



Formulae and Statistical Tables for GCE Mathematics and GCE Statistics

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For the new specifications for first teaching from September 2004

GCE Mathematics

ADVANCED SUBSIDIARY MATHEMATICS (5361)
ADVANCED SUBSIDIARY PURE MATHEMATICS (5366)
ADVANCED SUBSIDIARY FURTHER MATHEMATICS (5371)

ADVANCED MATHEMATICS (6361)
ADVANCED PURE MATHEMATICS (6366)
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GCE Statistics

ADVANCED SUBSIDIARY STATISTICS (5381)
ADVANCED STATISTICS (6381)

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PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a + (n-1)d]$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Summations

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Trigonometry – the Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial Series

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1.2\dots r}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and exponentials

$$a^x = e^{x \ln a}$$

Complex numbers

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n(\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n-1$

Maclaurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln \left\{ x + \sqrt{x^2 - 1} \right\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln \left\{ x + \sqrt{x^2 + 1} \right\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Conics

	Ellipse	Parabola	Hyperbola	Rectangular hyperbola
Standard form	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	$y^2 = 4ax$	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$xy = c^2$
Asymptotes	none	none	$\frac{x}{a} = \pm \frac{y}{b}$	$x = 0, y = 0$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Vectors

The resolved part of $\mathbf{a}$ in the direction of $\mathbf{b}$ is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The position vector of the point dividing AB in the ratio $\lambda : \mu$ is $\frac{\mu \mathbf{a} + \lambda \mathbf{b}}{\lambda + \mu}$

$$\text{Vector product: } \mathbf{a} \times \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

If A is the point with position vector $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ and the direction vector $\mathbf{b}$ is given by $\mathbf{b} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}$, then the straight line through A with direction vector $\mathbf{b}$ has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} = \lambda$$

The plane through A with normal vector $\mathbf{n} = n_1 \mathbf{i} + n_2 \mathbf{j} + n_3 \mathbf{k}$ has cartesian equation

$$n_1 x + n_2 y + n_3 z = d \text{ where } d = \mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points A, B and C has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector $\mathbf{a}$ and parallel to $\mathbf{b}$ and $\mathbf{c}$ has equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$$

Matrix transformations

Anticlockwise rotation through θ about O : $\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$

The matrices for rotations (in three dimensions) through an angle θ about one of the axes are

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \text{ for the } x\text{-axis}$$

$$\begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} \text{ for the } y\text{-axis}$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for the } z\text{-axis}$$

Differentiation

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan kx$	$k \sec^2 kx$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Integration

(+ constant; $a > 0$ where relevant)

$f(x)$	$\int f(x) dx$
$\tan x$	$\ln \sec x $
$\cot x$	$\ln \sin x $
$\operatorname{cosec} x$	$-\ln \operatorname{cosec} x + \cot x = \ln\left \tan\left(\frac{1}{2}x\right)\right $
$\sec x$	$\ln \sec x + \tan x = \ln\left \tan\left(\frac{1}{2}x + \frac{1}{4}\pi\right)\right $
$\sec^2 kx$	$\frac{1}{k} \tan kx$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\ln \cosh x$

INTEGRATION FORMULAE CONTINUE OVER THE PAGE

$\frac{1}{\sqrt{a^2 - x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right) \quad (x < a)$
$\frac{1}{a^2 + x^2}$	$\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$
$\frac{1}{\sqrt{x^2 - a^2}}$	$\cosh^{-1}\left(\frac{x}{a}\right) \text{ or } \ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$
$\frac{1}{\sqrt{a^2 + x^2}}$	$\sinh^{-1}\left(\frac{x}{a}\right) \text{ or } \ln\{x + \sqrt{x^2 + a^2}\}$
$\frac{1}{a^2 - x^2}$	$\frac{1}{2a} \ln \left \frac{a+x}{a-x} \right = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \quad (x < a)$
$\frac{1}{x^2 - a^2}$	$\frac{1}{2a} \ln \left \frac{x-a}{x+a} \right $

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Area of a sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

Arc length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

Surface area of revolution

$$S_x = 2\pi \int y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$S_x = 2\pi \int y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

Numerical integration

The trapezium rule: $\int_a^b y dx \approx \frac{1}{2} h \{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The mid-ordinate rule: $\int_a^b y dx \approx h(y_{\frac{1}{2}} + y_{\frac{3}{2}} + \dots + y_{n-\frac{3}{2}} + y_{n-\frac{1}{2}})$, where $h = \frac{b-a}{n}$

Simpson's rule: $\int_a^b y dx \approx \frac{1}{3} h \{(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2})\}$

where $h = \frac{b-a}{n}$ and n is even

Numerical solution of differential equations

For $\frac{dy}{dx} = f(x)$ and small h , recurrence relations are:

Euler's method: $y_{n+1} = y_n + hf(x_n); \quad x_{n+1} = x_n + h$

For $\frac{dy}{dx} = f(x, y)$:

Euler's method: $y_{r+1} = y_r + hf(x_r, y_r)$

Improved Euler method: $y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$, where $k_1 = hf(x_r, y_r), k_2 = hf(x_r + h, y_r + k_1)$

Numerical solution of equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

MECHANICS

Motion in a circle

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

Centres of mass

For uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere, radius r : $\frac{3}{8}r$ from centre

Hemispherical shell, radius r : $\frac{1}{2}r$ from centre

Circular arc, radius r , angle at centre 2α : $\frac{r \sin \alpha}{\alpha}$ from centre

Sector of circle, radius r , angle at centre 2α : $\frac{2r \sin \alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h : $\frac{1}{4}h$ above the base on the line from centre of base to vertex

Conical shell of height h : $\frac{1}{3}h$ above the base on the line from centre of base to vertex

Moments of inertia

For uniform bodies of mass m

Thin rod, length $2l$, about perpendicular axis through centre: $\frac{1}{3}ml^2$

Rectangular lamina about axis in plane bisecting edges of length $2l$: $\frac{1}{3}ml^2$

Thin rod, length $2l$, about perpendicular axis through end: $\frac{4}{3}ml^2$

Rectangular lamina about edge perpendicular to edges of length $2l$: $\frac{4}{3}ml^2$

Rectangular lamina, sides $2a$ and $2b$, about perpendicular axis through centre: $\frac{1}{3}m(a^2 + b^2)$

MOMENTS OF INERTIA FORMULAE CONTINUE OVER THE PAGE

Hoop or cylindrical shell of radius r about axis: mr^2

Hoop of radius r about a diameter: $\frac{1}{2}mr^2$

Disc or solid cylinder of radius r about axis: $\frac{1}{2}mr^2$

Disc of radius r about a diameter: $\frac{1}{4}mr^2$

Solid sphere, radius r , about diameter: $\frac{2}{5}mr^2$

Spherical shell of radius r about a diameter: $\frac{2}{3}mr^2$

Parallel axes theorem: $I_A = I_G + m(AG)^2$

Perpendicular axes theorem: $I_z = I_x + I_y$ (for a lamina in the x - y plane)

General motion in two dimensions

Radial velocity $\dot{r}$

Transverse velocity $r\dot{\theta}$

Radial acceleration $\ddot{r} - r\dot{\theta}^2$

Transverse acceleration $r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta})$

Moments as vectors

The moment about O of $\mathbf{F}$ acting through the point with position vector $\mathbf{r}$ is $\mathbf{r} \times \mathbf{F}$

Universal law of gravitation

$$\text{Force} = \frac{Gm_1m_2}{d^2}$$

PROBABILITY and STATISTICS

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \times P(B | A)$$

$$P(A_j | B) = \frac{P(A_j) \times P(B | A_j)}{\sum_{i=1}^n P(A_i) \times P(B | A_i)}$$

Expectation algebra

$$\text{Covariance: } \text{Cov}(X, Y) = E((X - \mu_X)(Y - \mu_Y)) = E(XY) - \mu_X \mu_Y$$

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \pm 2ab \text{Cov}(X, Y)$$

$$\text{Product moment correlation coefficient: } \rho = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

For independent random variables X and Y

$$E(XY) = E(X)E(Y)$$

$$\text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Discrete distributions

For a discrete random variable X taking values x_i with probabilities p_i

Expectation (mean): $E(X) = \mu = \sum x_i p_i$

Variance: $\text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2 = E(X^2) - \mu^2$

For a function $g(X)$: $E(g(X)) = \sum g(x_i) p_i$

Standard discrete distributions:

Distribution of X	$P(X = x)$	Mean	Variance
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ
Geometric $Geo(p)$ on $1, 2, \dots$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$

Continuous distributions

For a continuous random variable X having probability density function $f(x)$

Expectation (mean): $E(X) = \mu = \int x f(x) dx$

Variance: $\text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2 = E(X^2) - \mu^2$

For a function $g(X)$: $E(g(X)) = \int g(x) f(x) dx$

Cumulative distribution function: $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$

Standard continuous distributions:

Distribution of X	Probability density function	Mean	Variance
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$

Sampling distributions

For a random sample $X_1, X_2, \dots, X_n$ of n independent observations from a distribution having mean μ and variance σ^2

$\bar{X}$ is an unbiased estimator of μ , with $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

S^2 is an unbiased estimator of σ^2 , where $S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1)$$

$$\frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$$

$$\frac{\bar{X} - \mu}{\frac{S}{\sqrt{n}}} \sim t_{n-1} \quad (\text{also valid in matched-pairs situations})$$

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p , and $Y = \frac{X}{n}$, then

$$E(Y) = p \quad \text{and} \quad \text{Var}(Y) = \frac{p(1-p)}{n}$$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

$$\frac{S_x^2 / \sigma_x^2}{S_y^2 / \sigma_y^2} \sim F_{n_x-1, n_y-1}$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown),

$$\text{then } \frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y} \right)}} \sim t_{n_x+n_y-2} \quad \text{where} \quad S_p^2 = \frac{(n_x-1)S_x^2 + (n_y-1)S_y^2}{n_x + n_y - 2}$$

Correlation and regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \sum (x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{xy} = \sum (x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

The product moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\sum (x_i - \bar{x})^2\} \{\sum (y_i - \bar{y})^2\}}} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right) \left(\sum y_i^2 - \frac{(\sum y_i)^2}{n}\right)}}$$

Spearman's rank correlation coefficient is the product moment correlation coefficient between ranks

When there are no tied ranks it may be calculated using $r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$, where $a = \bar{y} - b\bar{x}$

Analysis of variance

One-factor model: $x_{ij} = \mu + \alpha_i + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim N(0, \sigma^2)$

$$\text{Total sum of squares } SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{n}$$

$$\text{Between groups sum of squares } SS_B = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{n}$$

Two-factor model (with m rows and n columns): $x_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim N(0, \sigma^2)$

$$\text{Total sum of squares, } SS_T = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{mn}$$

$$\text{Between rows sum of squares, } SS_R = \sum_i \frac{R_i^2}{n} - \frac{T^2}{mn}$$

$$\text{Between columns sum of squares, } SS_C = \sum_j \frac{C_j^2}{m} - \frac{T^2}{mn}$$

Distribution-free (non-parametric) tests

Goodness-of-fit tests and contingency tables: $\sum \frac{(O_i - E_i)^2}{E_i}$ is approximately distributed as χ^2

Wilcoxon signed rank test

T is the sum of the ranks of observations with the same sign

Mann-Whitney test

$U = T - \frac{n(n+1)}{2}$ where T is the sum of the ranks of the sample of size n

Kruskal-Wallis test

$$H = \frac{12}{N(N+1)} \sum_i \frac{T_i^2}{n_i} - 3(N+1)$$

where T_i is the sum of the ranks of a sample of size n_i and $N = \sum_i n_i$

H is approximately distributed as χ^2 with $k-1$ degrees of freedom
where k is the number of samples

TABLE 1 CUMULATIVE BINOMIAL DISTRIBUTION FUNCTION

The tabulated value is $P(X \leq x)$, where X has a binomial distribution with parameters n and p .

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
x	$n=2$																		x
0	0.9801	0.9604	0.9409	0.9216	0.9025	0.8836	0.8649	0.8464	0.8281	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	0
1	0.9999	0.9996	0.9991	0.9984	0.9975	0.9964	0.9951	0.9936	0.9919	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	1
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	2
x	$n=3$																		x
0	0.9703	0.9412	0.9127	0.8847	0.8574	0.8306	0.8044	0.7787	0.7536	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	0
1	0.9997	0.9988	0.9974	0.9953	0.9928	0.9896	0.9860	0.9818	0.9772	0.9720	0.9393	0.8960	0.8438	0.7840	0.7183	0.6480	0.5748	0.5000	1
2	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9993	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	2
3				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	3
x	$n=4$																		x
0	0.9606	0.9224	0.8853	0.8493	0.8145	0.7807	0.7481	0.7164	0.6857	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	0
1	0.9994	0.9977	0.9948	0.9909	0.9860	0.9801	0.9733	0.9656	0.9570	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	1
2	1.0000	1.0000	0.9999	0.9998	0.9995	0.9992	0.9987	0.9981	0.9973	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	2
3			1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995	0.9984	0.9961	0.9919	0.9850	0.9744	0.9590	0.9375	3
4									1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	4
x	$n=5$																		x
0	0.9510	0.9039	0.8587	0.8154	0.7738	0.7339	0.6957	0.6591	0.6240	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	0
1	0.9990	0.9962	0.9915	0.9852	0.9774	0.9681	0.9575	0.9456	0.9326	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	1
2	1.0000	0.9999	0.9997	0.9994	0.9988	0.9980	0.9969	0.9955	0.9937	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	2
3		1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	3
4						1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688	4
5											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	5
x	$n=6$																		x
0	0.9415	0.8858	0.8330	0.7828	0.7351	0.6899	0.6470	0.6064	0.5679	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	0
1	0.9985	0.9943	0.9875	0.9784	0.9672	0.9541	0.9392	0.9227	0.9048	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	1
2	1.0000	0.9998	0.9995	0.9988	0.9978	0.9962	0.9942	0.9915	0.9882	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	2
3		1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	3
4					1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9590	0.9308	0.8906	4
5										1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844	5
6											1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	6
x	$n=7$																		x
0	0.9321	0.8681	0.8080	0.7514	0.6983	0.6485	0.6017	0.5578	0.5168	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	0
1	0.9980	0.9921	0.9829	0.9706	0.9556	0.9382	0.9187	0.8974	0.8745	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	1
2	1.0000	0.9997	0.9991	0.9980	0.9962	0.9937	0.9903	0.9860	0.9807	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266	2
3		1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9988	0.9982	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	3
4				1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	4
5								1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	5
6											1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	6
7													1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	7
x	$n=8$																		x
0	0.9227	0.8508	0.7837	0.7214	0.6634	0.6096	0.5596	0.5132	0.4703	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	0
1	0.9973	0.9897	0.9777	0.9619	0.9428	0.9208	0.8965	0.8702	0.8423	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	1
2	0.9999	0.9996	0.9987	0.9969	0.9942	0.9904	0.9853	0.9789	0.9711	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	2
3	1.0000	1.0000	0.9999	0.9998	0.9996	0.9993	0.9987	0.9978	0.9966	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	3
4			1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	4
5							1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555	5
6											1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648	6
7												1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961	7
8														1.0000	1.0000	1.0000	1.0000	1.0000	8

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p	
x	$n=9$																			x
0	0.9135	0.8337	0.7602	0.6925	0.6302	0.5730	0.5204	0.4722	0.4279	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020	0	
1	0.9966	0.9869	0.9718	0.9522	0.9288	0.9022	0.8729	0.8417	0.8088	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195	1	
2	0.9999	0.9994	0.9980	0.9955	0.9916	0.9862	0.9791	0.9702	0.9595	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2318	0.1495	0.0898	2	
3	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9977	0.9963	0.9943	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539	3	
4		1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000		4	
5			1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461			5	
6				1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102					6	
7					1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805								7	
8						1.0000	1.0000	0.9999	0.9997	0.9992	0.9980								8	
9												1.0000	1.0000	0.9999	0.9997	0.9992	0.9980		9	
															1.0000	1.0000	1.0000	1.0000		
x	$n=10$																			x
0	0.9044	0.8171	0.7374	0.6648	0.5987	0.5386	0.4840	0.4344	0.3894	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010	0	
1	0.9957	0.9838	0.9655	0.9418	0.9139	0.8824	0.8483	0.8121	0.7746	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107	1	
2	0.9999	0.9991	0.9972	0.9938	0.9885	0.9812	0.9717	0.9599	0.9460	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547	2	
3	1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9964	0.9942	0.9912	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719	3	
4		1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9990	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770		4	
5			1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230			5	
6				1.0000	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281					6	
7					1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453							7	
8						1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893							8	
9							1.0000	0.9999	0.9997	0.9992	0.9990								9	
10												1.0000	1.0000	0.9999	0.9997	0.9990			10	
11															1.0000	1.0000	1.0000		11	
x	$n=11$																			x
0	0.8953	0.8007	0.7153	0.6382	0.5688	0.5063	0.4501	0.3996	0.3544	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005	0	
1	0.9948	0.9805	0.9587	0.9308	0.8981	0.8618	0.8228	0.7819	0.7399	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059	1	
2	0.9998	0.9988	0.9963	0.9917	0.9848	0.9752	0.9630	0.9481	0.9305	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327	2	
3	1.0000	1.0000	0.9998	0.9993	0.9984	0.9970	0.9947	0.9915	0.9871	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133	3	
4		1.0000	1.0000	0.9999	0.9997	0.9995	0.9990	0.9983	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744		4	
5			1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000			5	
6				1.0000	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256					6	
7					1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867							7	
8						1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673							8	
9							1.0000	1.0000	0.9998	0.9993	0.9978	0.9941							9	
10								1.0000	1.0000	0.9998	0.9993	0.9978	0.9941						10	
11														1.0000	1.0000	0.9998	0.9995		11	
12																1.0000			12	
x	$n=12$																			x
0	0.8864	0.7847	0.6938	0.6127	0.5404	0.4759	0.4186	0.3677	0.3225	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002	0	
1	0.9938	0.9769	0.9514	0.9191	0.8816	0.8405	0.7967	0.7513	0.7052	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032	1	
2	0.9998	0.9985	0.9952	0.9893	0.9804	0.9684	0.9532	0.9348	0.9134	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193	2	
3	1.0000	0.9999	0.9997	0.9990	0.9978	0.9957	0.9925	0.9880	0.9820	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730	3	
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9973	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938	4	
5			1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872		5	
6				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872	6	
7					1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062						7	
8						1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270						8	
9							1.0000	0.9998	0.9992	0.9972	0.9921	0.9807							9	
10								1.0000	0.9999	0.9997	0.9989	0.9968							10	
11									1.0000	1.0000	0.9999	0.9998							11	
12													1.0000	1.0000					12	
13																1.0000	1.0000		13	
x	$n=13$																			x
0	0.8775	0.7690	0.6730	0.5882	0.5133	0.4474	0.3893	0.3383	0.2935	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001	0	
1	0.9928	0.9730	0.9436	0.9068	0.8646	0.8186	0.7702	0.7206	0.6707	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0017	1	
2	0.9997	0.9980	0.9938	0.9865	0.9755	0.9608	0.9422	0.9201	0.8946	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0112	2	
3	1.0000	0.9999	0.9995	0.9986	0.9969	0.9940	0.9897	0.9837	0.9758	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0461	3	
4		1.0000	1.0000	0.9999	0.9997	0.9993	0.9987	0.9976	0.9959	0.9935	0.9658	0.9009	0.7940	0.6543	0.5005	0.3530	0.2279	0.1334	4	
5			1.0000	1.0000	0.9999	0.9999	0.9997	0.9995	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.2905		5	
6				1.0000	1.0000	1.0000	0.9999	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000			6	
7					1.0000	1.0000	1.0000	0.9999	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000		7	
8						1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.7095				8	
9							1.0000	0.9998	0.9990	0.9960	0.9874	0.9679	0.9302	0.8666					9	
10								1.0000	0.9999	0.9993	0.9975	0.9922	0.9797	0.95						

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p	
x	$n=14$																			x
0	0.8687	0.7536	0.6528	0.5647	0.4877	0.4205	0.3620	0.3112	0.2670	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001	0	
1	0.9916	0.9690	0.9355	0.8941	0.8470	0.7963	0.7436	0.6900	0.6368	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009	1	
2	0.9997	0.9975	0.9923	0.9833	0.9699	0.9522	0.9302	0.9042	0.8745	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065	2	
3	1.0000	0.9999	0.9994	0.9981	0.9958	0.9920	0.9864	0.9786	0.9685	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287	3	
4		1.0000	1.0000	0.9998	0.9996	0.9990	0.9980	0.9965	0.9941	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898	4	
5				1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120	5	
6						1.0000	1.0000	1.0000	0.9999	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953	6	
7									1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047	7	
8											1.0000	0.9996	0.9978	0.9917	0.9757	0.9417	0.8811	0.7880	8	
9												1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102	9	
10													1.0000	0.9998	0.9989	0.9961	0.9886	0.9713	10	
11														1.0000	0.9999	0.9994	0.9978	0.9935	11	
12															1.0000	0.9999	0.9997	0.9991	12	
13																1.0000	1.0000	0.9999	13	
14																		1.0000	14	
x	$n=15$																			x
0	0.8601	0.7386	0.6333	0.5421	0.4633	0.3953	0.3367	0.2863	0.2430	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000	0	
1	0.9904	0.9647	0.9270	0.8809	0.8290	0.7738	0.7168	0.6597	0.6035	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005	1	
2	0.9996	0.9970	0.9906	0.9797	0.9638	0.9429	0.9171	0.8870	0.8531	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037	2	
3	1.0000	0.9998	0.9992	0.9976	0.9945	0.9896	0.9825	0.9727	0.9601	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176	3	
4		1.0000	0.9999	0.9998	0.9994	0.9986	0.9972	0.9950	0.9918	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592	4	
5			1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509	5	
6					1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036	6	
7								1.0000	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000	7	
8											0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964	8	
9												1.0000	0.9999	0.9992	0.9963	0.9876	0.9662	0.9231	9	
10													1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	10	
11														1.0000	0.9999	0.9995	0.9981	0.9937	11	
12															1.0000	0.9999	0.9997	0.9989	12	
13																1.0000	1.0000	0.9999	13	
14																		1.0000	14	
x	$n=20$																			x
0	0.8179	0.6676	0.5438	0.4420	0.3585	0.2901	0.2342	0.1887	0.1516	0.1216	0.0388	0.0115	0.0032	0.0008	0.0002	0.0000	0.0000	0.0000	0	
1	0.9831	0.9401	0.8802	0.8103	0.7358	0.6605	0.5869	0.5169	0.4516	0.3917	0.1756	0.0692	0.0243	0.0076	0.0021	0.0005	0.0001	0.0000	1	
2	0.9990	0.9929	0.9790	0.9561	0.9245	0.8850	0.8390	0.7879	0.7334	0.6769	0.4049	0.2061	0.0913	0.0355	0.0121	0.0036	0.0009	0.0002	2	
3	1.0000	0.9994	0.9973	0.9926	0.9841	0.9710	0.9529	0.9294	0.9007	0.8670	0.6477	0.4114	0.2252	0.1071	0.0444	0.0160	0.0049	0.0013	3	
4		1.0000	0.9997	0.9990	0.9974	0.9944	0.9893	0.9817	0.9710	0.9568	0.8298	0.6296	0.4148	0.2375	0.1182	0.0510	0.0189	0.0059	4	
5			1.0000	0.9999	0.9997	0.9991	0.9981	0.9962	0.9932	0.9887	0.9327	0.8042	0.6172	0.4164	0.2454	0.1256	0.0553	0.0207	5	
6				1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9976	0.9781	0.9133	0.7858	0.6080	0.4166	0.2500	0.1299	0.0577	6	
7						1.0000	1.0000	0.9999	0.9998	0.9996	0.9941	0.9679	0.8982	0.7723	0.6010	0.4159	0.2520	0.1316	7	
8								1.0000	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.7624	0.5956	0.4143	0.2517	8	
9										1.0000	0.9998	0.9974	0.9861	0.9520	0.8782	0.7553	0.5914	0.4119	9	
10											1.0000	0.9994	0.9961	0.9829	0.9468	0.8725	0.7507	0.5881	10	
11												0.9999	0.9991	0.9949	0.9804	0.9435	0.8692	0.7483	11	
12													1.0000	0.9998	0.9987	0.9940	0.9790	0.9420	12	
13														1.0000	0.9997	0.9985	0.9935	0.9786	13	
14															1.0000	0.9997	0.9984	0.9936	14	
15																1.0000	0.9997	0.9985	15	
16																	1.0000	0.9997	16	
17																		1.0000	17	
18																			1.0000	18

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p	
x	$n=25$																			x
0	0.7778	0.6035	0.4670	0.3604	0.2774	0.2129	0.1630	0.1244	0.0946	0.0718	0.0172	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0	
1	0.9742	0.9114	0.8280	0.7358	0.6424	0.5527	0.4696	0.3947	0.3286	0.2712	0.0931	0.0274	0.0070	0.0016	0.0003	0.0001	0.0000	0.0000	1	
2	0.9980	0.9868	0.9620	0.9235	0.8729	0.8129	0.7466	0.6768	0.6063	0.5371	0.2537	0.0982	0.0321	0.0090	0.0021	0.0004	0.0001	0.0000	2	
3	0.9999	0.9986	0.9938	0.9835	0.9659	0.9402	0.9064	0.8649	0.8169	0.7636	0.4711	0.2340	0.0962	0.0332	0.0097	0.0024	0.0005	0.0001	3	
4	1.0000	0.9999	0.9992	0.9972	0.9928	0.9850	0.9726	0.9549	0.9314	0.9020	0.6821	0.4207	0.2137	0.0905	0.0320	0.0095	0.0023	0.0005	4	
5		1.0000	0.9999	0.9996	0.9988	0.9969	0.9935	0.9877	0.9790	0.9666	0.8385	0.6167	0.3783	0.1935	0.0826	0.0294	0.0086	0.0020	5	
6			1.0000	1.0000	0.9998	0.9995	0.9987	0.9972	0.9946	0.9905	0.9305	0.7800	0.5611	0.3407	0.1734	0.0736	0.0258	0.0073	6	
7				1.0000	0.9999	0.9998	0.9995	0.9989	0.9977	0.9745	0.8909	0.7265	0.5118	0.3061	0.1536	0.0639	0.0216		7	
8					1.0000	1.0000	0.9999	0.9998	0.9995	0.9920	0.9532	0.8506	0.6769	0.4668	0.2735	0.1340	0.0539		8	
9						1.0000	1.0000	0.9999	0.9979	0.9827	0.9287	0.8106	0.6303	0.4246	0.2424	0.1148			9	
10							1.0000	0.9995	0.9944	0.9703	0.9022	0.7712	0.5858	0.3843	0.2122				10	
11								0.9999	0.9985	0.9893	0.9558	0.8746	0.7323	0.5426	0.3450				11	
12								1.0000	0.9996	0.9966	0.9825	0.9396	0.8462	0.6937	0.5000				12	
13									0.9999	0.9991	0.9940	0.9745	0.9222	0.8173	0.6550				13	
14									1.0000	0.9998	0.9982	0.9907	0.9656	0.9040	0.7878				14	
15										1.0000	0.9995	0.9971	0.9868	0.9560	0.8852				15	
16											0.9999	0.9992	0.9957	0.9826	0.9461				16	
17											1.0000	0.9998	0.9988	0.9942	0.9784				17	
18												1.0000	0.9997	0.9984	0.9927				18	
19													0.9999	0.9996	0.9980				19	
20														1.0000	0.9999	0.9995			20	
21															1.0000	0.9999			21	
22																1.0000			22	
x	$n=30$																			x
0	0.7397	0.5455	0.4010	0.2939	0.2146	0.1563	0.1134	0.0820	0.0591	0.0424	0.0076	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0	
1	0.9639	0.8795	0.7731	0.6612	0.5535	0.4555	0.3694	0.2958	0.2343	0.1837	0.0480	0.0105	0.0020	0.0003	0.0000	0.0000	0.0000	0.0000	1	
2	0.9967	0.9783	0.9399	0.8831	0.8122	0.7324	0.6487	0.5654	0.4855	0.4114	0.1514	0.0442	0.0106	0.0021	0.0003	0.0000	0.0000	0.0000	2	
3	0.9998	0.9971	0.9881	0.9694	0.9392	0.8974	0.8450	0.7842	0.7175	0.6474	0.3217	0.1227	0.0374	0.0093	0.0019	0.0003	0.0000	0.0000	3	
4	1.0000	0.9997	0.9982	0.9937	0.9844	0.9685	0.9447	0.9126	0.8723	0.8245	0.5245	0.2552	0.0979	0.0302	0.0075	0.0015	0.0002	0.0000	4	
5		1.0000	0.9998	0.9989	0.9967	0.9921	0.9838	0.9707	0.9519	0.9268	0.7106	0.4275	0.2026	0.0766	0.0233	0.0057	0.0011	0.0002	5	
6			1.0000	0.9999	0.9994	0.9983	0.9960	0.9918	0.9848	0.9742	0.8474	0.6070	0.3481	0.1595	0.0586	0.0172	0.0040	0.0007	6	
7				1.0000	0.9999	0.9997	0.9992	0.9980	0.9959	0.9922	0.9302	0.7608	0.5143	0.2814	0.1238	0.0435	0.0121	0.0026	7	
8					1.0000	1.0000	0.9999	0.9996	0.9990	0.9980	0.9722	0.8713	0.6736	0.4315	0.2247	0.0940	0.0312	0.0081	8	
9						1.0000	0.9999	0.9998	0.9995	0.9903	0.9389	0.8034	0.5888	0.3575	0.1763	0.0694	0.0214		9	
10							1.0000	1.0000	0.9999	0.9971	0.9744	0.8943	0.7304	0.5078	0.2915	0.1350	0.0494		10	
11								1.0000	0.9992	0.9905	0.9493	0.8407	0.6548	0.4311	0.2327	0.1002			11	
12									0.9998	0.9969	0.9784	0.9155	0.7802	0.5785	0.3592	0.1808			12	
13									1.0000	0.9991	0.9918	0.9599	0.8737	0.7145	0.5025	0.2923			13	
14										0.9998	0.9973	0.9831	0.9348	0.8246	0.6448	0.4278			14	
15										0.9999	0.9992	0.9936	0.9699	0.9029	0.7691	0.5722			15	
16										1.0000	0.9998	0.9979	0.9876	0.9519	0.8644	0.7077			16	
17											0.9999	0.9994	0.9955	0.9788	0.9286	0.8192			17	
18											1.0000	0.9998	0.9986	0.9917	0.9666	0.8998			18	
19												1.0000	0.9996	0.9971	0.9862	0.9506			19	
20													0.9999	0.9991	0.9950	0.9786			20	
21														1.0000	0.9998	0.9984	0.9919		21	
22															1.0000	0.9996	0.9974		22	
23																0.9999	0.9993		23	
24																	1.0000	0.9998	24	
25																		1.0000	25	

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p	
x	$n=40$																			x
0	0.6690	0.4457	0.2957	0.1954	0.1285	0.0842	0.0549	0.0356	0.0230	0.0148	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0	
1	0.9393	0.8095	0.6615	0.5210	0.3991	0.2990	0.2201	0.1594	0.1140	0.0805	0.0121	0.0015	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	1	
2	0.9925	0.9543	0.8822	0.7855	0.6767	0.5665	0.4625	0.3694	0.2894	0.2228	0.0486	0.0079	0.0010	0.0001	0.0000	0.0000	0.0000	0.0000	2	
3	0.9993	0.9918	0.9686	0.9252	0.8619	0.7827	0.6937	0.6007	0.5092	0.4231	0.1302	0.0285	0.0047	0.0006	0.0001	0.0000	0.0000	0.0000	3	
4	1.0000	0.9988	0.9933	0.9790	0.9520	0.9104	0.8546	0.7868	0.7103	0.6290	0.2633	0.0759	0.0160	0.0026	0.0003	0.0000	0.0000	0.0000	4	
5		0.9999	0.9988	0.9951	0.9861	0.9691	0.9419	0.9033	0.8535	0.7937	0.4325	0.1613	0.0433	0.0086	0.0013	0.0001	0.0000	0.0000	5	
6		1.0000	0.9998	0.9990	0.9966	0.9909	0.9801	0.9624	0.9361	0.9005	0.6067	0.2859	0.0962	0.0238	0.0044	0.0006	0.0001	0.0000	6	
7			1.0000	0.9998	0.9993	0.9977	0.9942	0.9873	0.9758	0.9581	0.7559	0.4371	0.1820	0.0553	0.0124	0.0021	0.0002	0.0000	7	
8				1.0000	0.9999	0.9995	0.9985	0.9963	0.9919	0.9845	0.8646	0.5931	0.2998	0.1110	0.0303	0.0061	0.0009	0.0001	8	
9					1.0000	0.9999	0.9997	0.9990	0.9976	0.9949	0.9328	0.7318	0.4395	0.1959	0.0644	0.0156	0.0027	0.0003	9	
10						1.0000	0.9999	0.9998	0.9994	0.9985	0.9701	0.8392	0.5839	0.3087	0.1215	0.0352	0.0074	0.0011	10	
11							1.0000	1.0000	0.9999	0.9996	0.9880	0.9125	0.7151	0.4406	0.2053	0.0709	0.0179	0.0032	11	
12								1.0000	0.9999	0.9957	0.9568	0.8209	0.5772	0.3143	0.1285	0.0386	0.0083		12	
13									1.0000	0.9986	0.9806	0.8968	0.7032	0.4408	0.2112	0.0751	0.0192		13	
14										0.9996	0.9921	0.9456	0.8074	0.5721	0.3174	0.1326	0.0403		14	
15										0.9999	0.9971	0.9738	0.8849	0.6946	0.4402	0.2142	0.0769		15	
16										1.0000	0.9990	0.9884	0.9367	0.7978	0.5681	0.3185	0.1341		16	
17											0.9997	0.9953	0.9680	0.8761	0.6885	0.4391	0.2148		17	
18											0.9999	0.9983	0.9852	0.9301	0.7911	0.5651	0.3179		18	
19											1.0000	0.9994	0.9937	0.9637	0.8702	0.6844	0.4373		19	
20												0.9998	0.9976	0.9827	0.9256	0.7870	0.5627		20	
21												1.0000	0.9991	0.9925	0.9608	0.8669	0.6821		21	
22													0.9997	0.9970	0.9811	0.9233	0.7852		22	
23													0.9999	0.9989	0.9917	0.9595	0.8659		23	
24													1.0000	0.9996	0.9966	0.9804	0.9231		24	
25														0.9999	0.9988	0.9914	0.9597		25	
26														1.0000	0.9996	0.9966	0.9808		26	
27															0.9999	0.9988	0.9917		27	
28															1.0000	0.9996	0.9968		28	
29																0.9999	0.9989		29	
30																1.0000	0.9997		30	
31																	0.9999		31	
32																	1.0000		32	

p	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
x	$n=50$																		x
0	0.6050	0.3642	0.2181	0.1299	0.0769	0.0453	0.0266	0.0155	0.0090	0.0052	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	0.9106	0.7358	0.5553	0.4005	0.2794	0.1900	0.1265	0.0827	0.0532	0.0338	0.0029	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1
2	0.9862	0.9216	0.8108	0.6767	0.5405	0.4162	0.3108	0.2260	0.1605	0.1117	0.0142	0.0013	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	2
3	0.9984	0.9822	0.9372	0.8609	0.7604	0.6473	0.5327	0.4253	0.3303	0.2503	0.0460	0.0057	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	3
4	0.9999	0.9968	0.9832	0.9510	0.8964	0.8206	0.7290	0.6290	0.5277	0.4312	0.1121	0.0185	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	4
5	1.0000	0.9995	0.9963	0.9856	0.9622	0.9224	0.8650	0.7919	0.7072	0.6161	0.2194	0.0480	0.0070	0.0007	0.0001	0.0000	0.0000	0.0000	5
6		0.9999	0.9993	0.9964	0.9882	0.9711	0.9417	0.8981	0.8404	0.7702	0.3613	0.1034	0.0194	0.0025	0.0002	0.0000	0.0000	0.0000	6
7		1.0000	0.9999	0.9992	0.9968	0.9906	0.9780	0.9562	0.9232	0.8779	0.5188	0.1904	0.0453	0.0073	0.0008	0.0001	0.0000	0.0000	7
8			1.0000	0.9999	0.9992	0.9973	0.9927	0.9833	0.9672	0.9421	0.6681	0.3073	0.0916	0.0183	0.0025	0.0002	0.0000	0.0000	8
9				1.0000	0.9998	0.9993	0.9978	0.9944	0.9875	0.9755	0.7911	0.4437	0.1637	0.0402	0.0067	0.0008	0.0001	0.0000	9
10					1.0000	0.9998	0.9994	0.9983	0.9957	0.9906	0.8801	0.5836	0.2622	0.0789	0.0160	0.0022	0.0002	0.0000	10
11						1.0000	0.9999	0.9995	0.9987	0.9968	0.9372	0.7107	0.3816	0.1390	0.0342	0.0057	0.0006	0.0000	11
12							1.0000	0.9999	0.9996	0.9990	0.9699	0.8139	0.5110	0.2229	0.0661	0.0133	0.0018	0.0002	12
13								1.0000	0.9999	0.9997	0.9868	0.8894	0.6370	0.3279	0.1163	0.0280	0.0045	0.0005	13
14									1.0000	0.9999	0.9947	0.9393	0.7481	0.4468	0.1878	0.0540	0.0104	0.0013	14
15										1.0000	0.9981	0.9692	0.8369	0.5692	0.2801	0.0955	0.0220	0.0033	15
16											0.9993	0.9856	0.9017	0.6839	0.3889	0.1561	0.0427	0.0077	16
17											0.9998	0.9937	0.9449	0.7822	0.5060	0.2369	0.0765	0.0164	17
18											0.9999	0.9975	0.9713	0.8594	0.6216	0.3356	0.1273	0.0325	18
19											1.0000	0.9991	0.9861	0.9152	0.7264	0.4465	0.1974	0.0595	19
20												0.9997	0.9937	0.9522	0.8139	0.5610	0.2862	0.1013	20
21												0.9999	0.9974	0.9749	0.8813	0.6701	0.3900	0.1611	21
22												1.0000	0.9990	0.9877	0.9290	0.7660	0.5019	0.2399	22
23													0.9996	0.9944	0.9604	0.8438	0.6134	0.3359	23
24													0.9999	0.9976	0.9793	0.9022	0.7160	0.4439	24
25													1.0000	0.9991	0.9900	0.9427	0.8034	0.5561	25
26														0.9997	0.9955	0.9686	0.8721	0.6641	26
27														0.9999	0.9981	0.9840	0.9220	0.7601	27
28														1.0000	0.9993	0.9924	0.9556	0.8389	28
29															0.9997	0.9966	0.9765	0.8987	29
30															0.9999	0.9986	0.9884	0.9405	30
31															1.0000	0.9995	0.9947	0.9675	31
32																0.9998	0.9978	0.9836	32
33																0.9999	0.9991	0.9923	33
34																1.0000	0.9997	0.9967	34
35																	0.9999	0.9987	35
36																	1.0000	0.9995	36
37																		0.9998	37
38																		1.0000	38

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TABLE 2 CUMULATIVE POISSON DISTRIBUTION FUNCTION

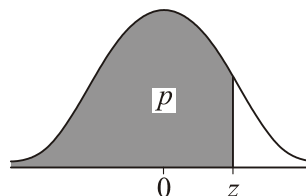
The tabulated value is $P(X \leq x)$, where X has a Poisson distribution with mean λ .

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.0	1.2	1.4	1.6	1.8	λ
x															x
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679	0.3012	0.2466	0.2019	0.1653	0
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358	0.6626	0.5918	0.5249	0.4628	1
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197	0.8795	0.8335	0.7834	0.7306	2
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810	0.9662	0.9463	0.9212	0.8913	3
4		1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9923	0.9857	0.9763	0.9636	4
5				1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994	0.9985	0.9968	0.9940	0.9896	5
6							1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9987	0.9974	6
7										1.0000	1.0000	0.9999	0.9997	0.9994	7
8												1.0000	1.0000	0.9999	8
9														1.0000	9
λ	2.0	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0	4.5	5.0	5.5	λ
x															x
0	0.1353	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183	0.0111	0.0067	0.0041	0
1	0.4060	0.3546	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074	0.0916	0.0611	0.0404	0.0266	1
2	0.6767	0.6227	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689	0.2381	0.1736	0.1247	0.0884	2
3	0.8571	0.8194	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735	0.4335	0.3423	0.2650	0.2017	3
4	0.9473	0.9275	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678	0.6288	0.5321	0.4405	0.3575	4
5	0.9834	0.9751	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156	0.7851	0.7029	0.6160	0.5289	5
6	0.9955	0.9925	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091	0.8893	0.8311	0.7622	0.6860	6
7	0.9989	0.9980	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599	0.9489	0.9134	0.8666	0.8095	7
8	0.9998	0.9995	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840	0.9786	0.9597	0.9319	0.8944	8
9	1.0000	0.9999	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942	0.9919	0.9829	0.9682	0.9462	9
10		1.0000	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981	0.9972	0.9933	0.9863	0.9747	10
11				1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994	0.9991	0.9976	0.9945	0.9890	11
12						1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9992	0.9980	0.9955	12
13								1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9983	13
14											1.0000	0.9999	0.9998	0.9994	14
15												1.0000	0.9999	0.9998	15
16													1.0000	0.9999	16
17														1.0000	17

λ	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	λ
x															x
0	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0
1	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	1
2	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	2
3	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	3
4	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	4
5	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	5
6	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	6
7	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	7
8	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	8
9	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	9
10	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	10
11	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	11
12	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	12
13	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	13
14	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	14
15	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	15
16	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	16
17	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	17
18	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	18
19		1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	19
20				1.0000	0.9999	0.9998	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	20
21					1.0000	0.9999	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	21
22						1.0000	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	22
23							1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	23
24								1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	24
25										0.9999	0.9997	0.9990	0.9974	0.9938	25
26										1.0000	0.9999	0.9995	0.9987	0.9967	26
27											0.9999	0.9998	0.9994	0.9983	27
28											1.0000	0.9999	0.9997	0.9991	28
29												1.0000	0.9999	0.9996	29
30													0.9999	0.9998	30
31													1.0000	0.9999	31
32														1.0000	32

TABLE 3 NORMAL DISTRIBUTION FUNCTION

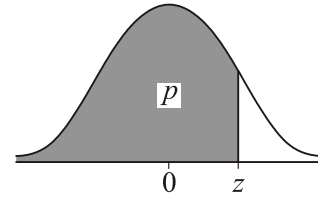
The table gives the probability, p , that a normally distributed random variable Z , with mean = 0 and variance = 1, is less than or equal to z .



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.53983	0.54380	0.54776	0.55172	0.55567	0.55962	0.56356	0.56749	0.57142	0.57535	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.61791	0.62172	0.62552	0.62930	0.63307	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.67364	0.67724	0.68082	0.68439	0.68793	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.70884	0.71226	0.71566	0.71904	0.72240	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.74215	0.74537	0.74857	0.75175	0.75490	0.6
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.77337	0.77637	0.77935	0.78230	0.78524	0.7
0.8	0.78814	0.79103	0.79389	0.79673	0.79955	0.80234	0.80511	0.80785	0.81057	0.81327	0.8
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.82894	0.83147	0.83398	0.83646	0.83891	0.9
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769	0.85993	0.86214	1.0
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900	0.88100	0.88298	1.1
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796	0.89973	0.90147	1.2
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466	0.91621	0.91774	1.3
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922	0.93056	0.93189	1.4
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	0.93943	0.94062	0.94179	0.94295	0.94408	1.5
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	0.95053	0.95154	0.95254	0.95352	0.95449	1.6
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	0.95994	0.96080	0.96164	0.96246	0.96327	1.7
1.8	0.96407	0.96485	0.96562	0.96638	0.96712	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	0.97441	0.97500	0.97558	0.97615	0.97670	1.9
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	0.97982	0.98030	0.98077	0.98124	0.98169	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	0.98422	0.98461	0.98500	0.98537	0.98574	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	0.98778	0.98809	0.98840	0.98870	0.98899	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	0.99061	0.99086	0.99111	0.99134	0.99158	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	0.99286	0.99305	0.99324	0.99343	0.99361	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	0.99461	0.99477	0.99492	0.99506	0.99520	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	0.99598	0.99609	0.99621	0.99632	0.99643	2.6
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	0.99702	0.99711	0.99720	0.99728	0.99736	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	0.99781	0.99788	0.99795	0.99801	0.99807	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	0.99841	0.99846	0.99851	0.99856	0.99861	2.9
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	0.99886	0.99889	0.99893	0.99896	0.99900	3.0
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	0.99918	0.99921	0.99924	0.99926	0.99929	3.1
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	0.99942	0.99944	0.99946	0.99948	0.99950	3.2
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	0.99960	0.99961	0.99962	0.99964	0.99965	3.3
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	0.99972	0.99973	0.99974	0.99975	0.99976	3.4
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	0.99981	0.99981	0.99982	0.99983	0.99983	3.5
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	0.99987	0.99987	0.99988	0.99988	0.99989	3.6
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	0.99991	0.99992	0.99992	0.99992	0.99992	3.7
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	0.99994	0.99994	0.99995	0.99995	0.99995	3.8
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99997	0.99997	3.9

TABLE 4 PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

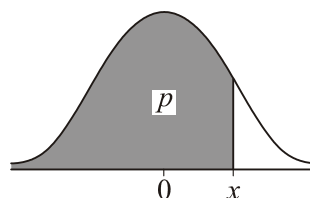
The table gives the values of z satisfying $P(Z \leq z) = p$, where Z is the normally distributed random variable with mean = 0 and variance = 1.



p	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	p
0.5	0.0000	0.0251	0.0502	0.0753	0.1004	0.1257	0.1510	0.1764	0.2019	0.2275	0.5
0.6	0.2533	0.2793	0.3055	0.3319	0.3585	0.3853	0.4125	0.4399	0.4677	0.4958	0.6
0.7	0.5244	0.5534	0.5828	0.6128	0.6433	0.6745	0.7063	0.7388	0.7722	0.8064	0.7
0.8	0.8416	0.8779	0.9154	0.9542	0.9945	1.0364	1.0803	1.1264	1.1750	1.2265	0.8
0.9	1.2816	1.3408	1.4051	1.4758	1.5548	1.6449	1.7507	1.8808	2.0537	2.3263	0.9
p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009	p
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392	0.95
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663	0.96
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335	0.97
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904	0.98
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902	0.99

TABLE 5 PERCENTAGE POINTS OF THE STUDENT'S t -DISTRIBUTION

The table gives the values of x satisfying $P(X \leq x) = p$, where X is a random variable having the Student's t -distribution with ν degrees of freedom.

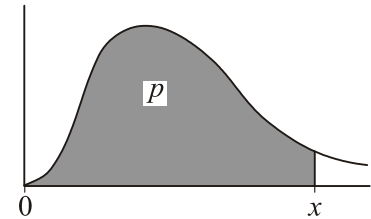


p	0.9	0.95	0.975	0.99	0.995
ν					
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.121	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763

p	0.9	0.95	0.975	0.99	0.995
ν					
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
45	1.301	1.679	2.014	2.412	2.690
50	1.299	1.676	2.009	2.403	2.678
55	1.297	1.673	2.004	2.396	2.668
60	1.296	1.671	2.000	2.390	2.660
65	1.295	1.669	1.997	2.385	2.654
70	1.294	1.667	1.994	2.381	2.648
75	1.293	1.665	1.992	2.377	2.643
80	1.292	1.664	1.990	2.374	2.639
85	1.292	1.663	1.998	2.371	2.635
90	1.291	1.662	1.987	2.368	2.632
95	1.291	1.661	1.985	2.366	2.629
100	1.290	1.660	1.984	2.364	2.626
125	1.288	1.657	1.979	2.357	2.616
150	1.287	1.655	1.976	2.351	2.609
200	1.286	1.653	1.972	2.345	2.601
∞	1.282	1.645	1.960	2.326	2.576

TABLE 6 PERCENTAGE POINTS OF THE χ^2 DISTRIBUTION

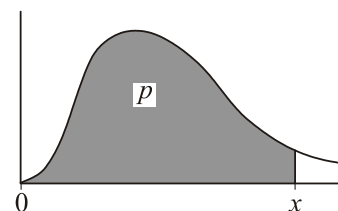
The table gives the values of x satisfying $P(X \leq x) = p$, where X is a random variable having the χ^2 distribution with ν degrees of freedom.



p	0.005	0.01	0.025	0.05	0.1	0.9	0.95	0.975	0.99	0.995	p
ν											ν
1	0.00004	0.0002	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	1
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	2
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838	3
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	4
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750	5
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	6
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	7
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	8
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	9
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	10
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	11
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300	12
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819	13
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319	14
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801	15
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267	16
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718	17
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156	18
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582	19
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997	20
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401	21
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796	22
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181	23
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559	24
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928	25
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290	26
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645	27
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993	28
29	13.121	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336	29
30	13.787	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672	30
31	14.458	15.655	17.539	19.281	21.434	41.422	44.985	48.232	52.191	55.003	31
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328	32
33	15.815	17.074	19.047	20.867	23.110	43.745	47.400	50.725	54.776	57.648	33
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.996	56.061	58.964	34
35	17.192	18.509	20.569	22.465	24.797	46.059	49.802	53.203	57.342	60.275	35
36	17.887	19.223	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581	36
37	18.586	19.960	22.106	24.075	26.492	48.363	52.192	55.668	59.892	62.883	37
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181	38
39	19.996	21.426	23.654	25.695	28.196	50.660	54.572	58.120	62.428	65.476	39
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766	40
45	24.311	25.901	28.366	30.612	33.350	57.505	61.656	65.410	69.957	73.166	45
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490	50
55	31.735	33.570	36.398	38.958	42.060	68.796	73.311	77.380	82.292	85.749	55
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952	60
65	39.383	41.444	44.603	47.450	50.883	79.973	84.821	89.177	94.422	98.105	65
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215	70
75	47.206	49.475	52.942	56.054	59.795	91.061	96.217	100.839	106.393	110.286	75
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321	80
85	55.170	57.634	61.389	64.749	68.777	102.079	107.522	112.393	118.236	122.325	85
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299	90
95	63.250	65.898	69.925	73.520	77.818	113.038	118.752	123.858	129.973	134.247	95
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169	100

TABLE 7 PERCENTAGE POINTS OF THE F -DISTRIBUTION

The tables give the values of x satisfying $P(X \leq x) = p$, where X is a random variable having the F -distribution with ν_1 degrees of freedom in the numerator and ν_2 degrees of freedom in the denominator.



F -Distribution ($p=0.995$)

Use for one-tail tests at significance level 0.5% or two-tail tests at significance level 1%.

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	ν_1
ν_2																					ν_2
1	16211	20000	21615	22500	23056	23437	23715	23925	24091	24224	24334	24426	24630	24836	24960	25044	25148	25211	25337	25464	1
2	198.5	199.0	199.2	199.2	199.3	199.3	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.4	199.5	199.5	199.5	199.5	199.5	199.5	2
3	55.55	49.80	47.47	46.19	45.39	44.84	44.43	44.13	43.88	43.69	43.52	43.39	43.08	42.78	42.59	42.47	42.31	42.21	42.02	41.83	3
4	31.33	26.28	24.26	23.15	22.46	21.97	21.62	21.35	21.14	20.97	20.82	20.70	20.44	20.17	20.00	19.89	19.75	19.67	19.50	19.32	4
5	22.78	18.31	16.53	15.56	14.94	14.51	14.20	13.96	13.77	13.62	13.49	13.38	13.15	12.90	12.76	12.66	12.53	12.45	12.30	12.14	5
6	18.635	14.544	12.917	12.028	11.464	11.073	10.786	10.566	10.391	10.250	10.133	10.034	9.814	9.589	9.451	9.358	9.241	9.170	9.026	8.879	6
7	16.236	12.404	10.882	10.050	9.522	9.155	8.885	8.678	8.514	8.380	8.270	8.176	7.968	7.754	7.623	7.534	7.422	7.354	7.217	7.076	7
8	14.688	11.042	9.596	8.805	8.302	7.952	7.694	7.496	7.339	7.211	7.104	7.015	6.814	6.608	6.482	6.396	6.288	6.222	6.088	5.951	8
9	13.614	10.107	8.717	7.956	7.471	7.134	6.885	6.693	6.541	6.417	6.314	6.227	6.032	5.832	5.708	5.625	5.519	5.454	5.322	5.188	9
10	12.826	9.427	8.081	7.343	6.872	6.545	6.302	6.116	5.968	5.847	5.746	5.661	5.471	5.274	5.153	5.071	4.966	4.902	4.772	4.639	10
11	12.226	8.912	7.600	6.881	6.422	6.102	5.865	5.682	5.537	5.418	5.320	5.236	5.049	4.855	4.736	4.654	4.551	4.488	4.359	4.226	11
12	11.754	8.510	7.226	6.521	6.071	5.757	5.525	5.345	5.202	5.085	4.988	4.906	4.721	4.530	4.412	4.331	4.228	4.165	4.037	3.904	12
13	11.374	8.186	6.926	6.233	5.791	5.482	5.253	5.076	4.935	4.820	4.724	4.643	4.460	4.270	4.153	4.073	3.970	3.908	3.780	3.647	13
14	11.060	7.922	6.680	5.998	5.562	5.257	5.031	4.857	4.717	4.603	4.508	4.428	4.247	4.059	3.942	3.862	3.760	3.697	3.569	3.436	14
15	10.798	7.701	6.476	5.803	5.372	5.071	4.847	4.674	4.536	4.424	4.329	4.250	4.070	3.883	3.766	3.687	3.585	3.523	3.394	3.260	15
20	9.944	6.986	5.818	5.174	4.762	4.472	4.257	4.090	3.956	3.847	3.756	3.678	3.502	3.318	3.203	3.123	3.022	2.959	2.828	2.690	20
25	9.475	6.598	5.462	4.835	4.433	4.150	3.939	3.776	3.645	3.537	3.447	3.370	3.196	3.013	2.898	2.819	2.716	2.652	2.519	2.377	25
30	9.180	6.355	5.239	4.623	4.228	3.949	3.742	3.580	3.450	3.344	3.255	3.179	3.006	2.823	2.708	2.628	2.524	2.459	2.323	2.176	30
40	8.828	6.066	4.976	4.374	3.986	3.713	3.509	3.350	3.222	3.117	3.028	2.953	2.781	2.598	2.482	2.401	2.296	2.230	2.088	1.932	40
50	8.626	5.902	4.826	4.232	3.849	3.579	3.376	3.219	3.092	2.988	2.900	2.825	2.653	2.470	2.353	2.272	2.164	2.097	1.951	1.786	50
100	8.241	5.589	4.542	3.963	3.589	3.325	3.127	2.972	2.847	2.744	2.657	2.583	2.411	2.227	2.108	2.024	1.912	1.840	1.681	1.485	100
∞	7.879	5.298	4.279	3.715	3.350	3.091	2.897	2.744	2.621	2.519	2.432	2.358	2.187	2.000	1.877	1.789	1.669	1.590	1.402	1.001	∞

F -Distribution ($p=0.99$)

Use for one-tail tests at significance level 1% or two-tail tests at significance level 2%.

ν_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	ν_1
ν_2																					ν_2
1	4052	5000	5403	5625	5764	5859	5928	5981	6022	6056	6083	6106	6157	6209	6240	6261	6287	6303	6334	6366	1
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.41	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50	2
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.13	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.24	26.13	3
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.45	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.58	13.46	4
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.96	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.13	9.02	5
6	13.745	10.925	9.780	9.148	8.746	8.466	8.260	8.102	7.976	7.874	7.790	7.718	7.559	7.396	7.296	7.229	7.143	7.091	6.987	6.880	6
7	12.246	9.547	8.451	7.847	7.460	7.191	6.993	6.840	6.719	6.620	6.538	6.469	6.314	6.155	6.058	5.992	5.908	5.858	5.755	5.650	7
8	11.259	8.649	7.591	7.006	6.632	6.371	6.178	6.029	5.911	5.814	5.734	5.667	5.515	5.359	5.263	5.198	5.116	5.065	4.963	4.859	8
9	10.561	8.022	6.992	6.422	6.057	5.802	5.613	5.467	5.351	5.257	5.178	5.111	4.962	4.808	4.713	4.649	4.567	4.517	4.415	4.311	9
10	10.044	7.559	6.552	5.994	5.636	5.386	5.200	5.057	4.942	4.849	4.772	4.706	4.558	4.405	4.311	4.247	4.165	4.115	4.014	3.909	10
11	9.646	7.206	6.217	5.668	5.316	5.069	4.886	4.744	4.632	4.539	4.462	4.397	4.251	4.099	4.005	3.941	3.860	3.810	3.708	3.602	11
12	9.330	6.927	5.953	5.412	5.064	4.821	4.640	4.499	4.388	4.296	4.220	4.155	4.010	3.858	3.765	3.701	3.619	3.569	3.467	3.361	12
13	9.074	6.701	5.739	5.205	4.862	4.620	4.441	4.302	4.191	4.100	4.025	3.960	3.815	3.665	3.571	3.507	3.425	3.375	3.272	3.165	13
14	8.862	6.515	5.564	5.035	4.695	4.456	4.278	4.140	4.030	3.939	3.864	3.800	3.656	3.505	3.412	3.348	3.266	3.215	3.112	3.004	14
15	8.683	6.359	5.417	4.893	4.556	4.318	4.142	4.004	3.895	3.805	3.730	3.666	3.522	3.372	3.278	3.214	3.132	3.081	2.977	2.868	15
20	8.096	5.849	4.938	4.431	4.103	3.871	3.699	3.564	3.457	3.368	3.294	3.231	3.088	2.938	2.843	2.778	2.695	2.643	2.535	2.421	20
25	7.770	5.568	4.675	4.177	3.855	3.627	3.457	3.324	3.217	3.129	3.056	2.993	2.850	2.699	2.604	2.538	2.453	2.400	2.289	2.169	25
30	7.562	5.390	4.510	4.018	3.699	3.473	3.304	3.173	3.067	2.979	2.906	2.843	2.700	2.549	2.453	2.386	2.299	2.245	2.131	2.006	30
40	7.314	5.179	4.313	3.828	3.514	3.291	3.124	2.993	2.888	2.801	2.727	2.665	2.522	2.369	2.271	2.203	2.114	2.058	1.938	1.805	40
50	7.171	5.057	4.199	3.720	3.408	3.186	3.020	2.890	2.785	2.698	2.625	2.562	2.419	2.265	2.167	2.098	2.007	1.949	1.825	1.683	50
100	6.895	4.824	3.984	3.513	3.206	2.988	2.823	2.694	2.590	2.503	2.430	2.368	2.223	2.067	1.965	1.893	1.797	1.735	1.598	1.427	100
∞	6.635	4.605	3.782	3.319	3.017	2.802	2.639	2.511	2.407	2.321	2.248	2.185	2.039	1.878	1.773	1.696	1.592	1.523	1.358	1.000	∞

F-Distribution ($p=0.975$)

Use for one-tail tests at significance level 2.5% or two-tail tests at significance level 5%.

V_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	V_1
V_2																					V_2
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	973.0	976.7	984.9	993.1	998.1	1001.4	1005.6	1008.1	1013.2	1018.3	1
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50	2
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.37	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.96	13.90	3
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.32	8.26	4
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.08	6.02	5
6	8.813	7.260	6.599	6.227	5.988	5.820	5.695	5.600	5.523	5.461	5.410	5.366	5.269	5.168	5.107	5.065	5.012	4.980	4.915	4.849	6
7	8.073	6.542	5.890	5.523	5.285	5.119	4.995	4.899	4.823	4.761	4.709	4.666	4.568	4.467	4.405	4.362	4.309	4.276	4.210	4.142	7
8	7.571	6.059	5.416	5.053	4.817	4.652	4.529	4.433	4.357	4.295	4.243	4.200	4.101	3.999	3.937	3.894	3.840	3.807	3.739	3.670	8
9	7.209	5.715	5.078	4.718	4.484	4.320	4.197	4.102	4.026	3.964	3.912	3.868	3.769	3.667	3.604	3.560	3.505	3.472	3.403	3.333	9
10	6.937	5.456	4.826	4.468	4.236	4.072	3.950	3.855	3.779	3.717	3.665	3.621	3.522	3.419	3.355	3.311	3.255	3.221	3.152	3.080	10
11	6.724	5.256	4.630	4.275	4.044	3.881	3.759	3.664	3.588	3.526	3.474	3.430	3.330	3.226	3.162	3.118	3.061	3.027	2.956	2.883	11
12	6.554	5.096	4.474	4.121	3.891	3.728	3.607	3.512	3.436	3.374	3.321	3.277	3.177	3.073	3.008	2.963	2.906	2.871	2.800	2.725	12
13	6.414	4.965	4.347	3.996	3.767	3.604	3.483	3.388	3.312	3.250	3.197	3.153	3.053	2.948	2.882	2.837	2.780	2.744	2.671	2.595	13
14	6.298	4.857	4.242	3.892	3.663	3.501	3.380	3.285	3.209	3.147	3.095	3.050	2.949	2.844	2.778	2.732	2.674	2.638	2.565	2.487	14
15	6.200	4.765	4.153	3.804	3.576	3.415	3.293	3.199	3.123	3.060	3.008	2.963	2.862	2.756	2.689	2.644	2.585	2.549	2.474	2.395	15
20	5.871	4.461	3.859	3.515	3.289	3.128	3.007	2.913	2.837	2.774	2.721	2.676	2.573	2.464	2.396	2.349	2.287	2.249	2.170	2.085	20
25	5.686	4.291	3.694	3.353	3.129	2.969	2.848	2.753	2.677	2.613	2.560	2.515	2.411	2.300	2.230	2.182	2.118	2.079	1.996	1.906	25
30	5.568	4.182	3.589	3.250	3.026	2.867	2.746	2.651	2.575	2.511	2.458	2.412	2.307	2.195	2.124	2.074	2.009	1.968	1.882	1.787	30
40	5.424	4.051	3.463	3.126	2.904	2.744	2.624	2.529	2.452	2.388	2.334	2.288	2.182	2.068	1.994	1.943	1.875	1.832	1.741	1.637	40
50	5.340	3.975	3.390	3.054	2.833	2.674	2.553	2.458	2.381	2.317	2.263	2.216	2.109	1.993	1.919	1.866	1.796	1.752	1.656	1.545	50
100	5.179	3.828	3.250	2.917	2.696	2.537	2.417	2.321	2.244	2.179	2.125	2.077	1.968	1.849	1.770	1.715	1.640	1.592	1.483	1.347	100
∞	5.024	3.689	3.116	2.786	2.567	2.408	2.288	2.192	2.114	2.048	1.993	1.945	1.833	1.708	1.626	1.566	1.484	1.428	1.296	1.000	∞

F-Distribution ($p=0.95$)

Use for one-tail tests at significance level 5% or two-tail tests at significance level 10%.

V_1	1	2	3	4	5	6	7	8	9	10	11	12	15	20	25	30	40	50	100	∞	V_1
V_2																					V_2
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.0	243.9	245.9	248.0	249.3	250.1	251.1	251.8	253.0	254.3	1
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.49	19.50	2
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.55	8.53	3
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.66	5.63	4
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.41	4.36	5
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.099	4.060	4.027	4.000	3.938	3.874	3.835	3.808	3.774	3.754	3.712	3.669	6
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.677	3.637	3.603	3.575	3.511	3.445	3.404	3.376	3.340	3.319	3.275	3.230	7
8	5.318	4.459	4.066	3.838	3.688	3.581	3.500	3.438	3.388	3.347	3.313	3.284	3.218	3.150	3.108	3.079	3.043	3.020	2.975	2.928	8
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.179	3.137	3.102	3.073	3.006	2.936	2.893	2.864	2.826	2.803	2.756	2.707	9
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	3.020	2.978	2.943	2.913	2.845	2.774	2.730	2.700	2.661	2.637	2.588	2.538	10
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.896	2.854	2.818	2.788	2.719	2.646	2.601	2.570	2.531	2.507	2.457	2.404	11
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.796	2.753	2.717	2.687	2.617	2.544	2.498	2.466	2.426	2.401	2.350	2.296	12
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.714	2.671	2.635	2.604	2.533	2.459	2.412	2.380	2.339	2.314	2.261	2.206	13
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.646	2.602	2.565	2.534	2.463	2.388	2.341	2.308	2.266	2.241	2.187	2.131	14
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.588	2.544	2.507	2.475	2.403	2.328	2.280	2.247	2.204	2.178	2.123	2.066	15
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.393	2.348	2.310	2.278	2.203	2.124	2.074	2.039	1.994	1.966	1.907	1.843	20
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.282	2.236	2.198	2.165	2.089	2.007	1.955	1.919	1.872	1.842	1.779	1.711	25
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.211	2.165	2.126	2.092	2.015	1.932	1.878	1.841	1.792	1.761	1.695	1.622	30
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.124	2.077	2.038	2.003	1.924	1.839	1.783	1.744	1.693	1.660	1.589	1.509	40
50	4.034	3.183	2.790	2.557	2.400	2.286	2.199	2.130	2.073	2.026	1.986	1.952	1.871	1.784	1.727	1.687	1.634	1.599	1.525	1.438	50
100	3.936	3.087	2.696	2.463	2.305	2.191	2.103	2.032	1.975	1.927	1.886	1.850	1.768	1.676	1.616	1.573	1.515	1.477	1.392	1.283	100
∞	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.880	1.831	1.789	1.752	1.666	1.571	1.506	1.459	1.394	1.350	1.243	1.000	∞

TABLE 8 CRITICAL VALUES OF THE PRODUCT MOMENT CORRELATION COEFFICIENT

The table gives the critical values, for different significance levels, of the product moment correlation coefficient, r , for varying sample sizes, n .

One tail Two tail	10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%	One tail Two tail
n						n
4	0.8000	0.9000	0.9500	0.9800	0.9900	4
5	0.6870	0.8054	0.8783	0.9343	0.9587	5
6	0.6084	0.7293	0.8114	0.8822	0.9172	6
7	0.5509	0.6694	0.7545	0.8329	0.8745	7
8	0.5067	0.6215	0.7067	0.7887	0.8343	8
9	0.4716	0.5822	0.6664	0.7498	0.7977	9
10	0.4428	0.5494	0.6319	0.7155	0.7646	10
11	0.4187	0.5214	0.6021	0.6851	0.7348	11
12	0.3981	0.4973	0.5760	0.6581	0.7079	12
13	0.3802	0.4762	0.5529	0.6339	0.6835	13
14	0.3646	0.4575	0.5324	0.6120	0.6614	14
15	0.3507	0.4409	0.5140	0.5923	0.6411	15
16	0.3383	0.4259	0.4973	0.5742	0.6226	16
17	0.3271	0.4124	0.4821	0.5577	0.6055	17
18	0.3170	0.4000	0.4683	0.5425	0.5897	18
19	0.3077	0.3887	0.4555	0.5285	0.5751	19
20	0.2992	0.3783	0.4438	0.5155	0.5614	20
21	0.2914	0.3687	0.4329	0.5034	0.5487	21
22	0.2841	0.3598	0.4227	0.4921	0.5368	22
23	0.2774	0.3515	0.4132	0.4815	0.5256	23
24	0.2711	0.3438	0.4044	0.4716	0.5151	24
25	0.2653	0.3365	0.3961	0.4622	0.5052	25
26	0.2598	0.3297	0.3882	0.4534	0.4958	26
27	0.2546	0.3233	0.3809	0.4451	0.4869	27
28	0.2497	0.3172	0.3739	0.4372	0.4785	28
29	0.2451	0.3115	0.3673	0.4297	0.4705	29
30	0.2407	0.3061	0.3610	0.4226	0.4629	30
31	0.2366	0.3009	0.3550	0.4158	0.4556	31
32	0.2327	0.2960	0.3494	0.4093	0.4487	32
33	0.2289	0.2913	0.3440	0.4032	0.4421	33
34	0.2254	0.2869	0.3388	0.3972	0.4357	34
35	0.2220	0.2826	0.3338	0.3916	0.4296	35
36	0.2187	0.2785	0.3291	0.3862	0.4238	36
37	0.2156	0.2746	0.3246	0.3810	0.4182	37
38	0.2126	0.2709	0.3202	0.3760	0.4128	38
39	0.2097	0.2673	0.3160	0.3712	0.4076	39
40	0.2070	0.2638	0.3120	0.3665	0.4026	40
41	0.2043	0.2605	0.3081	0.3621	0.3978	41
42	0.2018	0.2573	0.3044	0.3578	0.3932	42
43	0.1993	0.2542	0.3008	0.3536	0.3887	43
44	0.1970	0.2512	0.2973	0.3496	0.3843	44
45	0.1947	0.2483	0.2940	0.3457	0.3801	45
46	0.1925	0.2455	0.2907	0.3420	0.3761	46
47	0.1903	0.2429	0.2876	0.3384	0.3721	47
48	0.1883	0.2403	0.2845	0.3348	0.3683	48
49	0.1863	0.2377	0.2816	0.3314	0.3646	49
50	0.1843	0.2353	0.2787	0.3281	0.3610	50
60	0.1678	0.2144	0.2542	0.2997	0.3301	60
70	0.1550	0.1982	0.2352	0.2776	0.3060	70
80	0.1448	0.1852	0.2199	0.2597	0.2864	80
90	0.1364	0.1745	0.2072	0.2449	0.2702	90
100	0.1292	0.1654	0.1966	0.2324	0.2565	100

TABLE 9 CRITICAL VALUES OF SPEARMAN'S RANK CORRELATION COEFFICIENT

The table gives the critical values, for different significance levels, of Spearman's rank correlation coefficient, r_s , for varying sample sizes, n .

Since r_s is discrete, exact significance levels cannot be obtained in most cases. The critical values given are those with significance levels closest to the stated value.

One tail Two tail	10% 20%	5% 10%	2.5% 5%	1% 2%	0.5% 1%	One tail Two tail
<i>n</i>						<i>n</i>
4	1.0000	1.0000	1.0000	1.0000	1.0000	4
5	0.7000	0.9000	0.9000	1.0000	1.0000	5
6	0.6571	0.7714	0.8286	0.9429	0.9429	6
7	0.5714	0.6786	0.7857	0.8571	0.8929	7
8	0.5476	0.6429	0.7381	0.8095	0.8571	8
9	0.4833	0.6000	0.6833	0.7667	0.8167	9
10	0.4424	0.5636	0.6485	0.7333	0.7818	10
11	0.4182	0.5273	0.6091	0.7000	0.7545	11
12	0.3986	0.5035	0.5874	0.6713	0.7273	12
13	0.3791	0.4780	0.5604	0.6484	0.6978	13
14	0.3670	0.4593	0.5385	0.6220	0.6747	14
15	0.3500	0.4429	0.5179	0.6000	0.6536	15
16	0.3382	0.4265	0.5029	0.5824	0.6324	16
17	0.3271	0.4124	0.4821	0.5577	0.6055	17
18	0.3170	0.4000	0.4683	0.5425	0.5897	18
19	0.3077	0.3887	0.4555	0.5285	0.5751	19
20	0.2992	0.3783	0.4438	0.5155	0.5614	20
21	0.2914	0.3687	0.4329	0.5034	0.5487	21
22	0.2841	0.3598	0.4227	0.4921	0.5368	22
23	0.2774	0.3515	0.4132	0.4815	0.5256	23
24	0.2711	0.3438	0.4044	0.4716	0.5151	24
25	0.2653	0.3365	0.3961	0.4622	0.5052	25
26	0.2598	0.3297	0.3882	0.4534	0.4958	26
27	0.2546	0.3233	0.3809	0.4451	0.4869	27
28	0.2497	0.3172	0.3739	0.4372	0.4785	28
29	0.2451	0.3115	0.3673	0.4297	0.4705	29
30	0.2407	0.3061	0.3610	0.4226	0.4629	30
31	0.2366	0.3009	0.3550	0.4158	0.4556	31
32	0.2327	0.2960	0.3494	0.4093	0.4487	32
33	0.2289	0.2913	0.3440	0.4032	0.4421	33
34	0.2254	0.2869	0.3388	0.3972	0.4357	34
35	0.2220	0.2826	0.3338	0.3916	0.4296	35
36	0.2187	0.2785	0.3291	0.3862	0.4238	36
37	0.2156	0.2746	0.3246	0.3810	0.4182	37
38	0.2126	0.2709	0.3202	0.3760	0.4128	38
39	0.2097	0.2673	0.3160	0.3712	0.4076	39
40	0.2070	0.2638	0.3120	0.3665	0.4026	40
41	0.2043	0.2605	0.3081	0.3621	0.3978	41
42	0.2018	0.2573	0.3044	0.3578	0.3932	42
43	0.1993	0.2542	0.3008	0.3536	0.3887	43
44	0.1970	0.2512	0.2973	0.3496	0.3843	44
45	0.1947	0.2483	0.2940	0.3457	0.3801	45
46	0.1925	0.2455	0.2907	0.3420	0.3761	46
47	0.1903	0.2429	0.2876	0.3384	0.3721	47
48	0.1883	0.2403	0.2845	0.3348	0.3683	48
49	0.1863	0.2377	0.2816	0.3314	0.3646	49
50	0.1843	0.2353	0.2787	0.3281	0.3610	50
60	0.1678	0.2144	0.2542	0.2997	0.3301	60
70	0.1550	0.1982	0.2352	0.2776	0.3060	70
80	0.1448	0.1852	0.2199	0.2597	0.2864	80
90	0.1364	0.1745	0.2072	0.2449	0.2702	90
100	0.1292	0.1654	0.1966	0.2324	0.2565	100

TABLE 10 CRITICAL VALUES OF THE WILCOXON SIGNED RANK STATISTIC

The table gives the lower tail critical values of the statistic T .

The upper tail critical values are given by $\frac{1}{2}n(n+1) - T$.

T is the sum of the ranks of observations with the same sign.

Since T is discrete, exact significance levels cannot usually be obtained.

The critical values tabulated are those with significance levels closest to the stated value.

The critical region includes the tabulated value.

One tail	10%	5%	2.5%	1%	0.5%
Two tail	20%	10%	5%	2%	1%
n					
3	0				
4	1	0			
5	2	1	0		
6	4	2	1	0	
7	6	4	2	0	0
8	8	6	4	2	0
9	11	8	6	3	2
10	14	11	8	5	3
11	18	14	11	7	5
12	22	17	14	10	7
13	26	21	17	13	10
14	31	26	21	16	13
15	37	30	25	20	16
16	42	36	30	24	19
17	49	41	35	28	23
18	55	47	40	33	28
19	62	54	46	38	32
20	70	60	52	43	37

TABLE 11 CRITICAL VALUES OF THE MANN-WHITNEY STATISTIC

The table gives the lower tail critical values of the statistic U .

The upper tail critical values are given by $mn - U$.

$U = T - \frac{n(n+1)}{2}$ where T is the sum of the ranks of the sample of size n .

Since U is discrete, exact significance levels cannot be obtained.

The critical values tabulated are those with significance levels closest to the stated value.

The critical region includes the tabulated value.

One tail 5% Two tail 10%

m	2	3	4	5	6	7	8	9	10	11	12
n											
2		0	0	0	0	1	1	1	2	2	2
3	0	0	1	1	2	3	3	4	5	5	6
4	0	1	2	3	4	5	6	7	8	9	10
5	0	1	3	4	5	7	8	10	11	12	14
6	0	2	4	5	7	9	11	12	14	16	18
7	1	3	5	7	9	11	13	15	18	20	22
8	1	3	6	8	11	13	16	18	21	24	26
9	1	4	7	10	12	15	18	21	24	27	30
10	2	5	8	11	14	18	21	24	28	31	34
11	2	5	9	12	16	20	24	27	31	35	39
12	2	6	10	14	18	22	26	30	34	39	43

One tail 2.5% Two tail 5%

m	2	3	4	5	6	7	8	9	10	11	12
n											
2				0	0	0	0	0	1	1	1
3			0	0	1	2	2	3	3	4	4
4		0	1	2	2	3	4	5	6	7	8
5	0	0	2	3	4	5	6	7	9	10	11
6	0	1	2	4	5	7	8	10	12	13	15
7	0	2	3	5	7	9	11	13	15	17	18
8	0	2	4	6	8	11	13	15	18	20	22
9	0	3	5	7	10	13	15	18	21	23	26
10	1	3	6	9	12	15	18	21	24	27	30
11	1	4	7	10	13	17	20	23	27	30	34
12	1	4	8	11	15	18	22	26	30	34	38

TABLE 12 CONTROL CHARTS FOR VARIABILITY

For range charts, multiply σ by the appropriate value of D .

For standard deviation charts, multiply σ by the appropriate value of E .

To obtain an estimate of σ , multiply the mean range by the appropriate value of b .

Normal distribution is assumed.

Sample size	$D_{0.999}$	$D_{0.975}$	$D_{0.025}$	$D_{0.001}$	$E_{0.999}$	$E_{0.975}$	$E_{0.025}$	$E_{0.001}$	b
2			3.170	4.654			2.24	3.29	0.8862
3	0.060	0.303	3.682	5.063	0.03	0.16	1.92	2.63	0.5908
4	0.199	0.595	3.984	5.309	0.09	0.27	1.76	2.33	0.4857
5	0.367	0.850	4.197	5.484	0.15	0.35	1.67	2.15	0.4299
6	0.535	1.066	4.361	5.619	0.20	0.41	1.60	2.03	0.3946
7	0.691	1.251	4.494	5.730	0.25	0.45	1.55	1.93	0.3698
8	0.835	1.410	4.605	5.823	0.29	0.49	1.51	1.86	0.3512
10	1.085	1.674	4.784	5.973	0.36	0.55	1.45	1.76	0.3249
12	1.293	1.884	4.925	6.096	0.41	0.59	1.41	1.69	0.3069

TABLE 13 RANDOM NUMBERS

2 9 9 2 7	6 6 1 8 7	8 0 7 8 4	3 7 5 4 2	6 2 4 4 6	1 3 4 8 1	7 2 7 3 0	4 8 5 1 1	4 2 3 1 5
6 2 5 0 6	2 2 7 8 0	3 0 7 2 0	7 9 3 3 8	6 8 3 5 8	6 2 7 6 5	3 3 4 0 1	8 2 7 5 8	4 2 9 2 9
8 3 7 5 2	1 0 6 6 4	1 2 1 9 3	8 8 7 6 6	7 6 7 6 3	9 0 9 7 7	4 6 8 8 1	5 9 0 8 9	3 9 6 4 8
7 5 7 0 3	2 7 5 2 2	7 9 5 0 4	0 6 6 6 2	2 5 4 6 8	9 2 4 0 7	1 9 6 2 6	6 1 1 7 3	5 2 7 9 3
9 9 6 1 7	5 9 1 2 0	3 3 5 5 4	3 2 9 0 4	9 5 3 1 2	6 1 7 6 3	6 8 8 6 8	9 4 1 7 9	7 3 4 4 2
1 7 4 9 0	6 4 9 0 0	1 2 6 9 0	9 5 4 7 4	5 3 8 4 9	6 4 7 9 1	3 5 8 4 3	4 4 8 3 2	0 1 2 9 6
0 2 3 8 4	1 6 6 8 0	7 6 6 3 7	4 2 4 3 7	2 7 9 9 4	2 4 7 1 8	0 9 5 6 6	4 3 8 2 1	8 9 3 1 5
4 4 0 3 1	5 1 6 6 8	8 5 9 0 7	2 2 6 8 3	0 6 1 1 9	2 5 3 6 0	3 5 4 8 0	9 1 3 3 4	0 1 5 2 2
9 4 0 5 8	3 6 4 6 6	9 9 7 1 7	5 7 6 5 1	0 2 5 1 2	9 8 7 8 5	8 6 4 9 1	7 6 8 1 2	1 0 3 2 4
8 8 7 8 3	7 7 1 2 7	9 5 7 8 3	4 0 6 6 6	8 2 5 3 9	8 4 2 2 4	9 4 3 5 4	4 1 9 7 9	3 2 8 2 3
3 3 3 8 0	4 7 4 4 4	0 2 9 3 6	5 7 3 0 3	3 1 4 5 8	2 8 6 6 9	2 2 5 3 8	6 6 8 8 4	3 8 3 7 0
9 5 1 9 8	4 1 6 8 4	8 9 0 6 6	1 7 9 6 3	3 9 0 4 2	5 0 7 9 1	4 4 6 8 3	1 5 1 3 4	1 9 9 0 9
0 3 1 8 3	6 2 7 0 6	6 5 5 3 1	4 7 7 6 7	4 2 3 4 7	5 1 8 9 9	3 3 5 8 2	2 8 0 9 8	4 3 1 6 8
2 6 6 2 3	0 0 5 5 0	5 2 3 2 9	9 0 2 9 2	3 7 5 0 8	9 7 3 1 0	9 2 0 4 9	4 7 3 6 5	8 0 2 4 2
5 7 9 2 9	1 4 2 9 0	0 8 1 1 8	9 5 4 7 3	9 1 5 8 6	5 8 9 5 3	7 4 9 9 8	7 3 9 5 0	5 4 6 6 2
4 9 1 0 3	9 2 1 5 0	7 8 2 1 1	2 7 7 6 2	1 8 1 3 5	4 3 4 7 9	6 1 6 9 8	7 7 7 6 8	0 0 2 2 3
9 8 6 3 4	3 1 8 7 0	5 6 8 3 9	6 0 4 7 8	6 2 1 2 9	8 7 1 4 9	6 0 2 4 0	0 9 0 7 9	3 8 5 6 7
9 0 5 9 3	7 6 2 4 8	6 0 3 7 9	9 8 2 0 4	5 9 2 5 4	5 1 6 1 6	4 1 0 9 1	1 1 8 1 8	1 1 0 0 1
6 8 6 0 4	9 0 2 9 8	3 8 5 9 5	5 2 0 4 8	9 5 1 3 7	7 3 3 6 3	5 3 3 0 7	3 7 9 1 4	2 7 9 0 3
7 2 7 1 1	4 3 4 4 1	8 7 1 0 8	8 2 1 5 5	4 3 6 5 0	8 1 9 6 7	5 6 3 4 8	1 9 8 7 8	7 5 8 1 3
0 8 1 9 3	0 5 3 0 2	1 1 3 5 2	4 8 3 6 9	5 5 7 3 1	8 1 1 5 8	2 1 0 3 7	2 9 5 3 4	9 8 0 7 4
7 4 8 2 9	5 1 6 9 5	5 1 6 8 2	9 7 6 6 0	9 7 1 1 0	6 9 5 4 0	6 9 7 7 6	2 2 7 3 6	5 4 6 3 5
2 5 9 0 0	8 1 2 6 5	2 5 3 3 9	4 3 8 7 5	3 8 5 6 3	4 3 5 3 0	3 6 2 8 9	7 8 8 1 0	1 8 9 5 9
4 2 4 1 7	5 0 1 0 6	2 4 7 5 2	9 4 6 6 4	1 1 6 1 1	0 5 7 2 0	7 7 0 9 1	9 6 3 3 8	6 8 5 0 7
3 2 2 9 1	5 7 6 5 3	4 2 1 3 5	3 6 4 4 0	7 9 4 2 7	1 1 6 6 0	1 5 6 6 6	5 5 6 8 2	2 5 4 4 9
2 4 0 9 6	5 7 4 1 9	3 5 6 1 1	9 1 1 7 9	5 1 4 6 4	9 4 2 8 4	9 2 4 4 9	9 7 3 4 7	2 2 1 8 4
5 0 3 4 4	2 2 8 2 4	0 9 1 9 3	9 8 7 7 1	3 0 9 6 3	0 2 8 7 6	9 7 6 7 1	5 6 3 9 7	9 1 6 7 7
7 6 6 7 2	5 2 8 7 2	4 8 6 1 0	3 1 3 1 4	2 1 5 4 5	2 3 6 0 1	1 8 2 7 8	9 3 5 3 0	0 2 1 4 2
5 0 3 8 5	7 0 1 1 2	2 6 8 9 7	0 0 0 7 7	0 4 8 0 3	9 8 3 2 6	8 8 9 3 3	1 7 7 1 0	7 5 7 5 0
6 4 2 2 2	9 5 9 2 0	8 0 5 3 4	5 5 0 9 0	0 4 1 0 5	0 1 4 1 5	1 1 3 7 6	2 0 7 0 9	7 8 8 8 7
1 1 6 0 2	0 6 8 9 1	0 7 9 2 4	4 2 9 5 9	7 3 1 2 4	3 6 8 3 0	7 0 5 5 9	5 5 7 3 9	7 3 1 9 1
8 7 9 6 2	9 2 0 7 1	1 3 4 0 5	0 5 0 5 7	8 5 9 4 7	7 3 0 4 3	9 4 2 0 8	5 2 8 2 9	8 8 2 7 2
4 1 5 9 5	0 7 6 1 1	3 6 6 4 6	7 0 8 6 3	5 7 7 9 7	8 2 0 3 3	1 9 2 3 6	7 4 6 0 8	1 4 3 2 4
3 4 9 1 7	5 8 0 3 8	4 7 2 3 0	3 8 8 1 7	7 0 6 0 5	6 2 7 7 1	0 2 8 5 1	2 3 1 9 5	2 0 2 0 4
5 0 6 2 2	7 6 1 3 3	5 4 0 6 5	3 4 0 5 5	1 3 9 6 1	0 7 6 0 4	3 0 2 6 0	9 2 2 4 0	4 0 7 3 6
1 4 4 2 2	5 8 2 8 2	7 3 6 7 3	0 4 5 3 5	0 3 5 5 7	4 0 0 3 6	8 5 4 7 5	1 6 0 2 1	7 7 1 7 3
4 8 2 5 4	7 1 0 4 3	4 4 9 4 2	1 2 2 5 2	5 9 5 5 7	5 3 0 1 3	2 6 1 7 0	2 1 9 8 0	1 8 5 8 2
5 9 3 2 2	6 5 2 5 1	8 4 3 7 9	0 5 9 8 5	4 5 7 6 5	3 8 3 4 9	6 8 6 6 1	1 8 1 2 9	2 9 3 3 8
0 4 2 2 4	1 9 5 9 3	7 2 5 5 4	5 4 2 3 9	4 4 8 7 0	3 8 7 2 6	5 1 2 9 7	8 2 4 1 2	6 5 7 9 9
1 7 2 6 4	4 1 1 5 4	1 6 0 1 9	7 0 4 8 1	9 7 7 1 6	5 3 1 8 5	5 3 9 0 1	8 9 0 3 6	0 1 2 5 3
0 9 6 3 2	0 7 1 8 2	7 8 1 1 1	1 9 2 5 3	1 2 4 1 4	7 3 4 9 6	2 4 0 9 0	5 4 9 7 4	4 8 9 4 1
5 4 2 8 2	7 4 6 2 6	4 0 8 6 6	9 1 3 7 1	4 4 5 8 9	3 1 4 7 8	5 8 8 4 2	7 1 9 6 1	3 8 4 8 7
8 0 2 0 7	4 3 4 9 7	3 7 0 7 9	5 3 9 7 4	2 0 2 4 1	6 2 5 7 6	1 5 6 6 0	6 8 4 0 5	5 7 9 8 2
9 3 8 9 9	9 4 3 0 9	5 6 7 3 2	5 9 8 5 8	2 8 4 5 7	7 4 5 4 6	4 5 4 2 4	9 2 4 9 6	7 1 0 3 5
4 6 8 6 9	5 2 2 8 4	0 0 0 0 0	4 2 5 5 4	5 8 7 7 0	8 3 4 5 8	5 8 4 2 5	6 0 9 5 6	2 1 5 9 5