Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Gold Level (Harder) G2

Time: 1 hour 30 minutes

Materials required for examination   **Items included with question papers**
Mathematical Formulae (Green)   Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
65	58	47	42	36	28

1. $$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(*a*) Find the values of the constants *A*, *B* and *C*.

(4)

(*b*) (i) Hence find $\int f(x) \; dx$.

(ii) Find $\int_1^2 f(x) \; dx$, leaving your answer in the form *a* + ln *b*, where *a* and *b* are constants.

(6)

June 2012

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2. The current, *I* amps, in an electric circuit at time *t* seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{dI}{dt}$ when *t* = 3 .

Give your answer in the form ln *a*, where *a* is a constant.

(5)

January 2011

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3. (*a*) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \qquad |x| < 1$$

(6)

(*b*) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to √3.

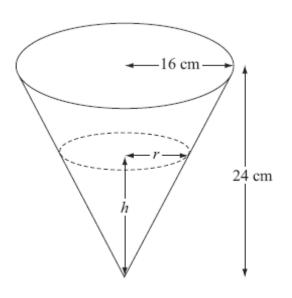Give your answer in the form $\frac{a}{b}$ where *a* and *b* are integers.

(3)

June 2013

2

4. Given that $y = 2$ at $x = \dfrac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}.$$

(5)

June 2012

5.

Figure 2

A container is made in the shape of a hollow inverted right circular cone. The height of the container is 24 cm and the radius is 16 cm, as shown in Figure 2. Water is flowing into the container. When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3.

(*a*) Show that $V = \dfrac{4\pi h^3}{27}$.

(2)

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula $V = \dfrac{1}{3}\pi r^2 h$.*]

Water flows into the container at a rate of 8 cm^3 s^{-1}.

(*b*) Find, in terms of π, the rate of change of h when $h = 12$.

(5)

January 2009

6.   (*a*) Find $\displaystyle\int \tan^2 x \ \mathrm{d}x$.

(2)

(*b*) Use integration by parts to find $\displaystyle\int \frac{1}{x^3} \ln x \ \mathrm{d}x$.

(4)

(*c*) Use the substitution $u = 1 + e^x$ to show that

$$\int \frac{e^{3x}}{1+e^x} \ \mathrm{d}x \ = \ \frac{1}{2} e^{2x} - e^x + \ln(1 + e^x) + k,$$

where k is a constant.

(7)

January 2009

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7. (*a*) Express $\dfrac{2}{4-y^2}$ in partial fractions.

(3)

(*b*) Hence obtain the solution of

$$2 \cot x \, \frac{\mathrm{d}y}{\mathrm{d}x} = (4 - y^2)$$

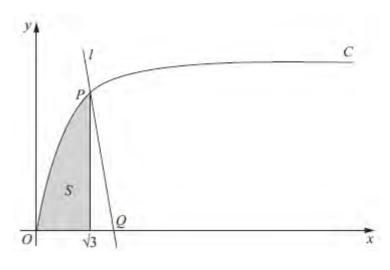for which $y = 0$ at $x = \dfrac{\pi}{3}$, giving your answer in the form $\sec^2 x = \mathrm{g}(y)$.

(8)

June 2008

8.

Figure 3

Figure 3 shows part of the curve C with parametric equations

$$x = \tan\theta, \qquad y = \sin\theta, \qquad 0 \le \theta < \frac{\pi}{2}.$$

The point P lies on C and has coordinates $\left(\sqrt{3}, \frac{1}{2}\sqrt{3}\right)$.

(*a*) Find the value of θ at the point P.

(2)

The line l is a normal to C at P. The normal cuts the x-axis at the point Q.

(*b*) Show that Q has coordinates $(k\sqrt{3}, 0)$, giving the value of the constant k.

(6)

The finite shaded region S shown in Figure 3 is bounded by the curve C, the line $x = \sqrt{3}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*c*) Find the volume of the solid of revolution, giving your answer in the form $p\pi\sqrt{3} + q\pi^2$, where p and q are constants.

(7)

June 2011

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$$1 = A(3x-1)^2 + Bx(3x-1) + Cx$$	B1
	$x \to 0 \qquad\qquad (1 = A)$	M1
	$x \to \frac{1}{3} \qquad\qquad 1 = \frac{1}{3}C \implies C = 3 \qquad\qquad$ any two constants correct	A1
	Coefficients of x^2	
	$\qquad\qquad\qquad 0 = 9A + 3B \implies B = -3 \qquad$ all three constants correct	A1 **(4)**
(b) (i)	$$\int \left(\frac{1}{x} - \frac{3}{3x-1} + \frac{3}{(3x-1)^2} \right) dx$$	
	$$= \ln x - \frac{3}{3}\ln(3x-1) + \frac{3}{(-1)3}(3x-1)^{-1} \quad (+C)$$	M1 A1ft A1ft
	$$\left(= \ln x - \ln(3x-1) - \frac{1}{3x-1} \quad (+C) \right)$$	
(ii)	$$\int_1^2 f(x)\,dx = \left[\ln x - \ln(3x-1) - \frac{1}{3x-1} \right]_1^2$$	
	$$= \left(\ln 2 - \ln 5 - \frac{1}{5} \right) - \left(\ln 1 - \ln 2 - \frac{1}{2} \right)$$	M1
	$$= \ln \frac{2 \times 2}{5} + \dots$$	M1
	$$= \frac{3}{10} + \ln\left(\frac{4}{5} \right)$$	A1 **(6)**
		[10]

2.	$$\frac{dI}{dt} = -16\ln(0.5)0.5^t$$	M1 A1
	At $t = 3 \qquad\qquad \dfrac{dI}{dt} = -16\ln(0.5)0.5^3$	M1
	$$= -2\ln 0.5 = \ln 4$$	M1 A1
		[5]

Question Number	Scheme		Marks
3. (a)	$\left\{\sqrt{\left(\dfrac{1+x}{1-x}\right)}\right\} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	$(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	B1
	$= \left(1+\left(\dfrac{1}{2}\right)x+\dfrac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2+\ldots\right)\times\left(1+\left(-\dfrac{1}{2}\right)(-x)+\dfrac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(-x)^2+\ldots\right)$		M1 A1 A1
	$= \left(1+\dfrac{1}{2}x-\dfrac{1}{8}x^2+\ldots\right)\times\left(1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+\ldots\right)$		
	$= 1+\dfrac{1}{2}x+\dfrac{3}{8}x^2+\dfrac{1}{2}x+\dfrac{1}{4}x^2-\dfrac{1}{8}x^2+\ldots$		M1
	$= 1+x+\dfrac{1}{2}x^2$	*Answer is given in the question.*	A1 *
			(6)
(b)	$\sqrt{\left(\dfrac{1+\left(\frac{1}{26}\right)}{1-\left(\frac{1}{26}\right)}\right)} = 1+\left(\dfrac{1}{26}\right)+\dfrac{1}{2}\left(\dfrac{1}{26}\right)^2$		M1
	ie: $\dfrac{3\sqrt{3}}{5} = \dfrac{1405}{1352}$		B1
	so, $\sqrt{3} = \dfrac{7025}{4056}$	$\dfrac{7025}{4056}$	A1 **cao**
			(3)
			[9]

4.	$\int y\,dy = \int \dfrac{3}{\cos^2 x}\,dx$	Can be implied. Ignore integral signs	B1	
	$= \int 3\sec^2 x\,dx$			
	$\dfrac{1}{2}y^2 = 3\tan x \quad (+C)$		M1 A1	
	$y=2, x=\dfrac{\pi}{4}$			
	$\dfrac{1}{2}2^2 = 3\tan\dfrac{\pi}{4}+C$		M1	
	Leading to			
	$C=-1$			
	$\dfrac{1}{2}y^2 = 3\tan x -1$	or equivalent	A1	**(5)**
				[5]

Question Number	Scheme		Marks
5 (a)	Similar triangles $\Rightarrow \dfrac{r}{h} = \dfrac{16}{24} \Rightarrow r = \dfrac{2h}{3}$	Uses similar triangles, ratios or trigonometry to find either one of these two expressions oe.	M1
	$V = \dfrac{1}{3}\pi r^2 h = \dfrac{1}{3}\pi \left(\dfrac{2h}{3}\right)^2 h = \dfrac{4\pi h^3}{27}$ **AG**	Substitutes $r = \frac{2h}{3}$ into the formula for the volume of water V.	A1
			(2)
(b)	From the question, $\dfrac{\mathrm{d}V}{\mathrm{d}t} = 8$	$\dfrac{\mathrm{d}V}{\mathrm{d}t} = 8$	B1
	$\dfrac{\mathrm{d}V}{\mathrm{d}h} = \dfrac{12\pi h^2}{27} = \dfrac{4\pi h^2}{9}$	$\dfrac{\mathrm{d}V}{\mathrm{d}h} = \dfrac{12\pi h^2}{27}$ or $\dfrac{4\pi h^2}{9}$	B1
		Candidate's $\dfrac{\mathrm{d}V}{\mathrm{d}t} \div \dfrac{\mathrm{d}V}{\mathrm{d}h}$;	M1;
	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t} \div \dfrac{\mathrm{d}V}{\mathrm{d}h} = 8 \times \dfrac{9}{4\pi h^2} = \dfrac{18}{\pi h^2}$	$8 \div \left(\dfrac{12\pi h^2}{27}\right)$ or $8 \times \dfrac{9}{4\pi h^2}$ or $\dfrac{18}{\pi h^2}$	A1
		oe	
	When $h = 12$, $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{18}{144\pi} = \dfrac{1}{8\pi}$	$\dfrac{18}{144\pi}$ or $\dfrac{1}{8\pi}$	A1 oe isw
	Note the answer must be a one term exact value. Note, also you can ignore subsequent working after $\dfrac{18}{144\pi}$.		(5)
			[7]

Question Number	Scheme	Marks
6 (a)	$\int \tan^2 x \, dx$ $\left[NB : \underline{\sec^2 A = 1 + \tan^2 A} \text{ gives } \underline{\tan^2 A = \sec^2 A - 1} \right]$ The correct <u>underlined identity</u>. $\quad = \int \sec^2 x - 1 \, dx$ $\quad = \underline{\tan x - x} \, (+c)$ Correct integration with/without + c	M1 oe A1 (2)
(b)	$\int \dfrac{1}{x^3} \ln x \, dx$ $\begin{cases} u = \ln x & \Rightarrow & \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = x^{-3} & \Rightarrow & v = \frac{x^{-2}}{-2} = \frac{-1}{2x^2} \end{cases}$ $= -\dfrac{1}{2x^2} \ln x - \int -\dfrac{1}{2x^2} \cdot \dfrac{1}{x} \, dx$ Use of 'integration by parts' formula in the correct direction. Correct direction means that $u = \ln x$. Correct expression. $= -\dfrac{1}{2x^2} \ln x + \dfrac{1}{2} \int \dfrac{1}{x^3} \, dx$ $= \underline{-\dfrac{1}{2x^2} \ln x + \dfrac{1}{2}\left(-\dfrac{1}{2x^2}\right)} \, (+c)$ An attempt to multiply through $\dfrac{k}{x^n}, n \in \mathbb{Z}, \; n \geqslant 2$ by $\frac{1}{x}$ and an attempt to ... ... "integrate"(process the result); <u>correct solution</u> with/without + c	M1 A1 M1 A1 oe (4)

7.	(a)	$\dfrac{2}{4-y^2} \equiv \dfrac{2}{(2-y)(2+y)} \equiv \dfrac{A}{(2-y)} + \dfrac{B}{(2+y)}$ so $2 \equiv A(2+y) + B(2-y)$	M1
		Let $y=-2$, $2 = B(4) \Rightarrow B = \frac{1}{2}$, Let $y=2$, $2 = A(4) \Rightarrow A = \frac{1}{2}$	M1
		giving $\dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}$	A1 cao (3)
	(b)	$\displaystyle\int \dfrac{2}{4-y^2}\,\mathrm{d}y = \int \dfrac{1}{\cot x}\,\mathrm{d}x$	B1
		$\displaystyle\int \dfrac{\frac{1}{2}}{(2-y)} + \dfrac{\frac{1}{2}}{(2+y)}\,\mathrm{d}y = \int \tan x\,\mathrm{d}x$	
		$\therefore -\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) + (c)$	B1 M1 A1 ft
		$y=0, x=\frac{\pi}{3} \Rightarrow -\frac{1}{2}\ln 2 + \frac{1}{2}\ln 2 = \ln\left(\frac{1}{\cos\left(\frac{\pi}{3}\right)}\right) + c$	M1
		$\left\{0 = \ln 2 + c \Rightarrow \underline{c = -\ln 2}\right\}$	
		$-\frac{1}{2}\ln(2-y) + \frac{1}{2}\ln(2+y) = \ln(\sec x) - \ln 2$	
		$\dfrac{1}{2}\ln\left(\dfrac{2+y}{2-y}\right) = \ln\left(\dfrac{\sec x}{2}\right)$	M1
		$\ln\left(\dfrac{2+y}{2-y}\right) = 2\ln\left(\dfrac{\sec x}{2}\right)$	
		$\ln\left(\dfrac{2+y}{2-y}\right) = \ln\left(\dfrac{\sec x}{2}\right)^2$	M1
		$\dfrac{2+y}{2-y} = \dfrac{\sec^2 x}{4}$	
		Hence, $\underline{\sec^2 x = \dfrac{8+4y}{2-y}}$	A1 (8)
			(11 marks)

Question Number	Scheme	Marks
8.	(a) $\quad \tan\theta = \sqrt{3} \ \ or \ \ \sin\theta = \dfrac{\sqrt{3}}{2}$	M1
	$\qquad\qquad\qquad\qquad \theta = \dfrac{\pi}{3}$ $\qquad\qquad$ awrt 1.05	A1 **(2)**
	(b) $\qquad\qquad\qquad \dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \sec^2\theta, \ \ \dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \cos\theta$	
	$\qquad\qquad\qquad\qquad \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cos\theta}{\sec^2\theta} \ \ \left(=\cos^3\theta\right)$	M1 A1
	At P, $\qquad\qquad\qquad m = \cos^3\left(\dfrac{\pi}{3}\right) = \dfrac{1}{8}$ $\qquad$ Can be implied	A1
	Using $mm' = -1$, $\qquad m' = -8$	M1
	For normal $\qquad y - \frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	M1
	At Q, $y = 0$ $\qquad -\frac{1}{2}\sqrt{3} = -8\left(x - \sqrt{3}\right)$	
	leading to $\qquad\qquad x = \frac{17}{16}\sqrt{3}$ $\qquad \left(k = \frac{17}{16}\right)$ $\qquad$ 1.0625	A1 **(6)**
	(c) $\quad \displaystyle\int y^2\,\mathrm{d}x = \int y^2\,\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\,\mathrm{d}\theta = \int \sin^2\theta\sec^2\theta\,\mathrm{d}\theta$	M1 A1
	$\qquad\qquad = \displaystyle\int \tan^2\theta\,\mathrm{d}\theta$	A1
	$\qquad\qquad = \displaystyle\int\left(\sec^2\theta - 1\right)\mathrm{d}\theta$	M1
	$\qquad\qquad = \tan\theta - \theta \ \ \left(+C\right)$	A1
	$V = \pi\displaystyle\int_0^{\frac{\pi}{3}} y^2\,\mathrm{d}x = \left[\tan\theta - \theta\right]_0^{\frac{\pi}{3}} = \pi\left[\left(\sqrt{3} - \tfrac{\pi}{3}\right) - \left(0 - 0\right)\right]$	M1
	$\qquad = \sqrt{3}\pi - \frac{1}{3}\pi^2 \qquad \left(p = 1, q = -\frac{1}{3}\right)$	A1 **(7)**
		[15]

Question 1

The majority of candidates gained full marks in part (a) and it was better done than similar questions in previous years. Most obtained the identity $1 = A(3x-1)^2 + Bx(3x-1) + Cx$ and found A and C by making an appropriate substitution. Finding B proved more difficult and the error $B = 0$ was not uncommon. In part (b)(i), most could gain the method mark, recognising that the integral of $\dfrac{1}{x}$ was $\ln x$, and those with incorrect non-zero values of A, B and C were able to benefit from follow through marks in this part. There were, however, many errors when finding $\displaystyle\int \dfrac{-3}{3x-1}\,dx$ and $\displaystyle\int \dfrac{3}{(3x-1)^2}\,dx$; $-3 \ln (3x-1)$ and $3 \ln (x-1)^2$ respectively, being common errors. The majority could start part (b)(ii) and, if they had correct values of A, B and C, then full marks for the question were common.

Question 2

Those who knew, and often quoted, a formula of the form $\dfrac{d}{dx}(a^x) = a^x \ln a$ usually found this question straightforward. Those who did not, tried a number of methods and these were frequently incorrect. Errors seen included $-16t(0.5)^{t-1}$, $-16(0.5)\ln t$ and $8^t \ln t$. Nearly all candidates substituted $t = 3$ into their $\dfrac{dI}{dt}$ but a significant number of candidates failed to give their answer in the form $\ln a$, as required by the question, leaving their answer in the form $n \ln a$.

Question 3

This question discriminated well across all abilities, with about 52% of candidates gaining at least 6 of the 9 marks available and about 20% of candidates gaining all 9 marks.

In part (a), the most popular method was to rewrite $\sqrt{\left(\dfrac{1+x}{1-x}\right)}$ as $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ and achieve the result $1 + x + \dfrac{1}{2}x^2$ by multiplying out the binomial expansion of $(1+x)^{\frac{1}{2}}$ with the binomial expansion of $(1-x)^{-\frac{1}{2}}$. Some candidates, however, were not able to formulate a strategy for expressing $\sqrt{\left(\dfrac{1+x}{1-x}\right)}$ in a form so that relevant binomial expansions could be applied. The most common mistake was to express $\sqrt{\left(\dfrac{1+x}{1-x}\right)}$ as $\dfrac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$ and then try to divide the two expressions, once expanded, but without success.

Although many candidates had written $(1 + x)^{\frac{1}{2}}(1 - x)^{-\frac{1}{2}}$, a significant number did not attempt to multiply the two resulting expansions together, with several attempting to divide their expansions and some deciding to add their expansions after observing $\frac{1}{2}x$ both expansions. Many candidates were able to use a correct method for expanding a binomial expression of the form $(1 + x)^n$ with some making sign errors when simplifying. The majority of candidates, however, who recognised the need to multiply the two expressions did so successfully, showing sufficient working and ignoring higher powers of x, to produce the given result.

Examples of alternative methods seen from a few candidates in part (a) included:

- $\sqrt{\left(\dfrac{1 + x}{1 - x}\right)} = \sqrt{\dfrac{(1 + x)(1 - x)}{(1 + x)(1 - x)}} = \sqrt{\dfrac{(1 - x^2)}{(1 - x)^2}} = (1 - x^2)^{\frac{1}{2}}(1 - x)^{-1}$, etc.

or

- $\sqrt{\left(\dfrac{1 + x}{1 - x}\right)} = \sqrt{\dfrac{(1 + x)(1 + x)}{(1 - x)(1 + x)}} = (1 + x)(1 - x^2)^{-\frac{1}{2}}$, etc.

In part (b), the main obstacle to success was the lack of realisation that a substitution of $x = \dfrac{1}{26}$ must be made into both sides of $\sqrt{\left(\dfrac{1 + x}{1 - x}\right)} = 1 + x + \dfrac{1}{2}x^2$. A significant number substituted into the RHS alone, assuming the LHS was $\sqrt{3}$ and claimed that $\sqrt{3} \approx \dfrac{1405}{1352}$. Even when candidates did substitute $x = \dfrac{1}{26}$ correctly into both sides, many neglected to equate both sides, and so had little chance of figuring out how to proceed to estimate $\sqrt{3}$. Those that equated both sides usually achieved the correct estimate of $\dfrac{7025}{4036}$.

Few candidates ignored the instruction given in part (b) and equated $\sqrt{\left(\dfrac{1 + x}{1 - x}\right)}$ to $\sqrt{3}$, deduced the value of $x = \dfrac{1}{2}$ and substituted this into the RHS in order to find an estimate for $\sqrt{3}$. Historically there have been a number of past examination questions that have asked candidates to "use a suitable value of x" and presumably these candidates had decided to do just that.

Question 4

This question was not well done and over one quarter of the candidates gained no marks on this question. In all of the other questions on the paper fewer than 8% of the candidates gained no marks. Many did not recognise that the question required a separation of the variables and they could make no progress.

The main error in separation was obtaining, through faulty algebra, $\int \frac{1}{y} dy$ on one side of the equation. A quite unexpected error was to see those who had, correctly, obtained $\int y \, dy$ integrate this to $\ln y$. The only explanation of this appears to be that many questions on this topic do result in logarithms and candidates were following an expected pattern rather than actually solving the question set. Those who could deal with the y side of the equation often had trouble with $\int \frac{1}{\cos^2 x} dx$, not rewriting this as $\int \sec^2 x \, dx$ and proceeding to $\tan x$. Many long and fruitless attempts were made using a double angle formula. Those who did integrate sometimes left it there, not realising that it was necessary to evaluate a constant of integration. About one third of candidates did obtain full marks.

Question 5

A considerable number of candidates did not attempt part (a), but of those who did, the most common method was to use similar triangles to obtain $r = \frac{2h}{3}$ and substitute r into $V = \frac{1}{3}\pi r^2 h$ to give $V = \frac{4}{27}\pi h^3$. Some candidates used trigonometry to find the semi-vertical angle of the cone and obtained $r = \frac{2h}{3}$ from this. A few candidates correctly used similar shapes to compare volumes by writing down the equation $\dfrac{V}{\frac{1}{3}\pi (16)^2 24} = \left(\dfrac{h}{24}\right)^3$.

Part (b) discriminated well between many candidates who were able to gain full marks with ease and some candidates who were able to gain just the first one or two marks. Some incorrectly differentiated $V = \frac{1}{3}\pi r^2 h$ to give $\dfrac{dV}{dh} = \frac{1}{3}\pi r^2$. Most of the successful candidates used the chain rule to find $\dfrac{dh}{dt}$ by applying $\dfrac{dV}{dt} \div \dfrac{dV}{dh}$. The final answer $\dfrac{1}{8\pi}$ was sometimes carelessly written as $\frac{1}{8}\pi$. Occasionally, some candidates solved the differential equation $\dfrac{dV}{dt} = 8$ and equated their solution to $\dfrac{4\pi h^3}{27}$ and then found $\dfrac{dt}{dh}$ or differentiated implicitly to find $\dfrac{dh}{dt}$.

Question 6

In part (a), a surprisingly large number of candidates did not know how to integrate $\tan^2 x$. Examiners were confronted with some strange attempts involving either double angle formulae or logarithmic answers such as $\ln(\sec^2 x)$ or $\ln(\sec^4 x)$. Those candidates who realised that the needed the identity $\sec^2 x = 1 + \tan^2 x$ sometimes wrote it down incorrectly.

Part (b) was probably the best attempted of the three parts in the question. This was a tricky integration by parts question owing to the term of $\frac{1}{x^3}$, meaning that candidates had to be especially careful when using negative powers. Many candidates applied the integration by parts formula correctly and then went on to integrate an expression of the form $\frac{k}{x^3}$ to gain 3 out of the 4 marks available. A significant number of candidates failed to gain the final accuracy mark owing to sign errors or errors with the constants α and β in $\frac{\alpha}{x^2}\ln x + \frac{\beta}{x^2} + c$. A minority of candidates applied the by parts formula in the 'wrong direction' and incorrectly stated that $\frac{dv}{dx} = \ln x$ implied $v = \frac{1}{x}$.

In part (c), most candidates correctly differentiated the substitution to gain the first mark. A significant proportion of candidates found the substitution to obtain an integral in terms of u more demanding. Some candidates did not realise that e^{2x} and e^{3x} are $(e^x)^2$ and $(e^x)^3$ respectively and hence $u^2 - 1$, rather than $(u - 1)^2$ was a frequently encountered error seen in the numerator of the substituted expression. Fewer than half of the candidates simplified their substituted expression to arrive at the correct result of $\int \frac{(u-1)^2}{u}\, du$. Some candidates could not proceed further at this point but the majority of the candidates who achieved this result were able to multiply out the numerator, divide by u, integrate and substitute back for u. At this point some candidates struggled to achieve the expression required. The most common misconception was that the constant of integration was a fixed constant to be determined, and so many candidates concluded that $k = -\frac{3}{2}$. Many candidates did not realise that $-\frac{3}{2}$ when added to c combined to make another arbitrary constant k.

Question 7

In part (a), many candidates realised that they needed to factorise the denominator to give two linear factors, and usually proceeded to give a fully correct solution. A few candidates, however, thought that $4 - y^2$ was an example of a repeated linear factor and tried to split up their fraction up accordingly. Some candidates struggled with factorising $4 - y^2$ giving answers such as $(4+y)(4-y)$ or $(y+2)(y-2)$. The majority of candidates were able to write down the correct identity to find their constants, although a noticeable number of candidates, when solving $4A = 2$ found $A = 2$.

A significant minority of candidates who completed part (a) correctly made no attempt at part (b). About half of the candidates in part (b) were able to separate out the variables correctly. Many of these candidates spotted the link with part (a). It was pleasing that candidates who progressed this far were able to correctly integrate $\tan x$ and correctly find the two ln terms by integrating their partial fraction. Common errors at this point were integrating $\tan x$ to give $\sec^2 x$ and the sign error involved when integrating $\frac{K}{2-y}$. A significant number of candidates at this point did not attempt to find a constant of integration. Other candidates substituted $x = \frac{\pi}{3}$ and $y = 0$ into an integrated equation which did not contain a constant of integration. A majority of candidates who found the constant of integration struggled to simplify their equation down to an equation with a single ln term on each side. The most common error of these candidates was to believe that $\ln A + \ln B = \ln C$ implies $A + B = C$.

Of all the 8 questions, this was the most demanding in terms of a need for accuracy. Fewer than 10% of candidates were able to score all 11 marks in this question, although statistics show that about half of the candidates were able to score at least 5 marks.

Question 8

The majority of candidates were able to complete part (a), although some candidates gave the answer in degrees rather than radians. Most could start part (b) correctly and, apart from a few errors in sign, obtain $\dfrac{dy}{dx} = \dfrac{\cos\theta}{\sec^2\theta}$, although this was often simplified to $\cos\theta$ rather than the correct $\cos^3\theta$. The majority of candidates were able to demonstrate the correct method of finding the equation of the normal and to complete the part by substituting $y = 0$ and solving for x. A small number of candidates eliminated θ, successfully differentiated the cartesian equation and completed the question.

Part (c) proved challenging for many candidates and a substantial number of candidates thought that the volume was given by $\int \sin^2\theta$, often ignoring $d\theta$ or dx. Among those who recognised that the appropriate integral was $\int \sin^2\theta\sec^2\theta\,d\theta$, many were unable to rewrite this in a form which could be integrated. $\ln\sec\theta$ and $\sin^3\theta$ were among the erroneous attempts seen. Those who realised that $\sin^2\theta\sec^2\theta = \tan^2\theta = \sec^2\theta - 1$ usually completed the question correctly, although a few used x limits rather than θ limits. There are a number of possible alternative approaches to this question and there were some successful attempts using integration by parts. A number of candidates attempted to use the cartesian form of the equation but few of these were able to establish a method of integrating $\dfrac{x^2}{1+x^2}$.

Statistics for C4 Practice Paper G2

Qu	Max score	Modal score	Mean %	ALL	A*	A	B	C	D	E	U
1	10		75	7.47	9.68	8.85	7.82	6.62	5.31	3.97	2.22
2	5		57	2.83	4.71	3.30	2.12	1.36	0.94	0.90	0.67
3	9	9	59	5.28	8.16	6.52	5.28	4.26	3.4	2.61	1.64
4	5		57	2.86	4.81	4.10	3.00	1.70	0.81	0.30	0.08
5	7		47	3.26		5.18	2.52	1.20	0.65	0.20	0.11
6	13		55	7.18		10.02	5.96	3.82	2.50	1.38	0.40
7	11		46	5.05		7.44	4.70	3.12	2.10	1.42	0.83
8	15		52	7.86	14.16	10.39	7.32	4.95	3.34	2.04	1.10
	75		56	41.79		55.80	38.72	27.03	19.05	12.82	7.05

Header above table: **Mean score for students achieving grade:**