OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS — 4727

Further Pure Mathematics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
None

Wednesday 20 May 2009
Afternoon

Duration: 1 hour 30 minutes

4727

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

1 Find the cube roots of $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$, giving your answers in the form $\cos\theta + \mathrm{i}\sin\theta$, where $0 \leqslant \theta < 2\pi$. **[4]**

2 It is given that the set of complex numbers of the form $r\mathrm{e}^{\mathrm{i}\theta}$ for $-\pi < \theta \leqslant \pi$ and $r > 0$, under multiplication, forms a group.

(i) Write down the inverse of $5\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$. **[1]**

(ii) Prove the closure property for the group. **[2]**

(iii) Z denotes the element $\mathrm{e}^{\mathrm{i}\gamma}$, where $\frac{1}{2}\pi < \gamma < \pi$. Express Z^2 in the form $\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta < 0$. **[2]**

3 A line l has equation $\dfrac{x-6}{-4} = \dfrac{y+7}{8} = \dfrac{z+10}{7}$ and a plane p has equation $3x - 4y - 2z = 8$.

(i) Find the point of intersection of l and p. **[3]**

(ii) Find the equation of the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = d$. **[5]**

4 The differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{1}{1-x^2}y = (1-x)^{\frac{1}{2}}, \quad \text{where } |x| < 1,$$

can be solved by the integrating factor method.

(i) Use an appropriate result given in the List of Formulae (MF1) to show that the integrating factor can be written as $\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$. **[2]**

(ii) Hence find the solution of the differential equation for which $y = 2$ when $x = 0$, giving your answer in the form $y = \mathrm{f}(x)$. **[6]**

5 The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = \mathrm{e}^{3x}.$$

(i) Find the complementary function. **[3]**

(ii) Explain briefly why there is no particular integral of either of the forms $y = k\mathrm{e}^{3x}$ or $y = kx\mathrm{e}^{3x}$. **[1]**

(iii) Given that there is a particular integral of the form $y = kx^2\mathrm{e}^{3x}$, find the value of k. **[5]**

6 The plane Π_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$.

(i) Express the equation of Π_1 in the form $\mathbf{r.n} = p$. **[4]**

The plane Π_2 has equation $\mathbf{r}.\begin{pmatrix} 7 \\ 17 \\ -3 \end{pmatrix} = 21$.

(ii) Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **[5]**

7 **(i)** Use de Moivre's theorem to prove that

$$\tan 3\theta \equiv \frac{\tan\theta(3 - \tan^2\theta)}{1 - 3\tan^2\theta}. \qquad \textbf{[4]}$$

(ii) **(a)** By putting $\theta = \frac{1}{12}\pi$ in the identity in part **(i)**, show that $\tan\frac{1}{12}\pi$ is a solution of the equation

$$t^3 - 3t^2 - 3t + 1 = 0. \qquad \textbf{[1]}$$

(b) Hence show that $\tan\frac{1}{12}\pi = 2 - \sqrt{3}$. **[4]**

(iii) Use the substitution $t = \tan\theta$ to show that

$$\int_0^{2-\sqrt{3}} \frac{t(3 - t^2)}{(1 - 3t^2)(1 + t^2)}\,dt = a\ln b,$$

where a and b are positive constants to be determined. **[5]**

8 A multiplicative group Q of order 8 has elements $\{e, p, p^2, p^3, a, ap, ap^2, ap^3\}$, where e is the identity. The elements have the properties $p^4 = e$ and $a^2 = p^2 = (ap)^2$.

(i) Prove that $a = pap$ and that $p = apa$. **[2]**

(ii) Find the order of each of the elements p^2, a, ap, ap^2. **[5]**

(iii) Prove that $\{e, a, p^2, ap^2\}$ is a subgroup of Q. **[4]**

(iv) Determine whether Q is a commutative group. **[4]**