GCE Examinations

Advanced Subsidiary / Advanced Level

Mechanics
Module M3

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

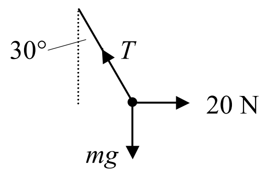
Accuracy marks (A) can only be awarded when a correct method has been used.

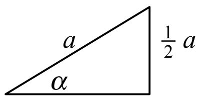
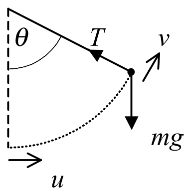
(B) marks are independent of method marks.

Written by Shaun Armstrong & Chris Huffer

M3 Paper B – Marking Guide

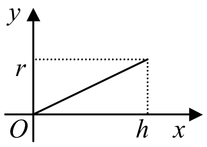
1.	*(a)*	$s = e^{0.3} - 1 = 0.3499$ m = 35 cm (nearest cm)	M1 A1	
	(b)	$v = \frac{ds}{dt} = 3e^{3t}$	M1 A1	
		when $t = 0$, $v = 3$ ms^{-1}	A1	
	(c)	$a = \frac{dv}{dt} = 9e^{3t}$ ms^{-2}	A1	
	(d)	e.g. model predicts increasing accel., more likely to be decreasing	B1	**(7)**

2.	*(a)*			
		resolve →: $20 - T\sin 30° = 0 \therefore T = 40$ N	M1 A2	
	(b)	let natural length be $l \therefore T = \frac{\lambda x}{l} = \frac{80(1.2-l)}{l}$	M1 A1	
		$T = 40 \therefore l = 2(1.2 - l)$ giving $l = 0.8$ m	A1	
		EPE $= \frac{\lambda x^2}{2l} = \frac{80 \times 0.4^2}{2 \times 0.8} = 8$ J	M1 A1	**(8)**

3.	*(a)*	 $\sin\alpha = \frac{1}{2} \therefore \alpha = 30°$	M1	
		$\angle BOA = 90 + \alpha = 120°$	A1	
	(b)	 resolve ↖: $T - mg\cos\theta = \frac{mv^2}{r}$	M1	
		at B, $T = 0$, $\theta = 120° \therefore v^2 = \frac{1}{2}ga$	A1	
		con. of ME: $\frac{1}{2}m(u^2 - v^2) = mg \times \frac{3}{2}a$	M1	
		$\therefore v^2 = u^2 - 3ga$	A1	
		combining, $\frac{1}{2}ga = u^2 - 3ga \therefore u^2 = \frac{7}{2}ga$	M1 A1	**(8)**

4.	*(a)*	amplitude $= \frac{1}{2}(14 - 6) = 4$ m	B1	
		period $= 2 \times 6\frac{1}{4} = 12\frac{1}{2}$ hours	B1	
	(b)	$x = a\cos\omega t \therefore {}^{-}1 = 4\cos\omega t$	M1 A1	
		$\cos\omega t = -\frac{1}{4} \therefore \omega t = 1.8235, \ldots$	A1	
		period $= \frac{2\pi}{\omega} = 12\frac{1}{2} \therefore \omega = \frac{4\pi}{25}$	M1	
		giving $t = 1.8235 \div \frac{4\pi}{25} = 3.6277$ hours	A1	
		depth 9 m at 11 am + 3.6277 hours = 2.38 pm (nearest min.)	A1	
	(c)	depth 9 m again when $\omega t = 2\pi - 1.8235 = 4.4597$	M1	
		$t = 4.4597 \div \frac{4\pi}{25} = 8.8723$	A1	
		wait until 11 am + 8.8723 hours = 7.52 pm	M1	
		$\therefore$ 2 hours 52 min. wait (nearest min.)	A1	**(12)**

5. *(a)*



let $y = \frac{r}{h}x$, ρ = mass per unit volume — M1

$\therefore\ \frac{1}{3}\rho\pi r^2 h\ \bar{x} = \int_0^h \rho\pi y^2 x\ \mathrm{d}x$ — A1

$\frac{1}{3}r^2 h\ \bar{x} = \int_0^h \frac{r^2}{h^2}x^3\ \mathrm{d}x = \frac{r^2}{h^2}\left[\frac{x^4}{4}\right]_0^h$ — M1 A1

$\frac{1}{3}r^2 h\ \bar{x} = \frac{1}{4}r^2 h^2\ \ \therefore\ \bar{x} = \frac{3}{4}h$ — M1 A1

(b)

portion	mass	y	my
cone	$\rho\times\frac{1}{3}\pi r^2\times r = \frac{1}{3}\rho\pi r^3$	$\frac{3}{4}r$	$\frac{1}{4}\rho\pi r^4$
hemisphere	$\frac{1}{2}\times\rho\times\frac{4}{3}\pi r^3 = \frac{2}{3}\rho\pi r^3$	$r + \frac{3}{8}r = \frac{11}{8}r$	$\frac{11}{12}\rho\pi r^4$
paperweight	$\rho\pi r^3$	$\bar{y}$	$\frac{7}{6}\rho\pi r^4$

ρ = mass per unit volume $\quad$ y coords. taken vert. from vertex — M2 A3

$\rho\pi r^3\times\bar{y} = \frac{7}{6}\rho\pi r^4\ \ \therefore\ \bar{y} = \frac{7}{6}r$ — M1 A1 **(13)**

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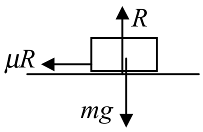
6. *(a)*



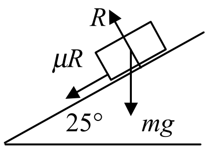
resolve $\uparrow$: $R - mg = 0,\ R = mg$ — M1

resolve $\leftarrow$: $\mu R = \frac{mv^2}{r},\ \ \frac{2}{5}R = \frac{mv^2}{40}$ — M1 A1

combining, $\frac{2}{5}mg = \frac{mv^2}{40}$ — M1

$\therefore\ v^2 = \frac{2}{5}g\times 40$ giving $v = 12.5\ \mathrm{ms}^{-1}$ (3sf) — A1

(b)



resolve $\uparrow$: $R\cos 25 - \mu R\sin 25 - mg = 0$ — M1 A1

$R = \frac{mg}{\cos 25 - \frac{2}{5}\sin 25}$

resolve $\leftarrow$: $R\sin 25 + \mu R\cos 25 = \frac{mv^2}{r}$ — M1 A1

combining, $\frac{mg(\sin 25 + \frac{2}{5}\cos 25)}{\cos 25 - \frac{2}{5}\sin 25} = \frac{mv^2}{40}$ — M1

giving $v^2 = \frac{40g(\sin 25 + \frac{2}{5}\cos 25)}{\cos 25 - \frac{2}{5}\sin 25}\ \ \therefore\ v = 20.43$ — A1

% increase $= \frac{20.43 - 12.52}{12.52}\times 100\% = 63\%$ (nearest WN) — M1 A1 **(13)**

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7. *(a)* $a = v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{2}{x+1}\times\frac{-2}{(x+1)^2} = \frac{-4}{(x+1)^3}$ — M2 A2

(b) $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{x+1}\ \ \therefore\ \int (x+1)\ \mathrm{d}x = \int 2\ \mathrm{d}t$ — M2

$\frac{1}{2}x^2 + x = 2t + c$ — A1

$t = 0, x = 0\ \ \therefore\ c = 0$ so $x^2 + 2x = 4t$ — M1

$t = T, x = d\ \ \therefore\ d^2 + 2d = 4T$ — A1

$t = T + 9, x = d + 4\ \ \therefore\ (d+4)^2 + 2(d+4) = 4(T+9)$ — M1

combining, $d^2 + 8d + 16 + 2d + 8 = d^2 + 2d + 36$ — M1

giving $8d = 12$ so $d = 1.5$ — A1

(c) $(1.5)^2 + 2(1.5) = 4T$ giving $T = \frac{21}{16}$ or 1.3125 s — M1 A1 **(14)**

Total **(75)**

Performance Record – M3 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	variable accel.	elastic string, EPE	motion in a vertical circle	SHM	centre of mass by integr., composite body	circular motion, banked track	variable accel.	
Marks	7	8	8	12	13	13	14	75
Student								