OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4734080113

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** Differentiate with respect to x the equation $a\tan y = x$ (where a is a constant), and hence show that the derivative of $\arctan\frac{x}{a}$ is $\frac{a}{a^2+x^2}$. **[3]**

(ii) By first expressing x^2-4x+8 in completed square form, evaluate the integral $\int_0^4 \frac{1}{x^2-4x+8}\,dx$, giving your answer exactly. **[4]**

(iii) Use integration by parts to find $\int \arctan x \, dx$. **[4]**

(b) **(i)** A curve has polar equation $r = 2\cos\theta$, for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. Show, by considering its cartesian equation, that the curve is a circle. State the centre and radius of the circle. **[5]**

(ii) Another circle has radius 2 and its centre, in cartesian coordinates, is (0, 2). Find the polar equation of this circle. **[2]**

2 **(a)** **(i)** Show that

$$1 + e^{j2\theta} = 2\cos\theta(\cos\theta + j\sin\theta).$$ **[2]**

(ii) The series C and S are defined as follows.

$$C = 1 + \binom{n}{1}\cos 2\theta + \binom{n}{2}\cos 4\theta + \ldots + \cos 2n\theta$$

$$S = \binom{n}{1}\sin 2\theta + \binom{n}{2}\sin 4\theta + \ldots + \sin 2n\theta$$

By considering $C + jS$, show that

$$C = 2^n \cos^n\theta \cos n\theta,$$

and find a corresponding expression for S. **[7]**

(b) **(i)** Express $e^{j2\pi/3}$ in the form $x + jy$, where the real numbers x and y should be given exactly. **[1]**

(ii) An equilateral triangle in the Argand diagram has its centre at the origin. One vertex of the triangle is at the point representing $2 + 4j$. Obtain the complex numbers representing the other two vertices, giving your answers in the form $x + jy$, where the real numbers x and y should be given exactly. **[6]**

(iii) Show that the length of a side of the triangle is $2\sqrt{15}$. **[2]**

3 You are given the matrix $\mathbf{M} = \begin{pmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$.

(i) Show that the characteristic equation of **M** is

$$\lambda^3 - 13\lambda + 12 = 0.$$ **[3]**

(ii) Find the eigenvalues and corresponding eigenvectors of **M**. **[12]**

(iii) Write down a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{M}^n = \mathbf{PDP}^{-1}.$$

(You are not required to calculate $\mathbf{P}^{-1}$.) **[3]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Show that the curve with equation

$$y = 3\sinh x - 2\cosh x$$

has no turning points.

Show that the curve crosses the x-axis at $x = \frac{1}{2}\ln 5$. Show that this is also the point at which the gradient of the curve has a stationary value. **[7]**

(ii) Sketch the curve. **[2]**

(iii) Express $(3\sinh x - 2\cosh x)^2$ in terms of $\sinh 2x$ and $\cosh 2x$.

Hence or otherwise, show that the volume of the solid of revolution formed by rotating the region bounded by the curve and the axes through 360° about the x-axis is

$$\pi(3 - \tfrac{5}{4}\ln 5).$$ **[9]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 This question concerns the curves with polar equation

$$r = \sec\theta + a\cos\theta, \qquad (*)$$

where a is a constant which may take any real value, and $0 \leqslant \theta \leqslant 2\pi$.

(i) On a single diagram, sketch the curves for $a = 0$, $a = 1$, $a = 2$. **[3]**

(ii) On a single diagram, sketch the curves for $a = 0$, $a = -1$, $a = -2$. **[2]**

(iii) Identify a feature that the curves for $a = 1$, $a = 2$, $a = -1$, $a = -2$ share. **[1]**

(iv) Name a distinctive feature of the curve for $a = -1$, and a different distinctive feature of the curve for $a = -2$. **[2]**

(v) Show that, in cartesian coordinates, equation (*) may be written

$$y^2 = \frac{ax^2}{x-1} - x^2.$$

Hence comment further on the feature you identified in part **(iii)**. **[5]**

(vi) Show algebraically that, when $a > 0$, the curve exists for $1 < x < 1 + a$.

Find the set of values of x for which the curve exists when $a < 0$. **[5]**

RECOGNISING ACHIEVEMENT

ERRATUM NOTICE

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

FOR THE ATTENTION OF

THE EXAMINATIONS OFFICER

To be opened on the day of the exam

Instructions to Invigilators:

Before the start of the examination hand out a copy of this erratum to each candidate.

Please ensure all candidates amend their copy of the question paper by crossing out question **5 part (vi)** before the start of the examination.

Instructions to Candidates:

Please turn to **Page 4 of the Question Paper, Question 5, part (vi):**

You should cross out question **5 part (vi).** The new question **5 part (vi)** is shown below:

(vi) Show algebraically that, when $a > 0$, the curve exists for $1 < x \leqslant 1 + a$.

Find the set of values of x for which the curve exists when $a < 0$. **[5]**

Any enquiry about this notice should be referred to the Customer Contact Centre on 01223 553 998 or general.qualifications@ocr.org.uk

OCR
RECOGNISING ACHIEVEMENT

Monday 14 January 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4756/01 Further Methods for Advanced Mathematics (FP2)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

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- MEI Examination Formulae and Tables (MF2)

Other materials required:
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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (54 marks)

1 (a) (i)	
1 (a) (ii)	

1 (a) (iii)	

1 (b) (i)	
1 (b) (ii)	

2 (a) (i)	
2 (a) (ii)	
	(answer space continued on next page)

2 (a) (ii)	**(continued)**
2 (b) (i)	

2 (b) (ii)	
2 (b) (iii)	

3 (i)	

3 (ii)	
	(answer space continued on next page)

3 (ii)	**(continued)**
3 (iii)	

Section B (18 marks)

4 (i)	

4 (ii)	
4 (iii)	
	(answer space continued on next page)

4 (iii)	**(continued)**

5 (i)	
5 (ii)	
5 (iii)	
5 (iv)	

5 (v)

5 (vi)	

OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4756:** Further Methods for Advanced Mathematics

Mark Scheme for January 2013

Oxford Cambridge and RSA Examinations

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

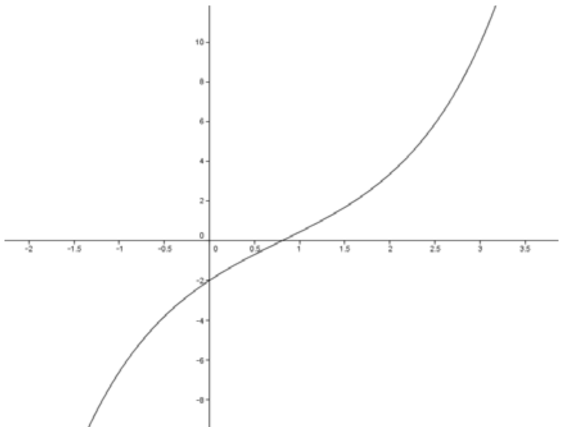
Question			Answer	Marks	Guidance	
1	(a)	(i)	$a\tan y = x \Rightarrow a\sec^2 y\dfrac{dy}{dx}=1$	M1	Differentiating with respect to x or y	$\dfrac{dx}{dy}=a\sec^2 y$
			$\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{a\sec^2 y}$	A1	For $\dfrac{dy}{dx}$	Or $a\dfrac{dy}{dx}=\dfrac{1}{\sec^2 y}$
			$\Rightarrow \dfrac{dy}{dx}=\dfrac{1}{a\left(1+\dfrac{x^2}{a^2}\right)}$			
			$\Rightarrow \dfrac{dy}{dx}=\dfrac{a}{a^2+x^2}$	A1(ag)	Completion www with sufficient detail	
				[3]		
1	(a)	(ii)	$x^2-4x+8=(x-2)^2+4$	B1		
			$\int_0^4 \dfrac{1}{x^2-4x+8}dx=\dfrac{1}{2}\left[\arctan\dfrac{x-2}{2}\right]_0^4$	M1	Integral of form a arctan bu or any appropriate substitution	$\dfrac{1}{2}\left[\arctan\dfrac{u}{2}\right]_{-2}^{2}$
				A1	Correct integral with consistent limits	
			$=\dfrac{1}{2}(\arctan(1)-\arctan(-1))$			
			$=\dfrac{\pi}{4}$	A1	Evaluated in terms of π	
				[4]		
1	(a)	(iii)	$\int 1\times\arctan x\,dx$	M1	Using parts with u = arctan x and v' = 1	Allow one other error
			$=x\arctan x-\int\dfrac{x}{1+x^2}dx$	A1		
				M1	$\int\dfrac{x}{1+x^2}dx=a\ln(1+x^2)$	
			$=x\arctan x-\dfrac{1}{2}\ln(1+x^2)+c$	A1	$a=\dfrac{1}{2}$. Condone omitted c	
				[4]		

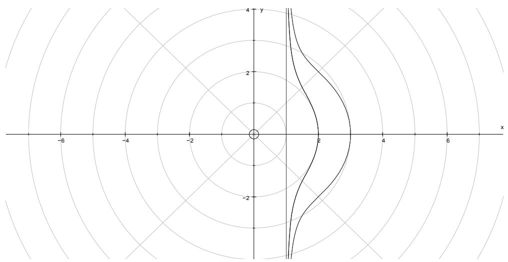
Question			Answer	Marks	Guidance	
1	(b)	(i)	$r = 2\cos\theta \Rightarrow r^2 = 2r\cos\theta$	M1	Using $r^2 = x^2 + y^2$ and $x = r\cos\theta$	
			$\Rightarrow x^2 + y^2 = 2x$	A1	A correct cartesian equation in any form	
			$\Rightarrow (x-1)^2 + y^2 = 1$	A1(ag)	Explaining that the curve is a circle	e.g. writing as $(x-\alpha)^2 + (y-\beta)^2 = r^2$
			OR $x = r\cos\theta \Rightarrow x = 2\cos^2\theta$ $y = r\sin\theta \Rightarrow y = 2\cos\theta\sin\theta = \sin 2\theta$ M1 $\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow x = \cos 2\theta + 1$ A1 $\Rightarrow (x-1)^2 + y^2 = 1$ A1(ag)		Using $x = r\cos\theta$, $y = r\sin\theta$ and linking x in terms of $\cos 2\theta$ Explaining that the curve is a circle	e.g. writing as $(x-\alpha)^2 + (y-\beta)^2 = r^2$
			Centre (1, 0) Radius 1	B1 B1 **[5]**	Independent Independent	
1	(b)	(ii)	$x^2 + (y-2)^2 = 4 \Rightarrow x^2 + y^2 = 4y$ $\Rightarrow r^2 = 4r\sin\theta$ $\Rightarrow r = 4\sin\theta$	 M1 A1 **[2]**	Using $r^2 = x^2 + y^2$ and $y = r\sin\theta$	For answer alone www: B1 for $r = k\sin\theta$, B1 for $k = 4$
2	(a)	(i)	$1 + e^{j2\theta} = 1 + \cos 2\theta + j\sin 2\theta$ $= 1 + (2\cos^2\theta - 1) + 2j\sin\theta\cos\theta$ $= 2\cos^2\theta + 2j\sin\theta\cos\theta$ $= 2\cos\theta(\cos\theta + j\sin\theta)$	M1 A1(ag)	Using $e^{2j\theta} = \cos 2\theta + j\sin 2\theta$ and double angle formulae Completion www	Allow one error
			OR $1 + e^{j2\theta} = e^{j\theta}(e^{-j\theta} + e^{j\theta})$ M1 $= (\cos\theta + j\sin\theta) \times 2\cos\theta$ A1(ag)		"Factorising" and complete replacement by trigonometric functions Completion www	
			OR $1 + e^{j2\theta} = 1 + (\cos\theta + j\sin\theta)^2$ $= 1 + \cos^2\theta - \sin^2\theta + 2j\sin\theta\cos\theta$ $= 2\cos^2\theta + 2j\sin\theta\cos\theta$ M1 $= 2\cos\theta(\cos\theta + j\sin\theta)$ A1(ag)		Using $e^{j\theta} = \cos\theta + j\sin\theta$ and $1 - \sin^2\theta = \cos^2\theta$ Completion www	
				[2]		

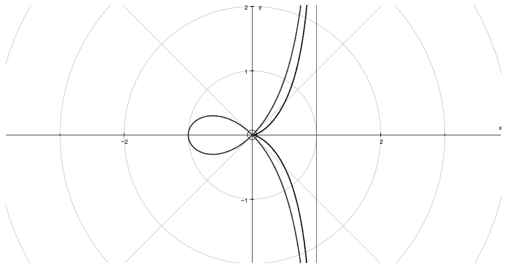
Question			Answer	Marks	Guidance	
2	(a)	(ii)	$C+\mathrm{j}S=1+\binom{n}{1}e^{\mathrm{j}2\theta}+\binom{n}{2}e^{\mathrm{j}4\theta}+\ldots+e^{\mathrm{j}2n\theta}$	M1	Forming $C+\mathrm{j}S$	
			$=\left(1+e^{\mathrm{j}2\theta}\right)^n$	M1 A1	Recognising as binomial expansion	
			$=2^n\cos^n\theta\left(\cos\theta+\mathrm{j}\sin\theta\right)^n$			
			$=2^n\cos^n\theta\left(\cos n\theta+\mathrm{j}\sin n\theta\right)$	M1 A1	Applying (i) and De Moivre o.e.	Dependent on M1M1 above
			$\Rightarrow C=2^n\cos^n\theta\cos n\theta$	A1(ag)	Completion www	Need to see $e^{\mathrm{j}n\theta}=\cos n\theta+\mathrm{j}\sin n\theta$ o.e.
			and $S=2^n\cos^n\theta\sin n\theta$	A1 **[7]**		
2	(b)	(i)	$e^{\mathrm{j}\frac{2\pi}{3}}=\cos\frac{2\pi}{3}+\mathrm{j}\sin\frac{2\pi}{3}=-\frac{1}{2}+\mathrm{j}\frac{\sqrt{3}}{2}$	B1 **[1]**	Must evaluate trigonometric functions	
2	(b)	(ii)	Other two vertices are $(2+4\mathrm{j})e^{\mathrm{j}\frac{2\pi}{3}}$	M1	Award for idea of rotation by $\frac{2\pi}{3}$	e.g. use of $\arctan 2+\frac{2\pi}{3}$ (3.202 rad) (must be 2)
			$=(2+4\mathrm{j})\left(-\frac{1}{2}+\mathrm{j}\frac{\sqrt{3}}{2}\right)$			
			$=\left(-1-2\sqrt{3}\right)+\mathrm{j}\left(-2+\sqrt{3}\right)$	A1A1	May be given as co-ordinates	
			and $(2+4\mathrm{j})e^{\mathrm{j}\frac{4\pi}{3}}=(2+4\mathrm{j})e^{-\mathrm{j}\frac{2\pi}{3}}$	M1	Award for idea of rotation by $-\frac{2\pi}{3}$	e.g. use of $\arctan 2+\frac{4\pi}{3}$ (5.296 rad) (must be 2)
			$=(2+4\mathrm{j})\left(-\frac{1}{2}-\mathrm{j}\frac{\sqrt{3}}{2}\right)$			
			$=\left(-1+2\sqrt{3}\right)+\mathrm{j}\left(-2-\sqrt{3}\right)$	A1A1 **[6]**	May be given as co-ordinates	If A0A0A0A0 award SC1 for awrt −4.46 − 0.27j and 2.46 − 3.73j

Question			Answer	Marks	Guidance	
2	(b)	(iii)	Length of (2 + 4j) = $\sqrt{20}$			
			So length of side = $2\sqrt{20}\cos\frac{\pi}{6} = 2\sqrt{20}\times\frac{\sqrt{3}}{2}$	M1	Complete method	Alternative: finding distance between (2, 4) and $\left(-1-2\sqrt{3}, -2+\sqrt{3}\right)$ o.e.
			$= 2\sqrt{15}$	A1(ag) **[2]**	Completion www	
3	(i)		$\mathbf{M}-\lambda\mathbf{I} = \begin{pmatrix} 1-\lambda & 3 & 0 \\ 3 & -2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{pmatrix}$			
			$\det(\mathbf{M}-\lambda\mathbf{I})$	M1	Forming $\det(\mathbf{M}-\lambda\mathbf{I})$	
			$=(1-\lambda)\left[(-2-\lambda)(1-\lambda)-1\right]-3\left[3(1-\lambda)\right]$	A1	Any correct form	Sarrus: $(1-\lambda)^2(-2-\lambda)-10(1-\lambda)$
			$=(1-\lambda)(\lambda^2+\lambda-3)-9(1-\lambda)$			or e.g. $\lambda-1+(1-\lambda)(\lambda^2+\lambda-11)$
			$\Rightarrow \lambda^3-13\lambda+12=0$	A1(ag) **[3]**	Condone omission of 0	
3	(ii)		$(\lambda-1)(\lambda^2+\lambda-12)=0$	M1	Factorising as far as quadratic	Allow one error
			$\Rightarrow (\lambda-1)(\lambda-3)(\lambda+4)=0$	A1		
			$\Rightarrow$ eigenvalues are 1, 3, −4	A1		
			$\lambda=1: \begin{pmatrix} 0 & 3 & 0 \\ 3 & -3 & -1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$	M2 M1	<u>For any one of λ = 1, 3, −4</u> Obtaining two independent equations Obtaining a non-zero eigenvector	From which an eigenvector could be found
			$\Rightarrow y=0, 3x-z=0$	A1	o.e.	Allow e.g. $3y=0, 3x-3y-z=0$
			$\Rightarrow$ eigenvector is $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$	A1		
			$\lambda=3: \begin{pmatrix} -2 & 3 & 0 \\ 3 & -5 & -1 \\ 0 & -1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$			
			$\Rightarrow -2x+3y=0, -y-2z=0$	A1	o.e.	

Question			Answer	Marks	Guidance	
			$\Rightarrow y=-2z,\ x=-3z$			
			$\Rightarrow$ eigenvector is $\begin{pmatrix}-3\\-2\\1\end{pmatrix}$	A1		
			$\lambda=-4:\ \begin{pmatrix}5&3&0\\3&2&-1\\0&-1&5\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$			
			$\Rightarrow 5x+3y=0,\ -y+5z=0$	A1	o.e.	
			$\Rightarrow y=5z,\ x=-3z$			
			$\Rightarrow$ eigenvector is $\begin{pmatrix}-3\\5\\1\end{pmatrix}$	A1		
				[12]		
3	**(iii)**		E.g. $\mathbf{P}=\begin{pmatrix}1&-3&-3\\0&-2&5\\3&1&1\end{pmatrix}$	B1	Use of eigenvectors (ft) as columns	
			$\mathbf{D}=\begin{pmatrix}1&0&0\\0&3^n&0\\0&0&(-4)^n\end{pmatrix}$	M1	Use of 1, 3, −4 (ft) in correct order	n not required for M1
				A1	Power n	-4^n A0
				[3]		

Question			Answer	Marks	Guidance	
4	(i)		$y = 3\sinh x - 2\cosh x$			$\frac{1}{2}e^{x} - \frac{5}{2}e^{-x}$
			$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 3\cosh x - 2\sinh x$	B1		$\frac{1}{2}e^{x} + \frac{5}{2}e^{-x}$
			At TPs, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \tanh x = \frac{3}{2}$	M1	Considering $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	$e^{2x} = -5$; $e^{x} > 0$ and $e^{-x} > 0$
			which has no (real) solutions	A1(ag)	Showing no real roots www	
			$y = 0 \Rightarrow \tanh x = \frac{2}{3}$	M1	Solving $y = 0$ as far as e^{2x} or tanh x etc.	$e^{2x} = 5$; $\cosh x = \frac{3}{\sqrt{5}}$; $\sinh x = \frac{2}{\sqrt{5}}$
			$\Rightarrow x = \frac{1}{2}\ln\frac{1+\frac{2}{3}}{1-\frac{2}{3}}$	M1	Solving as far as x	<u>Attempt to verify</u> Award M1 for substituting $x = \frac{1}{2}\ln 5$
			$\Rightarrow x = \frac{1}{2}\ln 5$	A1(ag)	Completion www	and M1 for clearly attempting to evaluate exactly
			$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3\sinh x - 2\cosh x = y$			
			so $y = 0 \Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$	B1(ag)		$3\sinh\left(\frac{1}{2}\ln 5\right) - 2\cosh\left(\frac{1}{2}\ln 5\right) = 0$ must be explained, e.g. connected with $y = 0$
				[7]		
4	(ii)			B2 **[2]**	For a curve with the following features: • increasing • intersecting the positive x-axis • (0, −2) indicated • gradient increasing with large $\lvert x\rvert$ • one point of inflection	Award B1 for a curve lacking one of these features

Question			Answer	Marks	Guidance	
4	(iii)		$(3\sinh x - 2\cosh x)^2$			
			$= 9\sinh^2 x - 12\sinh x\cosh x + 4\cosh^2 x$	B1		
			$= \frac{9}{2}(\cosh 2x - 1) - 6\sinh 2x + 2(\cosh 2x + 1)$	M1	Using double "angle" formulae or complete alternative	Condone sign errors but need $\frac{1}{2}$ s
			$= \frac{13}{2}\cosh 2x - 6\sinh 2x - \frac{5}{2}$	A1	Accept unsimplified	$\frac{1}{4}e^{2x} + \frac{25}{4}e^{-2x} - \frac{5}{2}$
			$V = \pi\int_0^{\frac{1}{2}\ln 5} y^2\,\mathrm{d}x$	M1	Attempting to integrate their y^2 (ignore limits)	
			$= \pi\left[\frac{13}{4}\sinh 2x - 3\cosh 2x - \frac{5}{2}x\right]_0^{\frac{1}{2}\ln 5}$	A2	Correct results and limits c.a.o. Ignore omitted π	Give A1 for one error, or for all three terms correct and incorrect limits
				M1	Substituting both of their limits	
			$= \pi\left[\frac{13}{4}\times\frac{12}{5} - 3\times\frac{13}{5} - \frac{5}{4}\ln 5 + 3\right]$	M1	Obtaining exact values of $\sinh(\ln 5)$ and $\cosh(\ln 5)$	$\sinh(\ln 5) = \frac{12}{5}$, $\cosh(\ln 5) = \frac{13}{5}$
			OR $= \pi\left[\frac{1}{8}e^{2x} - \frac{25}{8}e^{-2x} - \frac{5}{2}x\right]_0^{\frac{1}{2}\ln 5}$ A2		Correct results and limits	Give A1 for one error, or for all three terms correct and incorrect limits
			M1		Substituting both of their limits	
			$= \pi\left[\frac{5}{8} - \frac{5}{8} - \frac{5}{4}\ln 5 + 3\right]$ M1		Obtaining exact values of e^{2x} and e^{-2x}	$e^{2x} = 5$, $e^{-2x} = \frac{1}{5}$
			$= \pi\left[3 - \frac{5}{4}\ln 5\right]$	A1(ag) **[9]**	Completion www	
5	(i)			B2 B1 **[3]**	Three curves of correct shape Correctly identified	Give B1 for two correct curves $a = 0$, $a = 1$, $a = 2$ from left to right

Question			Answer	Marks	Guidance	
5	(ii)			B1 B1 **[2]**	Curve for $a = -1$ Curve for $a = -2$	Curve with cusp Curve with loop
5	(iii)		Asymptote	B1 **[1]**		
5	(iv)		$a = -1$: cusp $a = -2$: loop	B1 B1 **[2]**		
5	(v)		$r = \sec\theta + a\cos\theta \Rightarrow r\cos\theta = 1 + a\cos^2\theta$ $\Rightarrow x = 1 + a\left(\frac{x^2}{r^2}\right)$ $\Rightarrow x - 1 = a\left(\frac{x^2}{x^2+y^2}\right)$ $\Rightarrow x^2 + y^2 = a\left(\frac{x^2}{x-1}\right) \Rightarrow y^2 = a\left(\frac{x^2}{x-1}\right) - x^2$ Hence asymptote at $x = 1$	M1 M1 M1 A1(ag) B1 **[5]**	Using $x = r\cos\theta$ Using $r^2 = x^2 + y^2$ Making y^2 subject	
5	(vi)		Curve exists for $y^2 \ge 0$ $\Rightarrow a\left(\frac{1}{x-1}\right) - 1 \ge 0$ If $a > 0$ then $x - 1 > 0$ and so $a \ge x - 1$ i.e. $1 < x \le 1 + a$ If $a < 0$ then $x - 1 < 0$ and so $a \le x - 1$ i.e. $1 + a \le x < 1$	M1 M1 A1(ag) M1 A1 **[5]**	Considering $y^2 \ge 0$	

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OCR Report to Centres

January 2013

4756 Further Methods for Advanced Mathematics

General Comments

The level of performance in this paper was comparable with previous January series. The distribution of marks showed a strong negative skew, with over one-quarter of candidates scoring 60 marks or more and only about 4% scoring 20 marks or fewer. Question 3 was the best-scoring question, followed in order by Questions 1, 4 and 2. Only one candidate attempted Question 5, which was making its last appearance in this unit. There was no evidence of time trouble, but once again many candidates found it necessary to use supplementary sheets, often because they wished to replace an incorrect answer and had already filled the space. Centres should not issue candidates with graph paper for rough working. Presentation varied between the exemplary and the almost illegible: perhaps there were more badly-presented scripts in this series than before, and it was particularly difficult to follow some of the solutions to Q3(i).

"Standard" questions, such as the integrals in Q1(a), the derivation of the characteristic equation in Q3(i) and finding the eigenvectors in Q3(ii), were confidently and accurately handled by the vast majority of candidates. On the other hand, many candidates struggled with the geometry of complex numbers in Q2(b), where about half of all candidates scored zero in parts (ii) and (iii).

Candidates could have improved their performance if they had

- been more careful with their elementary algebra, errors included writing $-4\ ^{n}$ as -4^{n} in Q3(iii); $y-2\ ^{2}=y^{2}-2y+4$ in Q1(b)(ii) (and similar in Q4(iii)) and $\frac{1}{x-2\ ^{2}+4}=\frac{1}{x-2\ ^{2}}+\frac{1}{4}$ in Q1(a)(iii).
- made better use of the structure of a question, eg the use of arctan in the integral in Q1(a)(ii) is suggested by Q1(a)(i), Q2(b)(i) assists with Q2(b)(ii), and Q4(i) assists with Q4(ii).
- shown sufficient steps or sufficient clarity when establishing a given answer, eg in Q2(a)(ii), Q4(i) and Q4(iii).
- used simple methods, eg in Q4(i), a number of candidates reached $e^{2x}=5$, to which they applied the quadratic formula: $e^{x}\ ^{2}=5$ so $e^{x}=\frac{-0\pm\sqrt{0-4\times1\times\ -5}}{2\times1}$ etc.
- appreciated what is meant by "exact form", eg leaving $\cos\frac{2\pi}{3}+\mathrm{j}\sin\frac{2\pi}{3}$ as the final answer in Q2(b)(i), and conversely giving the complex numbers as decimals in Q2(b)(ii).
- read the questions more carefully, eg in Q3(iii).
-

Comments on Individual Questions

1 Calculus with inverse trigonometric functions; polar co-ordinates

Part (b) was generally found more challenging than part (a).

(a)(i) Most candidates found this an attractive start, although poor algebra or a lack of sufficient detail prevented many from scoring full marks. Some candidates mixed up *y* and *x* at an early stage. Some looked up or recalled the derivative of arcsin *x* and just applied the Chain Rule, which attracted no credit.

(ii) The vast majority could complete the square and then most took the hint provided by part (i). The most common error was to evaluate incorrectly at the lower limit, which often produced an answer of $\frac{\pi}{8}$. The argument that $\frac{1}{(x-2)^2+4}=\frac{1}{(x-2)^2}+\frac{1}{4}$ which gave, after integration, $\ln(x-2)^2+\ln 4$ or similar, was seen more often than expected. There were some elegant and fully correct answers by explicit substitution.

(iii) This integration by parts was handled very confidently. The vast majority produced a line containing $\int\frac{x}{1+x^2}\,\mathrm{d}x$ without trouble: the integral of this caused more difficulty, with many forgetting the $\frac{1}{2}$ and others writing $\int\frac{x}{1+x^2}\,\mathrm{d}x=x\int\frac{1}{1+x^2}\,\mathrm{d}x=x\arctan x$.

(b)(i) Although many candidates produced concise, efficient solutions, many confused *r* (as in the polar equation) with *r* (as in the radius of the circle) and, having obtained $x^2+y^2=4\cos^2\theta$, asserted that this was a circle centre (0, 0), radius $2\cos\theta$. Others appeared to be solving equations in *θ*. The correct centre and radius often appeared independently of a correct cartesian equation.

(ii) The usual approach was to obtain the cartesian equation as $x^2+(y-2)^2=4$, and then multiply out and obtain $x^2+y^2=4y$ and hence the correct polar equation. Unfortunately very many candidates obtained 2*y* rather than 4*y*, or started with a cartesian equation with 2 or even $\sqrt{2}$ on the right hand side.

2 Complex numbers and geometry

This question produced the lowest mean mark by some margin, and a fairly even distribution of marks between 0 and 18.

(a)(i) This was mainly handled very efficiently although the clarity of proofs often left something to be desired, with some candidates making liberal use of "invisible brackets". All three alternatives in the markscheme were seen regularly.

(ii) There were many admirably clear and concise solutions, although not all the required steps were always present to establish the given expression for *C*. Others were determined that *C* + j*S* should be a geometric series, and many even omitted the binomial coefficients to achieve this aim. The link with (a)(i) was not always recognised, especially among those who took the geometric route.

(b)(i) This was often, but not always, correct. Those who wrote $e^{\mathrm{j}\frac{2\pi}{3}}=\cos\frac{2\pi}{3}+\mathrm{j}\sin\frac{2\pi}{3}$ and then evaluated the trigonometric functions were much more frequently correct than those who drew diagrams, which often resulted in the minus sign being omitted from the real part.

(ii) Candidates found this part-question challenging. The structure of the question was, as always, intended to assist candidates and those who realised that they could multiply $2+4\mathrm{j}$ by the complex number in (b)(i) twice usually achieved the correct exact answers with little difficulty. Many related the situation to the three cube roots of $(2+4\mathrm{j})^3=-88-16\mathrm{j}$, but this rarely led to a correct answer in an acceptable exact form.

A very common approach was to find the modulus and argument of 2 + 4j, rotate through $\pm\frac{2\pi}{3}$ (which often appeared as $\frac{\pi}{3}$) and work with expressions of the form $\sqrt{20}\left(\cos\left(\arctan 2 \pm \frac{2\pi}{3}\right) + \mathrm{j}\sin\left(\arctan 2 \pm \frac{2\pi}{3}\right)\right)$ which rarely produced anything acceptably exact, although a little credit was given for answers such as −4.46 − 0.27j. One or two candidates battled away at this with compound angle formulae and deserve warm praise for perseverance, if not for elegance. There were a couple of successful solutions using matrices.

(iii) Success here really depended on something useful being produced in (b)(ii). Successful candidates used a variety of approaches, such as the cosine rule and the formula for the distance between two points. Many attempted to use the results of right-angle trigonometry in triangles without right angles.

3 Matrices: eigenvalues and eigenvectors

This question provided a good source of marks for almost all candidates, with the majority scoring 17 or 18 out of 18.

(i) This was often fully correct. Most expanded by the first row, although Sarrus' method was also popular. The algebra employed in obtaining the given result was usually correct and sufficiently clear, although very poor handwriting made some solutions difficult to decode and "invisible brackets" were commonly employed. A worrying assertion, seen frequently, was that $\det \mathbf{M} - \lambda\mathbf{I} = -\lambda^3 + 13\lambda - 12 = \lambda^3 - 13\lambda + 12$.

(ii) Solving the characteristic equation presented few problems, and was usually accomplished by spotting one root (usually $\lambda = 1$) and then obtaining the corresponding quadratic factor, although quite a few started with an expression of the form $\lambda + a \quad \lambda + b \quad \lambda + c$, multiplied out, and worked with equations such as $a + b + c = 0$ and $abc = 12$. This seems inefficient although it did sometimes work. Then the eigenvectors were usually produced without trouble, although $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ was frequently seen and equations such as $y = 0$ and $3x - z = 0$ were held to lead to $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ or similar as an eigenvector.

(iii) Most identified **P** correctly as a matrix of eigenvectors but many just gave **D** as $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -4 \end{pmatrix}$, without taking into account the power of *n*. Those who did often defied BIDMAS and gave -4^n as one of the elements.

4 Hyperbolic functions

This question was an effective discriminator, with the full range of marks being awarded in each part.

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ was usually found correctly and a variety of correct explanations were offered as why this could never be zero, the most common probably being that it leads to $e^{2x} = -5$ (having reached this, it was not necessary to consider the discriminant of a quadratic equation) or $\tanh x = \dfrac{3}{2}$ (although mention of $|x| < 1$ in this context was not credited). Then $y = 0$ was often solved efficiently, by exponentials (perhaps more common) or by obtaining $\tanh x = \dfrac{2}{3}$ and using the logarithmic form of artanh. Those who tried substituting the given value of x often needed to give much more detail in their verification: merely stating that $3\sinh\left(\tfrac{1}{2}\ln 5\right) - 2\cosh\left(\tfrac{1}{2}\ln 5\right) = 0$ attracted no credit. The final part of the question was frequently interpreted as "find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $\left(\tfrac{1}{2}\ln 5, 0\right)$" and much effort was wasted in evaluating $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ exactly. Those who just stated that $3\sinh\left(\tfrac{1}{2}\ln 5\right) - 2\cosh\left(\tfrac{1}{2}\ln 5\right) = 0$ without linking it with y or attempting an evaluation were not rewarded.

(ii) There were many fully correct curves, although some candidates clearly did not connect their sketch with information they had obtained in (i). Many did not mark (0,−2), had a curve bending the wrong way, had a stationary point (often on the *x*-axis), or had multiple points of inflection.

(iii) Multiplying out $(3\sinh x - 2\cosh x)^2$ caused a little trouble for some candidates, with $-6\sinh x\cosh x$ being a fairly common middle term. Then this expression had to be expressed in terms of cosh $2x$ and sinh $2x$. The correct answer was seen fairly frequently, but there were many factor and sign errors, particularly involving cosh $2x$. Many stopped at this point, but most knew that this expression had to be integrated to produce the required volume of revolution. Some were determined to use $\int \frac{1}{2} r^2 \,\mathrm{d}\theta$, thereby introducing a spurious $\dfrac{1}{2}$, and/or limits 0 and 2π. The integration itself was often done well but the lower limit of 0 was often neglected, and obtaining the given answer was not always done with the required clarity and transparency.

5 Investigations of curves

Only one candidate attempted this question in its final appearance in this unit.