GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper K

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

1. *(a)* Find the exact value of x such that

$$3 \arctan (x - 2) + \pi = 0.$$ **(3)**

(b) Solve, for $-\pi < \theta < \pi$, the equation

$$\cos 2\theta - \sin \theta - 1 = 0,$$

giving your answers in terms of π. **(5)**

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2. *(a)* Express

$$\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$$

as a single fraction in its simplest form. **(4)**

(b) Simplify

$$\frac{x^3 - 8}{3x^2 - 8x + 4}.$$ **(5)**

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3. Differentiate each of the following with respect to x and simplify your answers.

(a) $\cot x^2$ **(2)**

(b) $x^2 e^{-x}$ **(3)**

(c) $\dfrac{\sin x}{3 + 2\cos x}$ **(4)**

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4. *(a)* Find, as natural logarithms, the solutions of the equation

$$e^{2x} - 8e^x + 15 = 0.$$ **(4)**

(b) Use proof by contradiction to prove that $\log_2 3$ is irrational. **(6)**

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5. The function f is defined by

$$f : x \to 3e^{x-1}, \quad x \in \mathbb{R}.$$

(a) State the range of f. **(1)**

(b) Find an expression for $f^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$g : x \to 5x - 2, \quad x \in \mathbb{R}.$$

Find, in terms of e,

(c) the value of $gf(\ln 2)$, **(3)**

(d) the solution of the equation

$$f^{-1}g(x) = 4.$$ **(4)**

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6. $\qquad\qquad\qquad f(x) = 2x^2 + 3 \ln (2 - x), \quad x \in \mathbb{R}, \quad x < 2.$

(a) Show that the equation $f(x) = 0$ can be written in the form

$$x = 2 - e^{kx^2},$$

where k is a constant to be found. **(3)**

The root, α, of the equation $f(x) = 0$ is 1.9 correct to 1 decimal place.
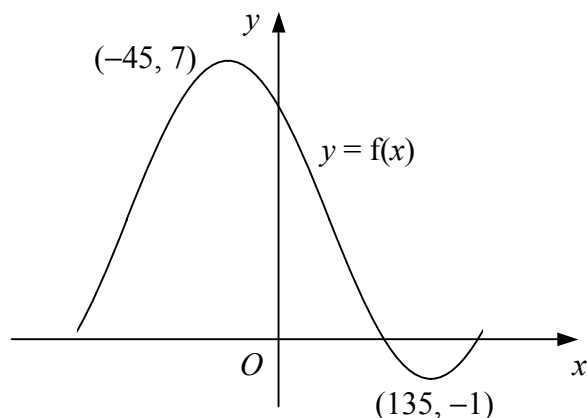
(b) Use the iteration formula

$$x_{n+1} = 2 - e^{kx_n^2},$$

with $x_0 = 1.9$ and your value of k, to find α to 3 decimal places and justify the accuracy of your answer. **(5)**

(c) Solve the equation $f'(x) = 0$. **(5)**

Turn over

7.

Figure 1

Figure 1 shows the curve $y = f(x)$ which has a maximum point at $(-45, 7)$ and a minimum point at $(135, -1)$.

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of

    (i) $y = f(|x|)$,

    (ii) $y = 1 + 2f(x)$.   **(6)**

Given that

$$f(x) = A + 2\sqrt{2} \cos x° - 2\sqrt{2} \sin x°, \quad x \in \mathbb{R}, \quad -180 \le x \le 180,$$

where A is a constant,

(b) show that $f(x)$ can be expressed in the form

$$f(x) = A + R \cos (x + \alpha)°,$$

    where $R > 0$ and $0 < \alpha < 90$,   **(3)**

(c) state the value of A,   **(1)**

(d) find, to 1 decimal place, the x-coordinates of the points where the curve $y = f(x)$ crosses the x-axis.   **(4)**

END