \alpha)$ with x-axis, then co-ordinates of B are $(a \cos (90° + \alpha), a \sin (90° + \alpha))$

$$= (-a \sin \alpha, a \cos \alpha)$$

Equation of the diagonal not passing through the origin is

$$(y - a \sin \alpha) = \frac{a \cos \alpha - a \sin \alpha}{-a \sin \alpha - a \cos \alpha}(x - a \cos \alpha)$$

$$\Rightarrow (y - a \sin \alpha) = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}(x - a \cos \alpha)$$

$$\Rightarrow (\sin \alpha + \cos \alpha)(y - a \sin \alpha) = (\sin \alpha - \cos \alpha)(x - a \cos \alpha)$$

$$\Rightarrow (\sin \alpha + \cos \alpha) y - a \sin \alpha (\sin \alpha + \cos \alpha) = (\sin \alpha - \cos \alpha) x - a \cos \alpha (\sin \alpha - \cos \alpha)$$

$$\Rightarrow y (\sin \alpha + \cos \alpha) + x (\cos \alpha - \sin \alpha) = a \sin \alpha (\sin \alpha + \cos \alpha) - a \cos \alpha (\sin \alpha - \cos \alpha)$$

$$= a[\sin^2 \alpha + \sin \alpha \cos \alpha - \cos \alpha \sin \alpha + \cos^2 \alpha]$$

$$\therefore y(\sin \alpha + \cos \alpha) + x (\cos \alpha - \sin \alpha) = a$$

49. Key Idea : *Two circles of radii r_1 and r_2 respectively intersect in two distinct points, if $r_1 - r_2 < C_1C_2 < r_1 + r_2$.*

The equation of first circle is

$$(x - 1)^2 + (y - 3)^2 = r^2$$

Centre $C_1(1, 3)$ and radius $r_1 = r$

and equation of second circle is

$$x^2 + y^2 - 8x + 2y + 8 = 0$$

Centre $C_2(4, -1)$ and radius $r_2 = \sqrt{4^2 + 1^2 - 8}$

$$= \sqrt{17 - 8} = 3$$

Two circles intersect in two distinct points, then

$$r_1 - r_2 < C_1C_2 < r_1 + r_2$$

$$\Rightarrow r - 3 < \sqrt{(4-1)^2 + (-1-3)^2} < r + 3$$

$$\Rightarrow r - 3 < \sqrt{9 + 16} < r + 3$$

$$\Rightarrow r - 3 < 5 < r + 3$$

$$\Rightarrow r - 3 < 5 \text{ and } 5 < r + 3$$

$$\Rightarrow r < 8 \quad \text{and } 2 < r$$

$$\Rightarrow 2 < r < 8$$

50. The equations of diameters of a circle are

$$2x - 3y = 5 \quad \text{...(i)}$$

and $$3x - 4y = 7 \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$x = 1, y = -1.$$

$\therefore$ Co-ordinates of centre are $(1, -1)$ and area of the circle $= \pi r^2$

$$\Rightarrow 154 = \frac{22}{7} r^2$$

$$\Rightarrow r = 7$$

Equation of the circle is

$$(x - 1)^2 + (y + 1)^2 = 7^2$$

$$\Rightarrow x^2 - 2x + 1 + y^2 + 2y + 1 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y = 47$$

51. Key Idea : *Equation of normal at a point $(at^2, 2at)$ on a parabola $y^2 = 4ax$ is $y = -tx + 2at + bt^3$.*

Equation of the normal at point $(bt_1^2, 2bt_1)$ on parabola is

$$y = -t_1 x + 2bt_1 + bt_1^3$$

It also passes through $(bt_2^2, 2bt_2)$, then

$$2bt_2 = -t_1 \cdot bt_2^2 + 2bt_1 + bt_1^3$$

$$2t_2 - 2t_1 = -t_1 (t_2^2 - t_1^2)$$

$$= -t_1 (t_2 + t_1)(t_2 - t_1)$$

$$\Rightarrow 2 = -t_1 (t_2 + t_1)$$

$$\Rightarrow t_2 + t_1 = -\frac{2}{t_1}$$

$$\Rightarrow t_2 = -t_1 - \frac{2}{t_1}$$

Note :

1. If a normal drawn at a point t_1 on the parabola meets the parabola again at a point t_2, then
$$t_2 = -t_1 - \frac{2}{t_1}.$$
2. Three normals can be drawn at a point on a parabola.
3. Foot of the normals drawn from a point are called conormal points.

52. Key Idea : *The foci of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $(\pm ae, 0)$ and foci of a hyperbola $\frac{x^2}{a'^2} - \frac{y^2}{b'^2} = 1$ are $(\pm a'e', 0)$.*

Equation of the hyperbola

$$\frac{x^2}{\left(\frac{12}{5}\right)^2} - \frac{y^2}{\left(\frac{9}{5}\right)^2} = 1$$

Eccentricity is given by

$$b'^2 = a'^2 (e'^2 - 1)$$

$$\Rightarrow \quad \frac{81}{25} = \frac{144}{25}(e'^2 - 1)$$

$$\Rightarrow \quad e'^2 = 1 + \frac{9}{16} = \frac{25}{16}$$

$$\Rightarrow \quad e' = \frac{5}{4}$$

Then, foci of a hyperbola are

$$(\pm a'e', 0) = \left(\pm \frac{12}{5} \times \frac{5}{4}, 0\right)$$

$$= (\pm 3, 0)$$

Equation of the ellipse is

$$\frac{x^2}{16} + \frac{y^2}{b^2} = 1$$

Foci of an ellipse are $(\pm ae, 0) = (\pm 4e, 0)$.

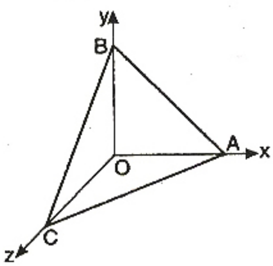
But given focus of ellipse and hyperbola coincide, then

$$4e = 3 \quad \Rightarrow \quad e = \frac{3}{4}$$

Also $b^2 = a^2 (1 - e^2)$

$$= 16\left(1 - \frac{9}{16}\right) = 16 - 9 = 7$$

53. Vector perpendicular to face $OAB = \vec{\mathbf{n}}_1$



$$= \vec{\mathbf{OA}} \times \vec{\mathbf{OB}} = (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= (5\hat{\mathbf{i}} - \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

Vector perpendicular to face $ABC = \vec{\mathbf{n}}_2$

$$= \vec{\mathbf{AB}} \times \vec{\mathbf{AC}} = (\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) \times (-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}})$$

$$= (\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 3\hat{\mathbf{k}})$$

Since angle between faces equals angle between their normals. Therefore

$$\cos\theta = \frac{\vec{\mathbf{n}}_1 \cdot \vec{\mathbf{n}}_2}{|\vec{\mathbf{n}}_1||\vec{\mathbf{n}}_2|}$$

$$= \frac{5 \times 1 + (-1) \times (-5) + (-3) \times (-3)}{\sqrt{5^2 + (-1)^2 + (-3)^2}\sqrt{1^2 + (-5)^2 + (-3)^2}}$$

$$= \frac{5 + 5 + 9}{\sqrt{35}\sqrt{35}} = \frac{19}{35}$$

$$\theta = \cos^{-1}\left(\frac{19}{35}\right)$$

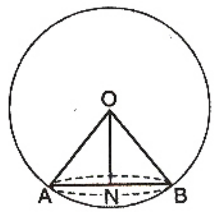
54. Equation of sphere is

$$x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$$

Centre of the sphere is $(-1, 1, 2)$

and radius $= \sqrt{(-1)^2 + (1)^2 + (2)^2 + 19}$

$$= \sqrt{25} = 5$$



Equation of plane is

$$x + 2y + 2z + 7 = 0$$

Length of the perpendicular from centre O on the plane is

$$ON = \frac{-1 \times 1 + 1 \times 2 + 2 \times 2 + 7}{\sqrt{1^2 + 2^2 + 2^2}}$$

$$= \frac{12}{3} = 4$$

In ΔOBN, $\quad OB^2 = ON^2 + NB^2$

$$\Rightarrow \quad 5^2 = 4^2 + NB^2$$

$$\Rightarrow \quad NB^2 = 25 - 16$$

$$\Rightarrow \quad NB^2 = 9$$

$$\Rightarrow \quad NB = 3$$

55. The lines

$$\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k} \qquad \text{...(i)}$$

and $$\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1} \qquad \text{...(ii)}$$

are coplanar, if

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

$$\Rightarrow \quad \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad -1(1 + 2k) - 1(1 + k^2) + 1(2 - k) = 0$$

$$\Rightarrow \quad -2k - 1 - 1 - k^2 + 2 - k = 0$$

$$\Rightarrow \quad -k^2 - 3k = 0$$

$$\Rightarrow \quad k = 0 \text{ or } -3$$

56. Equations of the lines are

$$x = ay + b, \quad z = cy + d$$

$$\Rightarrow \quad \frac{x-b}{a} = y, \quad \frac{z-d}{c} = y$$

$$\Rightarrow \quad \frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c} \quad \ldots(i)$$

and $x = a'y + b', \quad z = c'y + d'$

$$\frac{x-b'}{a'} = y, \quad \frac{z-d'}{c'} = y$$

$$\Rightarrow \quad \frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'} \quad \ldots(ii)$$

Eqs. (i) and (ii) will be perpendicular to each other, if

$$l_1l_2 + m_1m_2 + n_1n_2 = 0$$

$$\Rightarrow \quad aa' + 1 \times 1 + c \times c' = 0$$

$$\Rightarrow \quad aa' + cc' + 1 = 0$$

57. Equations of sphere and plane are

$$x^2 + y^2 + z^2 + 4x - 2y - 6z - 155 = 0$$

and $\quad 12x + 4y + 3z - 327 = 0$

Centre of the sphere is $(-2, 1, 3)$

and radius of sphere

$$= \sqrt{4+1+9+155} = \sqrt{169} = 13$$

Lenght of the perpendicular from centre on the plane

$$= \left|\frac{-2\times12 + 1\times4 + 3\times3 - 327}{\sqrt{144+16+9}}\right|$$

$$= \left|\frac{-11-327}{13}\right|$$

$$= \frac{338}{13} = 26$$

Shortest distance between the plane and sphere

$$= 26 - \text{ radius of sphere}$$

$$= 26 - 13 = 13$$

58. Consider OX, OY, OZ and Ox, Oy, Oz are two system of rectangular axes.

Equation of the plane corresponding to OX, OY, OZ as axes is

$$\frac{X}{a} + \frac{Y}{b} + \frac{Z}{c} = 1 \quad \ldots(i)$$

Similarly, equation of the plane corresponding to Ox, Oy, Oz as axes is

$$\frac{x}{a'} + \frac{y}{b'} + \frac{z}{c'} = 1 \quad \ldots(ii)$$

Length of perpendicular from origin to (i) and (ii) must be same.

i. e., $$\frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} = \frac{1}{\sqrt{\frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}}}$$

$$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{a'^2} + \frac{1}{b'^2} + \frac{1}{c'^2}$$

$$\Rightarrow \quad \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} - \frac{1}{a'^2} - \frac{1}{b'^2} - \frac{1}{c'^2} = 0$$

59. $(\vec{a} + \vec{b} + \vec{c})^2 = (\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c})$

$$\Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$\Rightarrow 0 = 1^2 + 2^2 + 3^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

$$\Rightarrow \quad 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = -14$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -7$$

60. $(\vec{u} + \vec{v} - \vec{w}) \cdot [(\vec{u} - \vec{v}) \times (\vec{v} - \vec{w})]$

$$= (\vec{u} + \vec{v} - \vec{w}) \cdot [\vec{u} \times \vec{v} - \vec{u} \times \vec{w} - \vec{v} \times \vec{v} + \vec{v} \times \vec{w}]$$

$$= \vec{u}\cdot\vec{u}\times\vec{v} - \vec{u}\cdot\vec{u}\times\vec{w} + \vec{u}\cdot\vec{v}\times\vec{w} + \vec{v}\cdot\vec{u}\times\vec{v} - \vec{v}\cdot\vec{u}\times\vec{w} + \vec{v}\cdot\vec{v}\times\vec{w} - \vec{w}\cdot\vec{u}\times\vec{v} + \vec{w}\cdot\vec{u}\times\vec{w} - \vec{w}\cdot\vec{v}\times\vec{w}$$

$$= \vec{u}\cdot\vec{v}\times\vec{w} - \vec{v}\cdot\vec{u}\times\vec{w} - \vec{w}\cdot\vec{u}\times\vec{v}$$

$$= \vec{u}\cdot\vec{v}\times\vec{w} + \vec{w}\cdot\vec{u}\times\vec{v} - \vec{w}\cdot\vec{u}\times\vec{v}$$

$$= \vec{u}\cdot\vec{v}\times\vec{w}$$

Since $[\vec{u}\ \vec{u}\ \vec{v}] = [\vec{u}\ \vec{u}\ \vec{w}]$

$$= [\vec{v}\ \vec{u}\ \vec{v}] = [\vec{v}\ \vec{v}\ \vec{w}]$$

$$= [\vec{w}\ \vec{u}\ \vec{w}] = [\vec{w}\ \vec{v}\ \vec{w}] = 0$$

Note :

If $\vec{u}, \vec{v}, \vec{w}$ are coplanar vectors, then $[\vec{u}\ \vec{v}\ \vec{w}] = 0$

61. Given

$\vec{OA} = 7\hat{i} - 4\hat{j} + 7\hat{k}, \quad \vec{OB} = \hat{i} - 6\hat{j} + 10\hat{k},$

$\vec{OC} = -\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{OD} = 5\hat{i} - \hat{j} + 5\hat{k}$

$$AB = \sqrt{(7-1)^2 + (-4+6)^2 + (7-10)^2}$$

$$= \sqrt{36 + 4 + 9} = \sqrt{49} = 7$$

$$BC = \sqrt{(1+1)^2 + (-6+3)^2 + (10-4)^2}$$

$$= \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

$$CD = \sqrt{(-1-5)^2 + (-3+1)^2 + (4-5)^2}$$

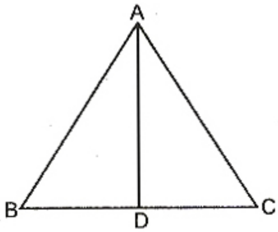
$$= \sqrt{36 + 4 + 1} = \sqrt{41}$$

$$DA = \sqrt{(5-7)^2 + (-1+4)^2 + (5-7)^2}$$

$$= \sqrt{4 + 9 + 4} = \sqrt{17}$$

Thus no option is correct.

62. Let D is mid point of BC.



$$\therefore \vec{\mathbf{AD}} = \frac{\vec{\mathbf{AB}} + \vec{\mathbf{AC}}}{2}$$

$$= \frac{(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}) + (5\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}})}{2}$$

$$= \frac{8\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{2} = 4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$$

$$\therefore \text{Length of } \vec{\mathbf{AD}} = |\vec{\mathbf{AD}}| = \sqrt{4^2 + (-1)^2 + 4^2}$$

$$= \sqrt{16 + 1 + 16} = \sqrt{33}$$

63. Total force

$$\vec{\mathbf{F}} = (4\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}) + (3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= 7\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$$

Displacement

$$\vec{\mathbf{d}} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}) - (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$$

$$= (4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

Total work done $= \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$

$$= (7\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}}) . (4\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$$

$$= 28 + 4 + 8$$

$$= 40 \text{ unit}$$

64. $\vec{\mathbf{u}} = \hat{\mathbf{i}} + \hat{\mathbf{j}}, \vec{\mathbf{v}} = \hat{\mathbf{i}} - \hat{\mathbf{j}}$ and $\vec{\mathbf{w}} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$,

$\hat{\mathbf{n}}$ is a unit vector such that

$$\vec{\mathbf{u}} \cdot \hat{\mathbf{n}} = 0$$

and $$\vec{\mathbf{v}} \cdot \hat{\mathbf{n}} = 0$$

Therefore, $\hat{\mathbf{n}}$ is perpendicular to both $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$

But $\vec{\mathbf{u}}, \vec{\mathbf{v}}$ lie in the xy plane.

$\Rightarrow \hat{\mathbf{n}}$ is a unit vector along z-axis

Hence, $|\vec{\mathbf{w}} \cdot \hat{\mathbf{n}}| = 3$

65. Median of new set remains the same as that of the original set.

66. Given $N = 15$

$$\Sigma x^2 = 2830, \qquad \Sigma x = 170$$

One observation 20 was replaced by 30, then

$$\Sigma x^2 = 2830 - 400 + 900 = 3330$$

$$\Sigma x = 170 - 20 + 30 = 180$$

$$\text{Variance, } \sigma^2 = \frac{\Sigma x^2}{N} - \left(\frac{\Sigma x}{N}\right)^2$$

$$= \frac{3330}{15} - \left(\frac{180}{15}\right)^2$$

$$= \frac{3330 - 15 \times 144}{15}$$

$$= \frac{3330 - 2160}{15} = \frac{1170}{15} = 78.0$$

67. The probability that Mr. A selected the loosing horse $= \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

The probability that Mr. A selected the winning horse $= 1 - \frac{3}{5} = \frac{2}{5}$.

68. $0 \le P(A) \le 1, 0 \le P(B) \le 1, 0 \le P(C) \le 1$

and $0 \le P(A) + P(B) + P(C) \le 1$

$$\therefore \qquad 0 \le \frac{3x+1}{3} \le 1$$

$$\Rightarrow \qquad -\frac{1}{3} \le x \le \frac{2}{3}$$

$$0 \le \frac{1-x}{4} \le 1$$

$$\Rightarrow \qquad -3 \le x \le 1$$

$$\text{and} \qquad 0 \le \frac{1-2x}{2} \le 1$$

$$\Rightarrow \qquad -\frac{1}{2} \le x \le \frac{1}{2}$$

$$\text{and } 0 \le \frac{3x+1}{3} + \frac{1-x}{4} + \frac{1-2x}{2} \le 1$$

$$0 \le 13 - 3x \le 12$$

$$\Rightarrow \qquad \frac{1}{3} \le x \le \frac{13}{3}$$

From all these conditions

$$\frac{1}{3} \le x \le \frac{1}{2}$$

69. Key Idea : *The mean and variance of a random variable in binomial distribution are np and npq and*

$$P(X = r) = {}^nC_r \, q^{n-r} p^r.$$

Since, $np = 4$ and $npq = 2$

$$q = \frac{1}{2}, \text{ but } p + q = 1 \Rightarrow p = \frac{1}{2}$$

$$n \times \frac{1}{2} = 4 \Rightarrow n = 8$$

We have, $P(X = r) = {}^nC_r p^r q^{n-r}$

$$\therefore \quad P(X = 1) = {}^8C_1 \left(\frac{1}{2}\right)^7 \cdot \left(\frac{1}{2}\right)^1$$

$$= 8 \times \frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$$

70. Key Idea : *Resultant force of two forces P and Q acting at an angle α, is given by*

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

If α is the angle between $\vec{\mathbf{P}}$ and $\vec{\mathbf{Q}}$, then

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha \quad \ldots\text{(i)}$$

If $\vec{\mathbf{Q}}$ is doubled, then $\vec{\mathbf{R}}$ is doubled

$$\Rightarrow \quad 4R^2 = P^2 + 4Q^2 + 4PQ\cos\alpha \quad \ldots\text{(ii)}$$

Again if the direction of $\vec{\mathbf{Q}}$ is reversed, then $\vec{\mathbf{R}}$ is again doubled

$$\Rightarrow \quad 4R^2 = P^2 + (-Q)^2 + 2P(-Q)\cos\alpha$$

$$4R^2 = P^2 + Q^2 - 2PQ\cos\alpha \quad \ldots\text{(iii)}$$

Adding (i) and (iii)

$$5R^2 = 2P^2 + 2Q^2 \quad \ldots\text{(iv)}$$

Multiplying by 2 in (iii) and adding (ii)

$$12R^2 = 3P^2 + 6Q^2$$

$$\Rightarrow \quad 4R^2 = P^2 + 2Q^2 \quad \ldots\text{(v)}$$

Subtracting (v) from (iv), we get

$$R^2 = P^2$$

Putting in (v)

$$4R^2 = R^2 + 2Q^2$$

$$\Rightarrow \quad 3R^2 = 2Q^2$$

$$\therefore \quad \frac{P^2}{2} = \frac{Q^2}{3} = \frac{R^2}{2}$$

$$\Rightarrow \quad P^2 : Q^2 : R^2 = 2 : 3 : 2$$

71. Since R_1 and R_2 respectively be the maximum ranges up and down an inclined plane, then

$$R_1 = \frac{u^2}{g(1 + \sin\beta)},$$

$$R_2 = \frac{u^2}{g(1 - \sin\beta)},$$

$$R = \frac{u^2}{g}.$$

Now, $\frac{1}{R_1} + \frac{1}{R_2} = \frac{g(1 + \sin\beta)}{u^2} + \frac{g(1 - \sin\beta)}{u^2}$

$$= \frac{g[1 + \sin\beta + 1 - \sin\beta]}{u^2}$$

$$= \frac{2g}{u^2} = \frac{2}{R}$$

$$\Rightarrow \frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

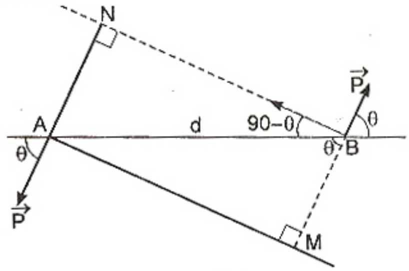
$\Rightarrow \frac{1}{R_1}, \frac{1}{R}, \frac{1}{R_2}$ are in AP

$\Rightarrow R_1, R, R_2$ are in HP.

72. Let $AB = d$.

AM is the perpendicular distance from A to B.

In ΔABM,



$$\sin\theta = \frac{AM}{AB}$$

$$AM = d\sin\theta$$

Moment of couple,

$$\vec{\mathbf{G}} = \vec{\mathbf{P}} \cdot AM = \vec{\mathbf{P}}\ d\sin\theta \quad \ldots\text{(i)}$$

Where $\vec{\mathbf{P}}$ is turned a right angle, then Moment of couple,

$$\vec{\mathbf{H}} = \vec{\mathbf{P}} \cdot d\sin(90° + \theta) = \vec{\mathbf{P}}\ d\cos\theta \quad \ldots\text{(ii)}$$

When $\vec{\mathbf{P}}$ is turned through an angle α, then Moment of couple

$$= \vec{\mathbf{P}} \cdot d\sin(\alpha + \theta)$$

$$= \vec{\mathbf{P}} \cdot d[\sin\alpha\cos\theta + \cos\alpha\sin\theta]$$

$$= (\vec{\mathbf{P}} \cdot d\cos\theta)\sin\alpha + (\vec{\mathbf{P}} \cdot d\sin\theta)\cos\alpha$$

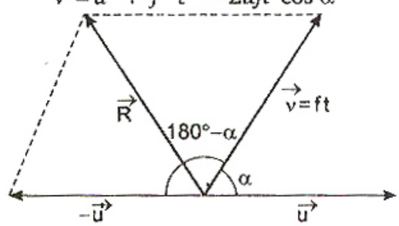
$$= \vec{\mathbf{G}}\cos\alpha + \vec{\mathbf{H}}\sin\alpha \quad \text{[Using (i) and (ii)]}$$

73. We have

$$R^2 = u^2 + f^2t^2 + 2uft\cos(180° - \alpha)$$

$$R^2 = u^2 + f^2t^2 - 2uft\cos\alpha$$

Let $\quad V = u^2 + f^2t^2 - 2uft\cos\alpha$



$$\frac{dV}{dt} = 0 + 2f^2t - 2uf \cos\alpha$$

$$\frac{d^2V}{dt^2} = 2f^2 = + \text{ve}$$

i.e., velocity will be least after a time.

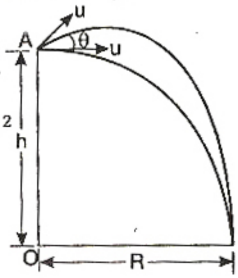
$$\frac{dV}{dt} = 0 = 2f^2t - 2uf \cos\alpha$$

$$\Rightarrow \quad 2f^2t = 2uf \cos\alpha$$

$$\Rightarrow \quad t = \frac{2uf \cos\alpha}{2f^2}$$

$$\Rightarrow \quad t = \frac{u \cos\alpha}{f}$$

74. When stone is projected horizontally, then



$$R = u \cos 0 \times t$$

and $h = u \sin 0 \times t + \frac{1}{2}gt^2$

$\Rightarrow \quad R = ut \quad$ and $\quad h = \frac{1}{2}gt^2$

$\Rightarrow \quad R = ut \quad$ and $t = \sqrt{\frac{2h}{g}}$

$$\Rightarrow \quad R = u\sqrt{\frac{2h}{g}} \qquad \ldots\text{(i)}$$

When stone is projected at an angle of θ to the horizontal, then

$$R = u \cos\theta \times t \qquad \ldots\text{(ii)}$$

and $\quad h = -u \sin\theta \times t + \frac{1}{2}gt^2 \qquad \ldots$(iii)

Using Eq. (i) in Eq. (ii), we get

$$u\sqrt{\frac{2h}{g}} = u\cos\theta \times t$$

$$\Rightarrow \quad t = \frac{1}{\cos\theta}\sqrt{\frac{2h}{g}} \qquad \ldots\text{(iv)}$$

Using Eq. (iv) in Eq. (iii), we get

$$h = \frac{-u\sin\theta}{\cos\theta}\sqrt{\frac{2h}{g}} + \frac{1}{2}g\frac{1}{\cos^2\theta}\left(\frac{2h}{g}\right)$$

$$\Rightarrow \quad h = -u\tan\theta\sqrt{\frac{2h}{g}} + \frac{h}{\cos^2\theta}$$

$$\Rightarrow \quad h\left[1 - \frac{1}{\cos^2\theta}\right] = -u\tan\theta\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad h(-\tan^2\theta) = -u\tan\theta\sqrt{\frac{2h}{g}}$$

$$\Rightarrow \quad \tan\theta = u\sqrt{\frac{2}{gh}}$$

75. Here $x_1 + x_2 = s$

and $\quad t_1 + t_2 = t$

We have from A to C

$$v^2 = u^2 + 2fx_1$$

$$v^2 = 0 + 2fx_1$$

$$\Rightarrow \quad x_1 = \frac{v^2}{2f} \qquad \ldots\text{(i)}$$

and $\quad v = u + ft$

$$\Rightarrow \quad v = 0 + ft_1$$

$$\Rightarrow \quad t_1 = \frac{v}{f} \qquad \ldots\text{(ii)}$$

From C to B, $\quad v^2 = u^2 + 2fs$

$$v'^2 = v^2 - 2rx_2$$

$$0 = v^2 - 2rx_2$$

$$\Rightarrow \quad x_2 = \frac{v^2}{2r} \qquad \ldots\text{(iii)}$$

and $v' = v - rt_2$

$$\Rightarrow \quad 0 = v - rt_2$$

$$\Rightarrow \quad t_2 = \frac{v}{r} \qquad \ldots\text{(iv)}$$

On adding Eqs. (i) and (iii)

$$x_1 + x_2 = \frac{v^2}{2}\left(\frac{1}{f} + \frac{1}{r}\right)$$

$$2s = v^2\left(\frac{1}{f} + \frac{1}{r}\right) \qquad \ldots\text{(v)}$$

On adding Eqs. (ii) and (iv)

$$\Rightarrow \quad t_1 + t_2 = v\left(\frac{1}{f} + \frac{1}{r}\right)$$

$$\Rightarrow \quad t = v\left(\frac{1}{f} + \frac{1}{r}\right)$$

$$\Rightarrow \quad t^2 = v^2\left(\frac{1}{f} + \frac{1}{r}\right)^2 \qquad \ldots\text{(vi)}$$

On dividing Eqs. (vi) by (v)

$$\frac{t^2}{2s} = \frac{v^2\left(\frac{1}{f} + \frac{1}{r}\right)^2}{v^2\left(\frac{1}{f} + \frac{1}{r}\right)}$$

$$= \left(\frac{1}{f} + \frac{1}{r}\right)$$

$$\Rightarrow \quad t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{r}\right)}$$