

1. The velocity v of a particle at time t is given by $v = \cosh 2t$.
When $t = t_1$, $v = 7$.
Find $\cosh t_1$ and hence or otherwise find t_1 . (6 marks)

2. With the usual notation, a curve has intrinsic equation $s = f(\psi)$. The curve passes through the point P with cartesian coordinates $(0, 3)$. At P , $\psi = 0$.
 - (a) Sketch a diagram to show the significance of s and ψ . (2 marks)
 - (b) If the radius of curvature of this curve at any point Q exceeds the arc length PQ by 1, find $f(\psi)$ in terms of ψ . (5 marks)

3. Using the substitution $u = x - 2$, or otherwise, find
 - (a) $\int \frac{1}{x^2 - 4x + 13} dx$, (b) $\int \frac{1}{\sqrt{12 + 4x - x^2}} dx$. (7 marks)

4. The parametric equations of the hyperbola $ax^2 - by^2 = 1$ are $x = 2 \sec \theta$, $y = 3 \tan \theta$, where $-\pi \leq \theta \leq \pi$.
 - (a) Find the values of a and b . (3 marks)
 - (b) Find an equation of the normal to the hyperbola at the point $(2 \sec \theta, 3 \tan \theta)$. (4 marks)
 - (c) Find the values of θ for which this normal passes through the point $(0, 1)$ and show that the normal cannot pass through $(1, 0)$. (4 marks)

5. (a) Given that $I_n = \int x \sinh^n x dx$, where n is a positive integer, show that

$$nI_n = x \sinh^{n-1} x \cosh x - \frac{1}{n} \sinh^n x - (n-1)I_{n-2}. \quad (9 \text{ marks})$$

 (b) Hence or otherwise find $\int_0^{\ln 2} x \sinh^2 x dx$. (5 marks)

6. The parametric equations of a curve are $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, where $a > 0$ and $0 \leq \theta \leq \frac{\pi}{3}$.
 - (a) Find the length of the curve. (9 marks)
The curve is rotated through 360° about the x -axis.
 - (b) Find the area of the curved surface that is formed. (5 marks)

7. Find, in the form $y = f(x)$, the general solution of each of the differential equations

(a) $\frac{dy}{dx} + y \sinh x = xe^{-\cosh x}$, **(6 marks)**

(b) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = \cosh 4x - \sinh 4x$. **(10 marks)**

[For (b), you may assume that a particular integral has the form $a \cosh 4x + b \sinh 4x$.]