Practice Paper A

CORE FOUR

crashMATHS

Name	

Duration	1 HOUR & 30 MINUTES
Total Marks Available	75 MARKS

Targets

For examiner's use only

Question Number	*Leave Blank*
1	
2	
3	
4	
5	
6	
7	
8	
/	
Total marks	

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1 By expressing $x^3 - 2x^2 - 5x + 6$ as a product of three linear factors, or otherwise,

express

$$\frac{2x+4}{x^3 - 2x^2 - 5x + 6}$$

as partial fractions. **(7)**

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Question 1 continued

TOTAL 7 MARKS

2 It is given that

$$f(x)=\frac{1}{\sqrt{4+x}}\ ,\ x>-4$$

(a) Find, in ascending powers of x, the first three terms in the binomial expansion of $f(x)$. **(5)**

(b) Using your expansion, estimate the value of $\sqrt{7}$. **(3)**

Given also that $\sqrt{7}\cong 2.645751$,

(c) State the number of significant figures to which your estimate is correct. **(1)**

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Question 2 continued

TOTAL 9 MARKS

3 (a) Show that

$$\int_0^{\frac{\pi}{4}} \tan x \, dx = \ln\left(\sqrt{2}\right)$$

(4)

(b) Hence, or otherwise, using the substitution $x = \sin u$, evaluate

$$\int_0^{\frac{\sqrt{2}}{2}} \frac{5x}{2\sqrt{1-x^2}} \, dx$$

(4)

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Question 3 continued

TOTAL 8 MARKS

4 (a) Using logarithims, prove that

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$ **(3)**

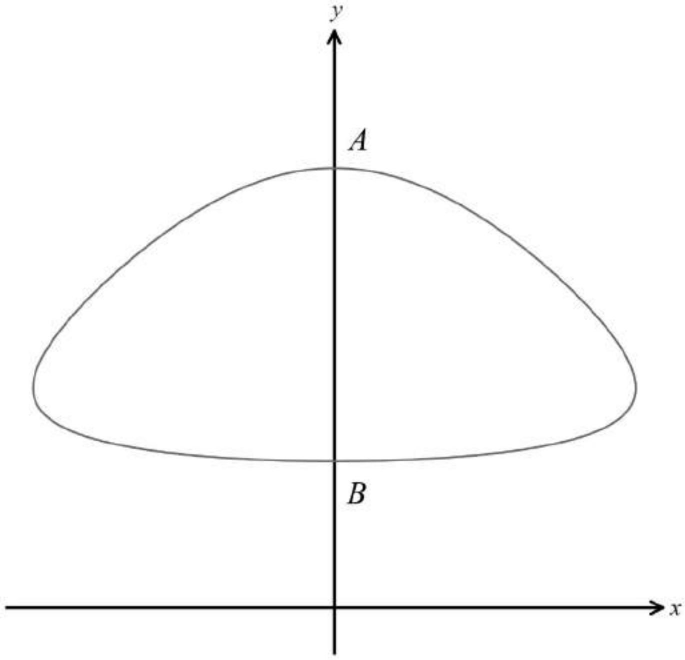
(b) Differentiate, with respect to x,

$$2^x - 2y^2 = xy$$ **(4)**

Question 4 continued

TOTAL 7 MARKS

5 The diagram below shows a sketch of the curve C.



The curve C is defined such that

$$x = \sin 3t\,,\ y\cos 3t + 2 = 2y\,,\ 0 \le t \le 2\pi$$

where t is a parameter.

(a) Find the coordinates of A and B. **(3)**

(b) Show that

$$\frac{dy}{dx} = -\frac{2\tan 3t}{(2-\cos 3t)^2}$$ **(5)**

(c) Find the equation of the tangent to C when $x = \frac{\sqrt{3}}{2}$. **(4)**

(d) Hence, state the equation of the tangent to C when $x = -\frac{\sqrt{3}}{2}$. **(1)**

The tangent to C at $x = \frac{\sqrt{3}}{2}$ crosses the x axis at the point P.

The tangent to C at $x = -\frac{\sqrt{3}}{2}$ crosses the x axis at the point Q.

(e) Verify that $AP = AQ$. **(4)**

Question 5 continued

Question 5 continued

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Question 5 continued

TOTAL 17 MARKS

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6 Two lines l_1 and l_2 are perpendicular and are defined such that

$$l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ a \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} b \\ 9 \\ 0 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -2 \\ 1 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ a \\ 4 \end{pmatrix}$$

where λ and μ are scalar parameters.

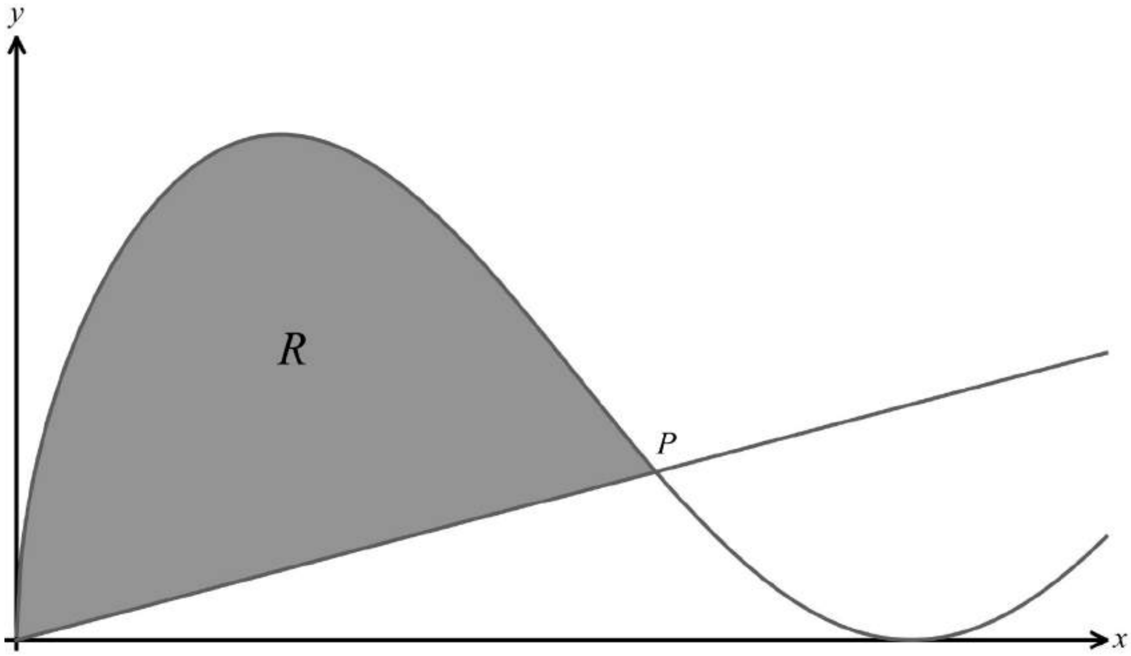
The two lines intersect at the point M with position vector $\frac{1}{4}\begin{pmatrix} -11 \\ 3 \\ 20 \end{pmatrix}$.

Find the values of a and b. **(6)**

Question 6 continued

TOTAL 6 MARKS

7 *[In this question, you may use the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone, if necessary.]*



The curve C has equation $y = \cos^2 x\sqrt{\sin x}$ and intersects the line l at the point P.

The line l has equation $y = \frac{9\sqrt{2}}{4\pi}x$

(a) Verify that P has coordinates $\left(\frac{\pi}{6}, \frac{3\sqrt{2}}{8}\right)$ **(2)**

The finite region R, as shown in the figure above, is bounded by the curve C, the x axis, the y axis and the line l. The shaded region is then rotated 2π radians about the x axis to form a solid of revolution.

(b) Find the volume of this solid of revolution. **(8)**

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Question 7 continued

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Question 7 continued

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Question 7 continued

TOTAL 10 MARKS

8 A microbiologist is going to conduct an experiment that involves growing a culture of bacteria. She assumes that the number of strands of bacteria, x, present in the culture t days after growth is increasing at a rate that is proportional to the number of strands present in the culture. She is going to use an initial culture containing 450 strands of bacteria. Previous research data shows that after 4 days, there will be 2103 strands of bacteria present.

(a) By forming and solving a differential equation, estimate the number of strands that will be present in her culture after a week. **(8)**

(b) Sketch a curve that shows the rate of increase of the number of bacterial strands in the culture. On your sketch, you should clearly the coordinates at which the curve crosses or meets the coordinate axis. **(3)**

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Question 8 continued

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Question 8 continued

Question 8 continued

TOTAL 11 MARKS