

**OXFORD CAMBRIDGE AND RSA EXAMINATIONS
AS GCE**

4725/01

MATHEMATICS

Further Pure Mathematics 1

QUESTION PAPER

WEDNESDAY 23 JANUARY 2013: Morning

**DURATION: 1 hour 30 minutes
plus your additional time allowance**

MODIFIED ENLARGED 18pt

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

**Printed Answer Book 4725/01
List of Formulae (MF1)**

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

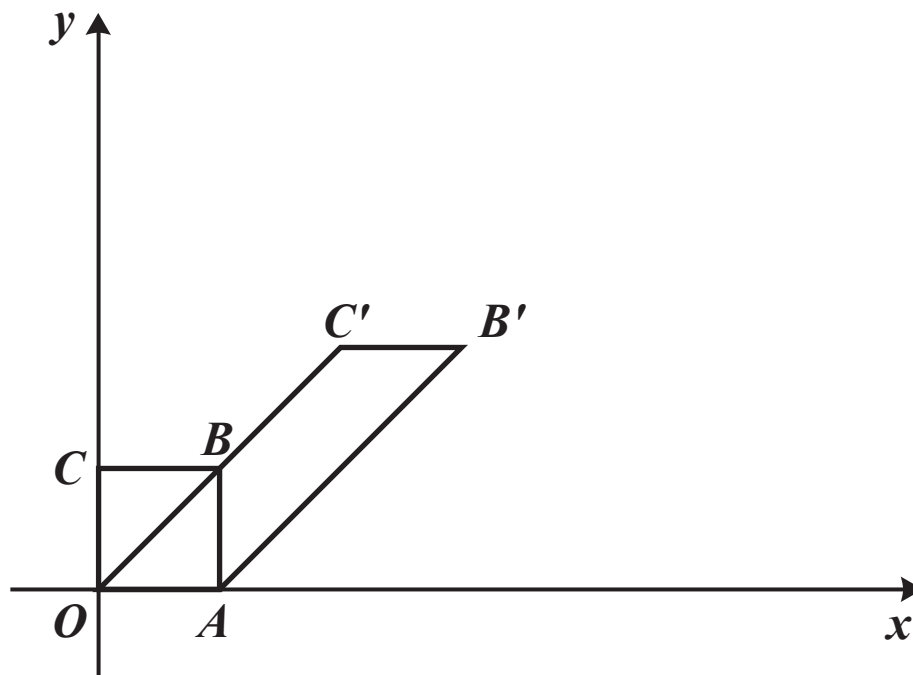
- **The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**
- **The Printed Answer Book consists of 12 pages. The Question Paper consists of 8 pages. Any blank pages are indicated.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

- 1 The matrix A is given by $A = \begin{pmatrix} a & 1 \\ 1 & 4 \end{pmatrix}$, where $a \neq \frac{1}{4}$, and I denotes the 2×2 identity matrix. Find
- (i) $2A - 3I$, [3]
 - (ii) A^{-1} . [2]
- 2 Find $\sum_{r=1}^n (r-1)(r+1)$, giving your answer in a fully factorised form. [6]
- 3 The complex number $2 - i$ is denoted by z .
- (i) Find $|z|$ and $\arg z$. [2]
 - (ii) Given that $az + bz^* = 4 - 8i$, find the values of the real constants a and b . [5]
- 4 The quadratic equation $x^2 + x + k = 0$ has roots α and β .
- (i) Use the substitution $x = 2u + 1$ to obtain a quadratic equation in u . [2]
 - (ii) Hence, or otherwise, find the value of $\left(\frac{\alpha-1}{2}\right)\left(\frac{\beta-1}{2}\right)$ in terms of k . [2]
- 5 By using the determinant of an appropriate matrix, find the values of λ for which the simultaneous equations
- $$\begin{aligned} 3x + 2y + 4z &= 5, \\ \lambda y + z &= 1, \\ x + \lambda y + \lambda z &= 4, \end{aligned}$$
- do not have a unique solution for x, y and z . [6]

6 Look at the following diagram.



The diagram above shows the unit square $OABC$, and its image $OAB'C'$ after a transformation. The points have the following coordinates: $A(1, 0)$, $B(1, 1)$, $C(0, 1)$, $B'(3, 2)$ and $C'(2, 2)$.

- (i) Write down the matrix, X , for this transformation. [2]
- (ii) The transformation represented by X is equivalent to a transformation P followed by a transformation Q . Give geometrical descriptions of a pair of possible transformations P and Q and state the matrices that represent them. [6]
- (iii) Find the matrix that represents transformation Q followed by transformation P . [2]

7 (i) Sketch on a single Argand diagram the loci given by

(a) $|z| = 2$, [2]

(b) $\arg(z - 3 - i) = \pi$. [3]

(ii) Indicate, by shading, the region of the Argand diagram for which

$|z| \leq 2$ and $0 \leq \arg(z - 3 - i) \leq \pi$. [2]

8 (i) Show that $\frac{1}{r} - \frac{3}{r+1} + \frac{2}{r+2} \equiv \frac{2-r}{r(r+1)(r+2)}$. [2]

(ii) Hence show that $\sum_{r=1}^n \frac{2-r}{r(r+1)(r+2)} = \frac{n}{(n+1)(n+2)}$. [5]

(iii) Find the value of $\sum_{r=2}^{\infty} \frac{2-r}{r(r+1)(r+2)}$. [2]

9 (i) Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. [3]

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$,

where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p . [5]

10 The sequence u_1, u_2, u_3, \dots is defined by $u_1 = 2$ and

$$u_{n+1} = \frac{u_n}{1 + u_n} \text{ for } n \geq 1.$$

- (i) Find u_2 and u_3 , and show that $u_4 = \frac{2}{7}$. [3]
- (ii) Hence suggest an expression for u_n . [2]
- (iii) Use induction to prove that your answer to part (ii) is correct. [5]

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