

(C1-7.1) Name:

### **Homework Questions 1 – Calculating an Estimate of a Gradient**

1. A is the point with coordinates (3,1) on the curve  $Y = (X - 2)^2$

Find the gradients of the chords joining A to:

a) Point B (4,4)

b) Point C (3.5, 2.25)

c) Point D (3.3, 1.69)

d) Point E (3.1, 1.21)

e) Point F (3.01, 1.0201)

f) Point G (3.001, 1.002001)

2. What do you deduce about the gradient of the tangent at the point (3,1) and explain why?

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## **Homework Questions 2 – Finding the Derived Function**

Differentiate the following, leave your answer in index form

1.  $f(x) = x^2$

$$f'(x) = 2x$$

2.  $f(x) = x^{-2}$

$$f'(x) = -2x^{-3}$$

3.  $f(x) = x^{-6}$

$$f'(x) = -6x^{-7}$$

4.  $f(x) = x^{\frac{1}{4}}$

$$f'(x) = \frac{1}{4}x^{-\frac{3}{4}}$$

5.  $f(x) = \sqrt[3]{x}$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$

6.  $f(x) = \sqrt[6]{x}$

$$f'(x) = \frac{1}{6}x^{-\frac{5}{6}}$$

7.  $f(x) = \frac{1}{x^4}$

$$f'(x) = -4x^{-5}$$

8.  $f(x) = \frac{1}{x^5}$

$$f'(x) = -5x^{-6}$$

9.  $f(x) = \frac{1}{\sqrt[4]{x}}$

$$f'(x) = -\frac{1}{4}x^{-\frac{5}{4}}$$

10.  $f(x) = \frac{1}{\sqrt[6]{x}}$

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$$f(x) = -\frac{1}{6}x^{-\frac{7}{6}}$$

11.  $f(x) = \frac{x^5}{x^2}$

$$f(x) = 3x^2$$

12.  $f(x) = \frac{x^7}{x^3}$

$$f(x) = 4x^3$$

13.  $f(x) = \frac{x^2}{x^5}$

$$f(x) = -3x^{-4}$$

14.  $f(x) = \frac{x}{x^3}$

$$f(x) = -2x^{-3}$$

15.  $f(x) = x^2 \times x^5$

$$f(x) = 7x^6$$

16.  $f(x) = x^4 \times x^5$

$$f(x) = 9x^8$$

17.  $f(x) = 5x^4$

$$f(x) = 20x^3$$

18.  $f(x) = 2x^3$

$$f(x) = 6x^2$$

19.  $f(x) = \frac{3}{x^2}$

$$f(x) = -6x^{-3}$$

20.  $f(x) = \frac{5}{\sqrt[3]{x^2}}$

$$f(x) = -\frac{10}{3}x^{-\frac{5}{3}}$$

(C1-7.3a) Name:

### Homework Questions 3 – Gradient Function

$\frac{dy}{dx}$

1. Find  $\frac{dy}{dx}$  when y equals:

a)  $y = 7x + 2$

$$\frac{dy}{dx} = 7$$

b)  $y = 5x^2 + 6x$

$$\frac{dy}{dx} = 10x + 6$$

c)  $y = 8x^2 + 2x + 3$

$$\frac{dy}{dx} = 16x + 2$$

d)  $y = 7x^2 + x + 2$

$$\frac{dy}{dx} = 14x + 1$$

e)  $y = 10x^2 + 3x + 4$

$$\frac{dy}{dx} = 20x + 3$$

f)  $y = \frac{x^2}{2} + 6x - 1$

$$\frac{dy}{dx} = x + 6$$

g)  $y = 6x^2 + \frac{x}{2} - 2$

$$\frac{dy}{dx} = 12x + 0.5$$

h)  $y = 8x^2 + 4x - 3$

$$\frac{dy}{dx} = 16x + 4$$

i)  $y = 2x^2 + 3x - 1$

$$\frac{dy}{dx} = 4x + 3$$

j)  $y = 6x^2 + 2x + 4$

$$\frac{dy}{dx} = 12x + 2$$

2. Find the gradient of the following curves at the given points

a)  $y = 4x^2$  at the point (2,16)

16

b)  $y = 2x^2 - 3x + 1$  at the point (1,0)

1

c)  $y = 5 - x^2 - 4x$  at the point (-1,8)

-2

d)  $y = \frac{x^2}{2} + 5x - 6$   
at the point (2,6)

7

e)  $y = \frac{2x^2}{3} + 4x - 10$   
at the point (3,8)

8

3. The curve  $y = 2x^2 + 4x - 12$  meets the line  $y=4$  at the points A and B

a) Find the coordinates of A and B

(2,4) (-4,4)

b) Find the gradient of the curve at the point A

-12

c) Find the gradient of the curve at the point B

12

d) Find the gradient of the line joining A to B

0

(C1-7.4a) Name:

### Homework Questions 4 – Using Standard Results to Differentiate

1. Use standard results to differentiate the following

a)  $y = x^3 + 2x^2$

$$\frac{dy}{dx} = 3x^2 + 4x$$

b)  $y = \frac{x^{-3}}{2}$

$$\frac{dy}{dx} = -\frac{3}{2}x^{-4}$$

c)  $y = 3x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = -\frac{3}{2}x^{-\frac{3}{2}}$$

d)  $y = x^{-\frac{1}{2}} + 2x^2$

$$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}} + 4x$$

e)  $y = 5x^2 + 3x^{-\frac{1}{3}} + 2$

$$\frac{dy}{dx} = 10x - x^{-\frac{4}{3}}$$

f)  $y = 3x^2 - 2x^{-1} + 5$

$$\frac{dy}{dx} = 6x^2 + 2x^{-2}$$

g)  $y = 6x^2 + \frac{x}{2} - 2$

$$\frac{dy}{dx} = 12x + 0.5$$

h)  $y = 8x^2 + 4x - 3$

$$\frac{dy}{dx} = 16x + 4$$

i)  $y = 2x^2 + 3x - 1$

$$\frac{dy}{dx} = 4x + 3$$

J)  $y = 6x^2 + 2x + 4$

$$\frac{dy}{dx} = 12x + 2$$

2. Find the gradient of the following curves at the given points

a)  $f(x) = \frac{1}{x^2}$  at the point (2, 0.25)

$$-\frac{1}{4}$$

b)  $f(x) = \frac{5}{\sqrt{x}}$  at the point where  $x=9$

$$-\frac{5}{54}$$

3. Find the coordinate of the point on the curve

a)  $y = x^2 - 3x + 1$  where the gradient is 7

$$(5, 11)$$

b)  $f(x) = 4x^2 - 7x + 3$  where the gradient is -3

$$(0.5, 0.5)$$

c)  $f(x) = x^2 + 5x + 3$  where the gradient is 1

$$(-2, -3)$$

d)  $y = 7x - 3x^2$  where the gradient is -5

$$(2, 2)$$

4. Find the coordinate of both points on the curve  $y = x - \frac{x^3}{3}$  where the gradient is 0

$$(1, 2/3) \quad (-1, -2/3)$$

5. Find the coordinate of both points on the curve  $y = x^3 - 9x^2 + 10x - 5$  where the gradient is -14

$$(4, -45) \quad (2, -13)$$

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### **Homework Questions 5 – Expanding and Simplifying**

Use standard results to differentiate after first expanding or simplifying the function

1.  $y = 3(2x - x^2)$

$$\frac{dy}{dx} = 6 - 6x$$

2.  $f(x) = x(x + 1)$

$$f'(x) = 2x + 1$$

3.  $f(x) = (2x + 3)^2$

$$f'(x) = 8x + 12$$

4.  $y = x(2x + 3)^2$

$$\frac{dy}{dx} = 12x^2 + 24x + 9$$

5.  $y = 3(x - 1)(x + 2)$

$$\frac{dy}{dx} = 6x + 3$$

6.  $f(x) = 2x^2(4x - 3)$

$$f'(x) = 24x^2 - 12x$$

7.  $f(x) = \frac{12x^5}{6x}$

$$f'(x) = 8x^3$$

8.  $y = \frac{x^2 - x}{x}$

$$\frac{dy}{dx} = 1$$

9.  $y = \frac{3x^2 - 4x}{x}$

$$\frac{dy}{dx} = 3$$

10.  $f(x) = \frac{3x^4 + 4x^2 + 6x}{2x}$

$$f'(x) = \frac{9}{4}x^2 + 2$$

11.  $f(x) = \sqrt[3]{x}$

$$f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$$



12.  $y = \sqrt[4]{x}$

$$\frac{dy}{dx} = \frac{1}{4} x^{-\frac{3}{4}}$$

13.  $y = 3x - \sqrt{x} + x^2$

$$\frac{dy}{dx} = 3 - \frac{1}{2} x^{-\frac{1}{2}} + 2x$$

14.  $f(x) = \frac{x^2 + x}{\sqrt{x}}$

$$f(x) = \frac{3}{2} x^{\frac{1}{2}} + \frac{1}{2} x^{-\frac{1}{2}}$$

15.  $f(x) = \frac{3x^4 - 2x^2}{\sqrt[3]{x}}$

$$f(x) = 11x^{\frac{8}{3}} - \frac{10}{3} x^{\frac{2}{3}}$$

16.  $f(x) = -\frac{2}{\sqrt{x}}$

$$f(x) = x^{-\frac{3}{2}}$$

17.  $y = \frac{x^2 - 3x}{2x^4}$

$$\frac{dy}{dx} = -x^{-3} + \frac{9}{2} x^{-4}$$

18.  $y = x^3 - \frac{1}{2x^2} + \frac{4}{x^3}$

$$\frac{dy}{dx} = 3x^2 + x^{-3} - 12x^{-4}$$

19.  $f(x) = (2x - 3)(x - 4)$

$$f(x) = 4x - 11$$

20.  $f(x) = (x + 4)^2 + 2x^2$

$$f(x) = 6x + 8$$

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### Homework Questions 6 - 2<sup>nd</sup> Order Derivatives

For each questions find the 2<sup>nd</sup> order derivative

1.  $y = 7x^3 - 2x^2 + 3$

$$\frac{d^2y}{dx^2} = 42x - 4$$

2.  $f(x) = 4x^3 - 2x$

$$f'(x) = 24x$$

3.  $f(x) = 8x^2 - 5x - 3$

$$f'(x) = 16$$

4.  $y = \sqrt{x} + x^2$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} + 2$$

5.  $y = \frac{1}{x^2} + \frac{1}{x^3}$

$$\frac{d^2y}{dx^2} = -\frac{1}{4}x^{-\frac{3}{2}} - \frac{2}{9}x^{-\frac{5}{3}}$$

6.  $f(x) = 6 - 5x + \frac{7}{x}$

$$f'(x) = 14x^{-3}$$

7.  $f(x) = 6\sqrt{x} - \frac{4}{x^2}$

$$f'(x) = -\frac{3}{2}x^{-\frac{3}{2}} - 24x^{-4}$$

8.  $y = (x + 2)(x - 3)$

$$\frac{d^2y}{dx^2} = 2$$

9.  $y = (x + 6)^2$

$$\frac{d^2y}{dx^2} = 2$$

10.  $f(x) = \frac{x^2}{3} - 2x^2 + x - \sqrt{x} + 5$

$$f'(x) = -\frac{10}{3} + \frac{1}{4}x^{-\frac{3}{2}}$$

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### **Homework Questions 7 - Rate of Change**

For the rate of change of the following by differentiating

1. find  $\frac{d\theta}{dt}$  where  $\theta = t^3 - 4t + 6$

$$\frac{d\theta}{dt} = 3t^2 - 4$$

2. find  $\frac{dx}{dy}$  where  $x = 7y^2 - 3y + 2$

$$\frac{dx}{dy} = 14y - 3$$

3. find  $\frac{dr}{dt}$  where  $r = 3t^2 - 2t + 1$

$$\frac{dr}{dt} = 6t - 2$$

4. find  $\frac{da}{dr}$  where  $a = \pi r^2$

$$\frac{da}{dr} = 2\pi r$$

5. find  $\frac{dt}{dy}$  where  $t = y^2(y + 3)$

$$\frac{dt}{dy} = 3y^2 + 6y$$

6. find  $\frac{ds}{dt}$  where  $s = 7t^2 - 6t$

$$\frac{ds}{dt} = 4t - 6$$

7. find  $\frac{da}{dx}$  where  $a = 8x^3 - 2x^2 - 3$

$$\frac{da}{dx} = 24x - 4x$$

8. find  $\frac{dy}{dx}$  where  $y = 5x^2 - \frac{1}{x^3} + x$

$$\frac{dy}{dx} = 10x + 3x^{-4}$$

9. find  $\frac{dy}{dx}$  where  $y = (x - 4)(2 - 3x)$

$$\frac{dy}{dx} = -6x + 14$$

10. find  $\frac{d\theta}{dt}$  where  $\theta = (t + 1)(t^2 - 4)$

$$\frac{d\theta}{dt} = 3t^2 + 2t - 4$$

(C1-7.8a) Name:

### **Homework Questions 8 – Equation of Tangents & Normals**

1. Find the equation of the tangent to the curve  $y = 3x^2 + 5x + 2$  at the point (3,44)

$$y = 23x - 25$$

2. For the curve given below, find the gradient of the tangent at the point (4,123)

$$y = 7x^2 + 4x - 5$$

$$60$$

3. Find the equation of the tangent to the function below at the point (1,11)

$$f(x) = 8x^2 + 3x$$

$$y = 19x - 8$$

4. If a tangent cuts a curve at (2,7) and has a gradient of -2,  
What is the equation of the tangent?

$$y = -2x + 11$$

5. Find the gradient of the function at the point (2,36)

If the equation of the curve is  $f(x) = 4x^2 + 9x + 2$

$$25$$

6. What are the coordinates of the point on the curve  $y = 4 - x^2$  where the gradient of  
the normal is  $\frac{1}{4}$

$$(2,0)$$

7. Find the equation of the normals to the curve  $y = x^2 - 5$  at the point (2,-1)

$$4y = -x - 2$$

8. What is the equation of the tangent to the curve  $y = 3x^2 - 9x$  at the point where  $x=4$

$$y = 15x - 48$$

9. What is the equation of the normal to the curve  $y = x - x^2 + 2x^3$  at the point where  $x=-1$

$$9y = -x - 37$$

10. Find the equation of the normal to the curve  $y = x^2 + 5x + 1$  at the point where the tangent gradient at this point is 2

$$2y = -x - 10$$