Mark Scheme (Results)

Summer 2012

AEA Mathematics (9801)

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- The second mark is dependent on gaining the first

Qu	Scheme	Mark	Notes
1. (a)	$x^2-2x+6=(x-1)^2+5$ <u>or</u> $2x-2=0$	M1	Differentiating or complete the square
	Sketch or work to show min at (1, 5)	A1	
	Range **$f \geq 5$** (Accept $y \geq 5$) (Answer only 3/3)	A1 **(3)**	$x \geq 5$ can score M1A1A0
(b)	gf(x) = $3+\sqrt{x^2-2x+6+4},=3+\sqrt{x^2-2x+10}$	M1,A1 **(2)**	
(c)	$\text{gf}(1)$ or $3+\sqrt{"5"+4}$	M1	Clear attempt to find gf(1) or correct express'
	Range of **gf$\geq$ 6**	A1	
	Domain = domain of f = **$x \geq 0$**	B1 **(3)**	
		[8]	

Qu	Scheme	Mark	Notes
2. (a)	sin(2x + x) = sin2$x$cos$x$ + cos2$x$sin$x$	M1	Use of sin (A+B)
	$= 2\sin x\cos^2 x+\left(\sin x-2\sin^3 x\right)$	M1	Use of sin2x and cos2x
	$=2\sin x-2\sin^3 x+\sin x-2\sin^3 x=3\sin x-4\sin^3 x$	A1cso	
Use of i	$\sin 3x=3\cos^2 x\sin x,-\sin^3 x$ for M1, M1	**(3)**	
(b)	$6\sin x-2\sin 3x=6\sin x-2\left(3\sin x-4\sin^3 x\right)=[8\sin^3 x]$	M1	Attempt to use (a)
	$I=\int \cos x 4\sin^2 x\ \mathrm{d}x$	A1	For $4\sin^2 x\cos x$ only
	$=\dfrac{4\sin^3 x}{3}$ (+c) (o.e.) e.g. $\frac{2}{3}\sin 2x\cos x-\frac{4}{3}\sin x\cos 2x$ (+c)	A1 **(3)**	
(c)	$\int\left(3\sin 2x-2\sin 3x\cos x\right)^{\frac{1}{3}}\mathrm{d}x=\int\left(6\sin x\cos x-2\sin 3x\cos x\right)^{\frac{1}{3}}\ \mathrm{d}x$	M1	Use of sin2x
	$=\int\cos^{\frac{1}{3}} x 2\sin x\ \mathrm{d}x$ <u>or</u> $\int\left(8\cos x\sin^3 x\right)^{\frac{1}{3}}\ \mathrm{d}x$	A1	Use of (a) to simplify integrand
	$=-\dfrac{3}{2}\cos^{\frac{4}{3}} x$ (+c)	M1	Attempt int. $\rightarrow k\cos^{\frac{4}{3}} x$
		A1 **(4)**	
		[10]	

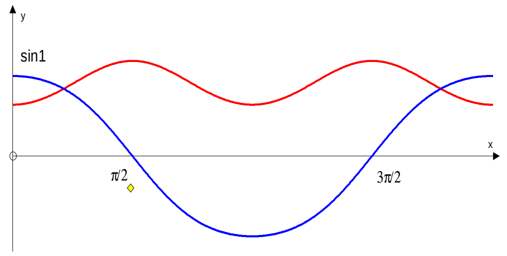
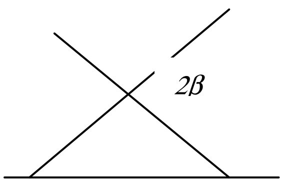
Qu	Scheme	Mark	Notes
3. (a)	RHS = GP a = 2, r = cos2θ $S_\infty =,\dfrac{2}{1-\cos 2\theta}$	M1,A1	Identify GP and attempt sum to ∞ for M1
	$\cos 2\theta=1-2\sin^2\theta \Rightarrow (\text{RHS})=\text{cosec}^2\theta$ (Allow $\frac{k}{\sin^2\theta}$)	M1	Use cos2θ to simplify
	$\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta} \Rightarrow$ (LHS) $=\dfrac{2\tan^2\theta}{1-\tan^2\theta}$	M1	Use of tan2θ on LHS
	Equating: $\dfrac{2\tan^2\theta}{1-\tan^2\theta}=1+\cot^2\theta=\dfrac{1+\tan^2\theta}{\tan^2\theta}$	M1	Equate LHS=RHS and use formula to get eqn in tanθ or single trig func.
	so $3\tan^4\theta-1=0$	A1	Correct eqn (either line)
	$\tan^4\theta=\frac{1}{3} \Rightarrow \tan\theta=\left(\frac{1}{3}\right)^{\frac{1}{4}}$	dM1	Solve their equn leading to tanθ=…Dep on 4th M
	$\tan\theta=3^{-\frac{1}{4}}$ or $p=-\dfrac{1}{4}$	A1 (8)	
(b)	$1>3^{-\frac{1}{4}}>3^{-\frac{1}{2}} \Rightarrow \tan\frac{\pi}{4}>\tan\theta>\tan\frac{\pi}{6}$	M1	
	$\Rightarrow \frac{\pi}{4}>\theta>\frac{\pi}{6}$	A1 (2)	cso
		[10]	

Qu	Scheme	Mark	Notes
4. (a)	$\overrightarrow{AB} = \begin{pmatrix} 8 \\ -3 \\ 5 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 3 \\ -2 \\ -6 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 11 \\ -5 \\ -1 \end{pmatrix}$ $\boxed{\text{Only 3 vertex-vertex distances in a cube}}$	M1	Attempt all of these three vectors or two and show perpendicular
	$\lvert AB\rvert = \sqrt{98}, \lvert BC\rvert = \sqrt{49}, \lvert AC\rvert = \sqrt{147}$	M1A1	For S+ M1 for attempting one A1 for all 2 or 3 correct
	BC is shortest so must be side length	M1	Select shortest
	Volume = $7^3 = 343$	A1 **(5)**	Requires all M marks
(b)	$\overrightarrow{PQ} \bullet \overrightarrow{PR} = 21 + 4 + 0 = 25$	M1	Attempt scalar product
	$\cos(QPR) = \dfrac{25}{\sqrt{50}\sqrt{25+\alpha^2}} = \dfrac{1}{2}$	M1	Use of cos 60 and scalar product formula to get an equation for α
	$\alpha = 5$ (Allow $\pm$ 5)	A1 **(3)**	
(c)	For 60^o angle, $PQ=PR = \sqrt{50}$ must be a diagonal of a face	M1	Recognize PQ or PR is face diagonal. OK on fig.
	Therefore side must be 5 (since face diagonal is side$\times\sqrt{2}$)	A1	
	Diagonal is therefore $\underline{5\sqrt{3}}$	A1**(3)**	
		[11]	

Qu	Scheme	Mark	Notes
5. (a)	$\log_a x^n = (\log_a x)^n \Rightarrow n\log_a x = (\log_a x)^n$	M1	Use of the power rule to form an equation
	$n = (\log_a x)^{n-1} \Rightarrow \log_a x = n^{\frac{1}{n-1}}$	M1	Attempt root to get an expression for log
	$x = a^{n^{\frac{1}{n-1}}}$ (o.e.)	A1 **(3)**	
(b) (i)	$(\log_a x)^3 + (\log_a x)^2 - 5\log_a x = 0$ or $(\log_a x)^3 - 6\log_a x + 5 = 0$	M1	Use n = 3 to get either
	Let $u = \log_a x$ and solve $u^2 + u - 5 = 0 \rightarrow u = \dfrac{-1 \pm \sqrt{21}}{2}$	M1	Attempt to solve relevant quadratic.
	$x_1 = a^{\frac{-1+\sqrt{21}}{2}}, x_2 = a^{\frac{-1-\sqrt{21}}{2}}$	A1	
(b)(ii)	$\log_a\left(\dfrac{x_1}{x_2}\right) = \log_a x_1 - \log_a x_2 = \dfrac{-1+\sqrt{21}}{2} - \dfrac{-1-\sqrt{21}}{2}$	M1	Use log$x$ - log$y$ rule and attempt to sub values for x
	$= \sqrt{21}$	A1 **(5)**	
(c)	LHS = $\log_a x(1+2+...+n)$	M1	Attempt to use power rule on all of LHS
	$= \log_a x\left(\dfrac{n(n+1)}{2}\right)$	A1	
	RHS = $\dfrac{\log_a x\left[(\log_a x)^n - 1\right]}{\log_a x - 1}$	M1 A1	Identify and attempt sum of GP
	Equate: $\cancel{\log_a x}\left(\dfrac{n(n+1)}{2}\right) = \dfrac{\cancel{\log_a x}\left[(\log_a x)^n - 1\right]}{\log_a x - 1}$	dM1	Equate and attempt to simplify to given answer. Dep on bothMs
	$\log_a x[n(n+1)] - (n^2+n) = 2(\log_a x)^n - 2$ leading to answer	A1 **(6)**	cso
		[14]	

Qu	Scheme	Mark	Notes
6. (a)	$P\ (-a,\ 0)\qquad Q\ (b,0)$	B1B1 **(2)**	Allow B1B0 for (0, -*a*) etc
(b)	$I=\int (x+a)\ \mathrm{d}\left[\frac{(x-b)^3}{3}\right],=\left[(x+a)\frac{(x-b)^3}{3}\right]_{-a}^{b}-\int\frac{(x-b)^3}{3}\ \mathrm{d}x$	M1, A1-A1	M1 for correct attempt by parts
	$=\ 0,-\left[\frac{(x-b)^4}{12}\right]_{-a}^{b}=\left(0\right)--\frac{\left(-a-b\right)^4}{12}=\frac{(a+b)^4}{12}$	B1, M1 A1cso **(6)**	M1 for second stage integration
(c)	$y'=(x-b)^2+(x+a)2(x-b)$	M1	Some correct diff'n
	$y'=0\Rightarrow 0=(x-b)[x-b+2x+2a]$	M1	Attempt to solve *y'*= 0
	$x=\frac{b-2a}{3}$	A1	
	y co-ord of *S* is: $y_S=\frac{(a+b)}{3}\left(\frac{-2a-2b}{3}\right)^2=\frac{4}{27}\left(a+b\right)^3$	dM1	Sub to get *y* co-ord of *S* Dep on 2nd M1
	Area of *PQRST* = $y_S\times(a+b),=\frac{4}{27}\left(a+b\right)^4$	dM1A1	M1 using correct formula Dep on 3rd M1
	Ratio = $\frac{\frac{(a+b)^4}{12}}{\frac{4}{27}\left(a+b\right)^4}$, $=\underline{\frac{27}{48}=\frac{9}{16}}$	dM1,A1 **(8)**	M1 dep on 2nd and 3rd M1. Must eliminate $(a+b)^4$
		[16]	
ALT (b)	<u>Expand</u> $I=\int\left(x^3+ax^2-2bx^2-2abx+b^2x+ab^2\right)\ \mathrm{d}x$	M1A1	M1 for 6 terms (3 corr) A1 for all correct
	$=\left(\frac{b^4}{12}+\frac{4ab^3}{12}\right)-\left(-\frac{a^4}{12}-\frac{4a^3b}{12}-\frac{6a^2b^2}{12}\right)\rightarrow$ answer	M1B1 A1 A1cso	M1 some integration B1 some use of *b* & -*a* A1 one bracket correct

Awarding of S and T marks		
Questions	**Marks**	
2, 3, 4	S1	For a fully correct solution that is succinct or includes an S+ point
5, 6, 7	S2	For a fully correct solution that is succinct and includes some S+ points
5, 6, 7	S1	For a fully correct solution that is succinct but does not mention any S+ points
5, 6, 7	S1	For a fully correct solution that is slightly laboured but includes an S+ point
5, 6, 7	S1	For a score of *n* -1 but solution is otherwise succinct or contains an S+ point
6	S1	For a score of *n* – 2 but solution is otherwise succinct and includes an S+ point
Maximum S score is 6		
ALL	T1	For at least half marks on all questions

Qu	Scheme	Mark	Notes
7.(a)	Max of cos$u$ is 1 when u = 0, u = cos$x$ = 0 when $x=\frac{\pi}{2}$ or $\frac{3\pi}{2}$	M1	Method to get at least one of these values Implied by correct P or R
	$P(\frac{\pi}{2},1)$ $R(\frac{3\pi}{2},1)$ [Require 1 not cos(0)]	A1A1	**Condone degrees in any part**
	cos(-1) = cos(1) so $Q(\pi,\cos 1)$ [Accept cos(-1)]	B1 **(4)**	
(b)		B1	Shape (one –ve min)
	Accept points NOT marked on graph	B1	sin1 seen at ends and cos1< sin1 < 1
		B1,B1	$\frac{\pi}{2}$, $\frac{3\pi}{2}$
		B1 **(5)**	(π, sin(-1))
(c)	$\cos(\cos x)=\sin(\cos x)\Rightarrow 1=\tan(\cos x)$	M1	Use of sin/cos= tan
	$\cos x=\frac{\pi}{4}$ (or $\frac{5\pi}{4}$) so $x=\alpha=\arccos\left(\frac{\pi}{4}\right)$	A1cso **(2)**	Allow verify but needs a comment " so $\alpha=$ …"
(d)	d = cos(cosα) = $\cos\left(\frac{\pi}{4}\right)$	M1	
	$S\left(\arccos\left(\frac{\pi}{4}\right),\frac{1}{\sqrt{2}}\right)$ Accept $d=\frac{1}{\sqrt{2}}$ (o.e.)	A1	
	$T\left(2\pi-\arccos\left(\frac{\pi}{4}\right),\frac{1}{\sqrt{2}}\right)$	B1ft **(3)**	ft their y co-ord of S
(e)	$y'=\sin(\cos x)\sin x$	M1A1	M1 for attempt at chain rule
	$m=\sin\left(\frac{\pi}{4}\right)\sin\alpha$	M1	Substitution attempt
	$m=\frac{1}{\sqrt{2}}\times\frac{\sqrt{16-\pi^2}}{4}$	M1	Attempt sinα in π
	$m=\sqrt{\frac{16-\pi^2}{32}}$ so $\beta=\arctan\left(\sqrt{\frac{16-\pi^2}{32}}\right)$	A1cso **(5)**	
(f)	For C_2 : $y'=-\cos(\cos x)\sin x$	M1	Attempt y'
	$m'=-\cos\left(\frac{\pi}{4}\right)\sin\alpha,\ =-\tan\beta$ (o.e.) e.g. $-\sqrt{\frac{16-\pi^2}{32}}$	M1A1	M1 for sub of α
		M1	Attempt to find angle between two tangents to get 2β or π - 2β
	Obtuse angle is $\pi-2\beta$	A1 **(5)**	Allow 180 – 2β
	[$\tan\beta=\sqrt{\frac{16-\pi^2}{32}}<1\Rightarrow\beta<\frac{\pi}{4}$ so 2β is acute for S+]	**[24]**	

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