PURE MATHS 4 (A) TEST PAPER 2 : ANSWERS AND MARK SCHEME

1.	$x > -2$ and $2x > x + 2$ or $x < -2$ and $2x < x + 2$	B1 B1	
	Solution set is $x < -2,\ x > 2$	M1 A1 A1	5
2.	$x(x^2 - 4x + 8) = 0$ $\quad x = 0$ or $x = (4 \pm \sqrt{(-16)})/2$	M1 M1	
	$x = 0$ or $x = 2 \pm 2\mathrm{i}$	A1 A1 A1	5
3.	(a) $f(0) = 1,\ f(1) = -0{\cdot}68$, so root between 0 and 1	M1 A1	
	(b) $f'(x) = \cos x\, e^{\sin x} - 3$ $\quad 0{\cdot}5 - f(0{\cdot}5)/f'(0{\cdot}5)$	B1 M1	
	$= 0{\cdot}5 - 0{\cdot}11514/(-1{\cdot}58257) = 0{\cdot}573$ (to 3 d.p.)	A1 A1	6
4.	(a) Let $t = r - 3$; then $r^2 - 6r + 5 = (r-3)^2 - 4 = t^2 - 4$	M1 A1 A1	
	(b) Sum $= \frac{1}{6}(n-3)(n-3+1)(2(n-3)+1) - 4(n-3)$	M1 A1 A1	
	$= \frac{1}{6}(n-3)[(n-2)(2n-5) - 24] = \frac{1}{6}(n-3)(2n^2 - 9n - 14)$	M1 A1	8
5.	(a) $\frac{dy}{dx} - \frac{1}{3}y = \frac{e^x}{3}$ $\quad$ Integrating factor $= e^{\int(-1/3)\,dx} = e^{-x/3}$	B1 M1 A1	
	$e^{-x/3}\frac{dy}{dx} - \frac{1}{3}e^{-x/3}y = \frac{e^{2x/3}}{3}$ $\quad \frac{d}{dx}\left(ye^{-x/3}\right) = \frac{e^{2x/3}}{3}$	M1 A1	
	$ye^{-x/3} = \frac{e^{2x/3}}{2} + c$ $\quad y = \frac{e^x}{2} + ce^{x/3}$ $\quad y(0) = 1 : c = \frac{1}{2}$	M1 A1	
	$y = \frac{1}{2}(e^x + e^{x/3})$ $\quad$ (b) Curve sketched, increasing from (0, 1)	A1; B3	11
6.	(a) Points shown at (2, –3) and (8, 14)	B1 B1	
	(b) $z_2 = (8 + 14\mathrm{i})/(2 - 3\mathrm{i}) = (-26 + 52\mathrm{i})/13 = -2 + 4\mathrm{i}$ $\quad a = -2,\ b = 4$	M1 M1 A1 A1	
	(c) $\lvert z_1z_2 \rvert = \sqrt{260},\ \lvert z_1 \rvert = \sqrt{13},\ \lvert z_2 \rvert = \sqrt{20}$, so $\lvert z_1z_2 \rvert = \lvert z_1 \rvert\, \lvert z_2 \rvert$	M1 A1 A1	
	(d) $z_2z_2^* = \lvert z_2 \rvert^2 = 16 + 4 = 20$	M1 A1	11
7.	(a) (i) $\frac{dx}{dt} = e^t$ $\quad \frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{1}{e^t}\frac{dy}{dt} = e^{-t}\frac{dy}{dt}$	M1 A1	
	(ii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{dy}{dx}\right)\frac{dt}{dx} = \frac{d}{dt}\left(e^{-t}\frac{dy}{dt}\right)\frac{dt}{dx}$	M1 A1 A1	
	$= e^{-t}\left(e^{-t}\frac{d^2y}{dt^2} - e^{-t}\frac{dy}{dt}\right) = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$	M1 A1	
	(b) $e^{2t}e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) - e^te^{-t}\frac{dy}{dt} + y = 0$ $\quad \frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 0$	M1 A1 A1	
	Aux. eqn $u^2 - 2u + 1 = 0$ has root $u = 1$, so $y = (at + b)e^t$	M1 A1	
	$t = \ln x$, so $y = x(a \ln x + b)$	A1	13
8.	(a) Curves sketched	B2 B2	
	(b) When $a\cos 2\theta = 2a\sin^2\theta$, $\ 1 - 2\sin^2\theta = 2\sin^2\theta$ $\quad \sin^2\theta = 1/4$	M1 A1 A1	
	In given range, $\sin\theta = 1/2$ $\quad$ Point is $(\pi/6,\ a/2)$	M1 A1	
	(c) Area $= \frac{1}{2}\int_0^{\pi/4} a^2\cos^2 2\theta\ d\theta = \frac{a^2}{4}\int_0^{\pi/4} 1 + \cos 4\theta\ d\theta$	M1 A1 M1 A1	
	$= \frac{a^2}{16}[4\theta + \sin 4\theta]_0^{\pi/4} = \frac{\pi a^2}{4}$	A1 M1 A1	16