GCE Examinations

Pure Mathematics
Module P5

Advanced Subsidiary / Advanced Level

Paper C

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and / or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 7 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.

Written by Rosemary Smith & Shaun Armstrong

1. The curve C has intrinsic equation

$$s = 4\sec^3 \psi, \qquad 0 \le \psi < \frac{\pi}{2}.$$

Find the radius of curvature of C at the point where $\psi = \frac{\pi}{4}$. **(5 marks)**

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2. Solve the equation

$$5 \coth x + 1 = 7 \operatorname{cosech} x,$$

giving your answer in terms of natural logarithms. **(7 marks)**

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3. *(a)* Show that $\frac{\mathrm{d}}{\mathrm{d}x}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$. **(3 marks)**

(b) The curve with equation

$$y = \arccos x - \frac{1}{2}\ln(1-x^2), \qquad -1 < x < 1,$$

has a stationary point in the interval $0 < x < 1$.

Find the exact coordinates of this stationary point. **(7 marks)**

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4. *(a)* Express $3 - 6x - 9x^2$ in the form $a - (bx + c)^2$ where a, b and c are constants. **(2 marks)**

Hence, or otherwise, find

(b) $\int \frac{1}{\sqrt{3-6x-9x^2}}\ \mathrm{d}x$, **(4 marks)**

(c) $\int_{-\frac{1}{3}}^{0} \frac{1}{3-6x-9x^2}\ \mathrm{d}x$,

expressing your answer to part *(c)* in terms of natural logarithms. **(6 marks)**

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5. $$\mathrm{f}(x) = \operatorname{artanh}\left(\frac{x^2 - 1}{x^2 + 1}\right), \quad x > 0.$$

(a) Using the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, express $\tanh x$ in terms of e^x and e^{-x}.

(1 mark)

(b) Hence prove that

$$\mathrm{f}(x) = \ln x.$$ **(6 marks)**

(c) Hence, or otherwise, show that the area bounded by the curve $y = \operatorname{artanh}\left(\frac{x^2 - 1}{x^2 + 1}\right)$, the positive x-axis and the line $x = 2\mathrm{e}$ is $2\mathrm{e}\ln 2 + 1$.

(5 marks)

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6. The ellipse C has equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

(a) Find an equation of the normal to C at the point P $(5\cos\theta,\ 3\sin\theta)$. **(5 marks)**

The normal to C at P meets the coordinate axes at Q and R.

Given that $ORSQ$ is a rectangle, where O is the origin,

(b) show that, as θ varies, the locus of S is an ellipse and find its equation in Cartesian form.

(8 marks)

Turn over

7. $$I_n(x) = \int_0^x \cos^n 2t \; dt, \quad n \geq 0.$$

(a) Show that

$$nI_n(x) = \frac{1}{2}\sin 2x \cos^{n-1} 2x + (n-1)I_{n-2}(x), \quad n \geq 2.$$ **(7 marks)**

(b) Find $I_0\left(\frac{\pi}{4}\right)$ in terms of π. **(2 marks)**

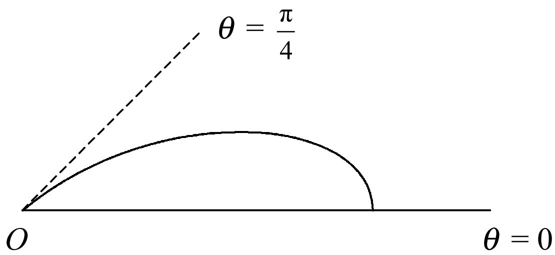

Fig. 1

Figure 1 shows the curve with polar equation

$$r = a\cos^2 2\theta, \quad 0 \leq \theta \leq \frac{\pi}{4},$$

where a is a positive constant.

(c) Using your answers to parts *(a)* and *(b)*, or otherwise, calculate the area bounded by the curve and the half-lines $\theta = 0$ and $\theta = \frac{\pi}{4}$.

(7 marks)

END