

ADVANCED GCE UNIT MATHEMATICS (MEI)

4767/01

Statistics 2

MONDAY 21 MAY 2007

Morning

Time: 1 hour 30 minutes

Additional Materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer all the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

1 The random variable *X* represents the time taken in minutes for a haircut at a barber's shop. *X* is Normally distributed with mean 11 and standard deviation 3.

(i) Find
$$P(X < 10)$$
. [4]

(ii) Find the probability that exactly 3 out of 8 randomly selected haircuts take less than 10 minutes.

[3]

[5]

(iii) Use a suitable approximating distribution to find the probability that at least 50 out of 100 randomly selected haircuts take less than 10 minutes. [4]

A new hairdresser joins the shop. The shop manager suspects that she takes longer on average than the other staff to do a haircut. In order to test this, the manager records the time taken for 25 randomly selected cuts by the new hairdresser. The mean time for these cuts is 12.34 minutes. You should assume that the time taken by the new hairdresser is Normally distributed with standard deviation 3 minutes.

- (iv) Write down suitable null and alternative hypotheses for the test. [3]
- (v) Carry out the test at the 5% level. [5]
- A medical student is trying to estimate the birth weight of babies using pre-natal scan images. The actual weights, $x \log x$, and the estimated weights, $y \log x$, of ten randomly selected babies are given in the table below.

	2.61									
У	3.2	2.6	3.5	3.1	2.8	2.7	3.4	3.3	4.4	4.1

- (i) Calculate the value of Spearman's rank correlation coefficient.
- (ii) Carry out a hypothesis test at the 5% level to determine whether there is positive association between the student's estimates and the actual birth weights of babies in the underlying population.

 [5]
- (iii) Calculate the value of the product moment correlation coefficient of the sample. You may use the following summary statistics in your calculations:

$$\Sigma x = 31.63$$
, $\Sigma y = 33.1$, $\Sigma x^2 = 101.92$, $\Sigma y^2 = 112.61$, $\Sigma xy = 106.51$. [5]

(iv) Explain why, if the underlying population has a bivariate Normal distribution, it would be preferable to carry out a hypothesis test based on the product moment correlation coefficient.

Comment briefly on the significance of the product moment correlation coefficient in relation to that of Spearman's rank correlation coefficient. [4]

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- 3 The number of calls received at an office per 5 minutes is modelled by a Poisson distribution with mean 3.2.
 - (i) Find the probability of
 - (A) exactly one call in a 5-minute period,
 - (B) at least 6 calls in a 5-minute period.

[4]

- (ii) Find the probability of
 - (A) exactly one call in a 1-minute period,
 - (B) exactly one call in each of five successive 1-minute periods.

[4]

(iii) Use a suitable approximating distribution to find the probability of at most 45 calls in a period of 1 hour. [4]

Two assumptions required for a Poisson distribution to be a suitable model are that calls arrive

- at a uniform average rate,
- independently of each other.
- (iv) Comment briefly on the validity of each of these assumptions if the office is
 - (A) the enquiry department of a bank,
 - (B) a police emergency control room.

[4]

4 The sexes and ages of a random sample of 300 runners taking part in marathons are classified as follows.

0	Observed		ex	Row totals
Observed		Male	Female	Now totals
	Under 40	70	54	124
Age group	40–49	76	36	112
Sroup	50 and over	52	12	64
Column totals		198	102	300

- (i) Carry out a test at the 5% significance level to examine whether there is any association between age group and sex. State carefully your null and alternative hypotheses. Your working should include a table showing the contributions of each cell to the test statistic. [10]
- (ii) Does your analysis support the suggestion that women are less likely than men to enter marathons as they get older? Justify your answer. [3]

For marathons in general, on average 3% of runners are 'Female, 50 and over'. The random variable X represents the number of 'Female, 50 and over' runners in a random sample of size 300.

(iii) Use a suitable approximating distribution to find $P(X \ge 12)$. [5]

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Mark Scheme 4767 June 2007

(8)	2		
(i)	$X \sim N(11,3^2)$ $P(X < 10) = P\left(Z < \frac{10-11}{3}\right)$	M1 for standardizing	
	$= P(Z < -0.333)$ $= \Phi(-0.333) = 1 - \Phi(0.333)$	M1 for use of tables with their <i>z</i> -value M1 <i>dep</i> for correct tail A1CAO (must include use	4
	= 1 - 0.6304 = 0.3696	of differences)	
(ii)	P(3 of 8 less than ten) = $\binom{8}{3} \times 0.3696^3 \times 0.6304^5 = 0.2815$	M1 for coefficient M1 for 0.3696 ³ × 0.6304 ⁵ A1 FT (min 2sf)	3
(iii)	$\mu = np = 100 \times 0.3696 = 36.96$ $\sigma^{2} = npq = 100 \times 0.3696 \times 0.6304 = 23.30$ $Y \sim N(36.96,23.30)$ $P(Y \ge 50) = P\left(Z > \frac{49.5 - 36.96}{\sqrt{23.30}}\right)$ $= P(Z > 2.598) = 1 - \Phi(2.598) = 1 - 0.9953$ $= 0.0047$	M1 for Normal approximation with correct (FT) parameters B1 for continuity corr. M1 for standardizing and using correct tail A1 CAO (FT 50.5 or omitted CC)	4
(iv)	H ₀ : $\mu = 11$; H ₁ : $\mu > 11$ Where μ denotes the mean time taken by the new hairdresser	B1 for H_{0} , as seen. B1 for H_{1} , as seen. B1 for definition of μ	3
(v)	Test statistic = $\frac{12.34 - 11}{3/\sqrt{25}} = \frac{1.34}{0.6}$ = 2.23	M1 must include $\sqrt{25}$ A1 (FT their μ)	
	5% level 1 tailed critical value of $z = 1.645$ 2.23 > 1.645, so significant. There is sufficient evidence to reject H_0	B1 for 1.645 M1 for sensible comparison leading to a conclusion	
	It is reasonable to conclude that the new hairdresser does take longer on average than other staff.	A1 for conclusion in words in context (FT their μ)	5
			19
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(i)	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	M1 for ranking (allow all ranks reversed) M1 for d^2 A1 for $\Sigma d^2 = 68$ M1 for method for r_s A1 f.t. for $ r_s < 1$ NB No ranking scores zero	5
(ii)	H_0 : no association between x and y H_1 : positive association between x and y Looking for positive association (one—tail test): critical value at 5% level is 0.5636 Since $0.588 > 0.5636$, there is sufficient evidence to reject H_0 , i.e. conclude that there is positive association between true weight x and estimated weight y .	B1 for H_0 , in context. B1 for H_1 , in context. NB $H_0 H_1$ not ito ρ B1 for ± 0.5636 M1 for sensible comparison with c.v., provided $ r_s < 1$ A1 for conclusion in words & in context, f.t. their r_s and sensible cv	5
(iii)	$\Sigma x = 31.63, \ \Sigma y = 33.1, \ \Sigma x^2 = 101.92, \ \Sigma y^2 = 112.61,$ $\Sigma xy = 106.51.$ $S_{xy} = \Sigma xy - \frac{1}{n} \Sigma x \Sigma y = 106.51 - \frac{1}{10} \times 31.63 \times 33.1$ $= 1.8147$ $S_{xx} = \Sigma x^2 - \frac{1}{n} (\Sigma x)^2 = 101.92 - \frac{1}{10} \times 31.63^2 = 1.8743$ $S_{yy} = \Sigma y^2 - \frac{1}{n} (\Sigma y)^2 = 112.61 - \frac{1}{10} \times 33.1^2 = 3.049$ $r = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{1.8147}{\sqrt{1.8743 \times 3.049}} = 0.759$	M1 for method for S_{xy} M1 for method for at least one of S_{xx} or S_{yy} A1 for at least one of S_{xy} , S_{xx} , S_{yy} correct. M1 for structure of r A1 (awrt 0.76)	5
(iv)	Use of the PMCC is better since it takes into account not just the ranking but the actual value of the weights. Thus it has more information than Spearman's and will therefore provide a more discriminatory test. Critical value for rho = 0.5494 PMCC is very highly significant whereas Spearman's is only just significant.	E1 for has values, not just ranks E1 for contains more information Allow alternatives. B1 for a cv E1 dep	4 19

	(A) $P(X=1) = 0.1712 - 0.0408 = 0.1304$	M1 for tables	
(i)	$_{32}3.2^{1}$	A1 (2 s.f. WWW)	
	$OR = e^{-3.2} \frac{3.2^1}{1!} = 0.1304$		
	(B) $P(X \ge 6) = 1 - P(X \le 5) = 1 - 0.8946$	M1	
	= 0.1054	A1	4
(ii)	(A) $\lambda = 3.2 \div 5 = 0.64$	B1 for mean (SOI)	
	0.064^{1}	M1 for probability	
	$P(X=1) = e^{-0.64} \frac{0.64^{1}}{1!} = 0.3375$	A1	
	(B) P(exactly one in each of 5 mins)	B1 (FT to at least 2 s.f.)	4
	$= 0.3375^5 = 0.004379$	BI (FI to at least 2 s.i.)	
(iii)	Mean no. of calls in 1 hour = $12 \times 3.2 = 38.4$	B1 for Normal approx.	
	Using Normal approx. to the Poisson,	with correct parameters	
	$X \sim N(38.4, 38.4)$	(SOI)	
	$P(X \le 45.5) = P\left(Z \le \frac{45.5 - 38.4}{\sqrt{38.4}}\right)$	B1 for continuity corr.	4
	$\sqrt{38.4}$ = P(Z \le 1.146) = \Phi(1.146) = 0.874 (3 s.f.)	M1 for probability using	
	1(2 _ 1.110)	correct tail A1 CAO, (but FT 44.5 or	
		omitted CC)	
(iv)	(A) Suitable arguments for/against each assumption:	E1, E1	
(14)		L1, D1	
	(B) Suitable arguments for/against each assumption:	E1, E1	4
			16

	o association be ome association	~ ~			B1 (in context)	
	Expected	Sex		Row		
	Lapecteu	Male	Female	totals		
	Under 40	81.84	42.16	124		
Age group	40 – 49	73.92	38.08	112		
Broak	50 and over	42.24	21.76	64		
Co	lumn totals	198	102	300	M1 A1 for expected	
		Sex			values (to 2dp)	
	ntribution to est statistic				NG 6 111 11 11 11 11 11 11 11 11 11 11 11	
	est statistic	Male	Female		M1 for valid attempt at (O–E) ² /E	
	Under 40	1.713	3.325			
Age	40 40	0.059	0.114	1	M1dep for summation	6
group	50 and over	2.255	4.378	1		
2				<u> </u>	A1CAO for X^2	
$X^2 =$	11.84				ATCAO IOI X	4
Critic Resul There	to Ξ_2^2 al value at 5% l t is significant is some associated. H ₀ H ₁ reversed, of	ation betwee	B1 for 2 deg of f B1 CAO for cv B1 dep on their cv & X ² E1 (conclusion in context)			
The a 40 ag be exp	nalysis suggests e group and less pected if there veverse is true fo these data do su	s in the 50 ar were no assort or males. Support the su				
n = 3 EITH $\lambda = n\mu$ Using = OR : ν	00, $p = 0.03$ so ER : use Poisson	in approximation 12) = 1 - F .197 roximation No. $3 > \frac{11.5 - 9}{\sqrt{8.73}}$	$P(X \le 11)$ $I(9, 8.73)$		EITHER: B1 for Poisson B1dep for Poisson(9) M1 for using tables to find $1 - P(X \le 11)$ A1 OR: B1 for Normal B1dep for parameters M1 for using tables with correct tail (cc not required for M1) A1	:

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General Comments

As with previous years, the majority of candidates were well prepared for this examination. Candidates are improving in their ability to carry out hypothesis tests, using correct notation and suitably thorough explanation. Most demonstrate good understanding of the Normal distribution; very few candidates use incorrect tail-probabilities in probability calculations compared with previous years. Marks for explanation and interpretation continue to be elusive to even the most able candidates.

Comments on Individual Questions

Section A

- 1 (i) Well answered. Many candidates lost marks through inappropriate use of continuity corrections. Most managed to calculate a probability using the correct tail of the Normal distribution.
 - (ii) Well answered. A few candidates omitted the binomial coefficient. Some found three eighths of their previous answer. Otherwise, most gained full marks.
 - (iii) The majority of candidates gained at least 3 of the 4 marks available. Many lost a single mark through inaccurate use of Normal tables, failure to use a continuity correction or using the continuity correction, 50.5. A small number attempted to use a Poisson approximation, gaining no credit.
 - (iv) Most candidates obtained two marks for providing correct hypotheses in terms of μ . The mark for defining μ proved harder to obtain. Many made no attempt to define μ at all; some of those who did, seemed unable to relate μ to the "new hairdresser". As with previous years, this mark still proves to be rarely given.
 - (v) Well answered. A variety of approaches were seen; the most common being as outlined in the mark scheme. A small number of students were penalised heavily for treating the sample mean as a single observation, thus avoiding use of the standard error 3/√25. Most candidates obtained at least 4 of the 5 available marks. A few lost the final mark through failing to answer in context. In such questions, the concluding statement should always refer to the context in which the question is set.
- 2 (a)(i) Well answered. Most achieved full marks. Some candidates made mistakes with ranking or with calculating d^2 , thus losing at least one mark. A number of candidates omitted the 6 from their calculation of r_s . Those failing to use ranks scored no marks on this part of the question.
 - (ii) Most candidates are now describing their hypotheses in tests for association, as outlined in the specification. Many failed to give their hypotheses in context, as required; in this particular question, "between x and y" was sufficient. Several lost a mark for omitting the word "positive" from their alternative hypothesis; a further mark was lost if "positive" was omitted from their conclusion. In the remainder of the question, most scored full marks, but marks were lost for failing to provide a conclusion in context.

- (iii) Well answered, with most candidates scoring full marks.
- (iv) Poorly done. Many answers merely repeated the wording given in the question without actually explaining why the pmcc test is preferable. Many candidates appeared not to realise that two explanations were required in this part of the question. For the second explanation, very few managed to refer to a critical value; most answers simply compared the values of the correlation coefficients with each other.
- 3 (i) A Most candidates scored full marks. A small number misinterpreted the question, finding P(X = 5) instead of P(X = 1).
 - (i) B Most candidates scored full marks. A small number used 1 $P(X \le 6)$, losing both marks.
 - (ii) A Most candidates scored full marks.
 - (ii) B Well answered. Some candidates misinterpreted the question and found P(X = 1), using B(5, 0,3375)
 - (iii) Well answered. Common mistakes involved incorrect, or omitted, continuity corrections. Most candidates worked to an acceptable degree of accuracy.
 - (iv) A In answering questions such as this one, candidates should aim to provide a decision together with a reason to support it. Many candidates provided indecisive comments. Other candidates merely stated that calls would (or would not) arrive independently and at a uniform average rate, making no attempt to interpret what this meant. It is clear that most candidates have a poor understanding of what is meant by uniform average rate.
- Well answered. In stating hypotheses, some candidates lost a mark for failing to provide context. Calculations of expected frequencies were handled accurately, on the whole, leading to full marks for the test statistic; however, some candidates lost an accuracy mark through premature approximation. Most candidates had little trouble picking up the final 4 marks, although a significant number thought they should carry out a two-tailed test and were, consequently, penalised. A small number mentioned correlation in their conclusions.
 - (ii) It proved difficult for candidates to obtain full marks for this part of the question. Better attempts saw candidates comparing observed and expected frequencies. Those who referred to the contributions to the test statistic tended to write nonsense unless they demonstrated an appreciation of the difference between positive and negative contributions.
 - (iii) Well answered, with most gaining full marks. The Poisson approximation proved more popular and successful than the Normal approximation. Of those using the Normal approximation, several applied incorrect continuity corrections.