1. Show that the matrix $\mathbf{A} = \begin{pmatrix} 2 & 4 \\ -1 & 6 \end{pmatrix}$ has only one eigenvalue, and find an eigenvector of **A**. **(6 marks)**

2. Prove by induction that $3^n > 3n$ for all integers n greater than 1. **(6 marks)**

3. By expressing arsinh x in terms of natural logarithms, or otherwise, find the first non-zero term in the Maclaurin series expansion of arsinh x in ascending powers of x. **(6 marks)**

4. A complex number z satisfies the equation $\arg\left(\frac{z-i}{z+1}\right) = \frac{\pi}{4}$.

   Show that the locus of the point representing z is a circle. Find the centre and radius of this circle and sketch it in an Argand diagram. **(8 marks)**

5. (a) Find, in terms of k, the inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 0 \\ 2 & k & 2 \\ -3 & 4 & 0 \end{pmatrix}.$$ **(6 marks)**

   (b) Hence or otherwise solve for x, y and z the equations
   $-x + 2y = 3,\ 2x + ky + 2z = 5,\ -3x + 4y = 7.$ **(3 marks)**

6. The equation of a straight line l in 3-dimensional space is $(\mathbf{r} - \mathbf{i} - \mathbf{j}) \times (3\mathbf{i} + 4\mathbf{k}) = \mathbf{0}$.
   (a) Find two vectors **r** which satisfy this equation. **(3 marks)**
   (b) Hence or otherwise find the equation of l in the parametric form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$. **(2 marks)**
   (c) Find a cartesian equation of the plane which contains l and the origin. **(5 marks)**

7. Given that $\frac{d^2y}{dx^2} = x^2 - y^2$, and that when $x = 0$, $y = 1$ and $\frac{dy}{dx} = -1$,

   (a) obtain a series expansion for y in ascending powers of x as far as the term in x^4. **(8 marks)**

   (b) Find an approximate value of y when $x = 0{\cdot}05$, giving your answer to 4 decimal places. **(2 marks)**

   (c) Use the result $\left(\frac{d^2y}{dx^2}\right)_0 \approx \frac{y_1 - 2y_0 + y_{-1}}{h^2}$ with $h = 0{\cdot}05$ and the value obtained in (b) for y_0, to estimate the value of y, to three decimal places, when $x = 0{\cdot}1$. **(4 marks)**

8. (a) Use de Moivre's theorem to prove that $\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ and express $\sin 4\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$. **(6 marks)**

   (b) Deduce that $\cot 4\theta = \frac{\cot^4\theta - 6\cot^2\theta + 1}{4\cot\theta(\cot^2\theta - 1)}$. **(3 marks)**

   (c) Hence find the possible values of $\cot\theta$ when $\cot 4\theta = 0$. Deduce the exact value of $\cot\frac{\pi}{8}$. **(7 marks)**