A-LEVEL
Mathematics
Statistics 3 – MS03
Mark scheme

6360
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Version/Stage: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

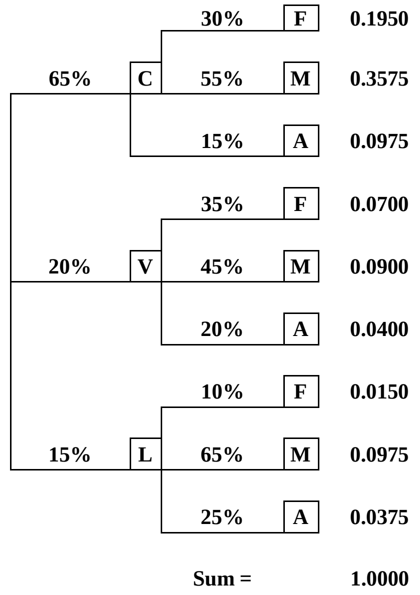
Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1 **(a)**	$96\% \ \Rightarrow \ z = \underline{\mathbf{2.05 \text{ to } 2.06}}$	B1		AWFW (2.0537)
	$\hat{p} = \frac{23}{200} = \underline{\mathbf{0.115}}$	B1		CAO; or equivalent
	Approximate CI for p: $\hat{p} \pm z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	M1		Used
	$0.115 \pm 2.0537\sqrt{\frac{0.115 \times 0.885}{200}}$	AF1		F on $\hat{p}$ and z
	$\underline{\mathbf{0.115 \pm 0.046}}$ **or** $\underline{\mathbf{(0.069,\ 0.161)}}$	A1	5	CAO/AWRT AWRT
(b)	$2 \text{ in } 50 = \frac{2}{50} = \underline{\mathbf{0.04 < LCL \text{ or } CI}}$	BF1		F on LCL or CI
	Thus evidence to **reject supplier's claim**	Bdep1	2	Dependent on BF1 Accept fairly definitive conclusion
		Total	7	

Q	Solution	Marks	Total	Comments
2	H_0: $\mu_B = \mu_G$ H_1: $\mu_B \neq \mu_G$	B1		At least H_1; allow suffices of 1 & 2 or X & Y, etc
	SL $\alpha = 0.05$ (5%) CV $z = (\pm)\underline{\mathbf{1.96}}$	B1		AWRT (1.9600)
	$z = \frac{\lvert \bar{b} - \bar{g} \rvert}{\sqrt{\frac{\sigma_B^2}{n_B} + \frac{\sigma_G^2}{n_G}}} = \frac{\lvert 21.35 - 21.90 \rvert}{\sqrt{\frac{0.5625}{20} + \frac{0.9025}{15}}}$	M1 M1		Numerator Denominator
	$= (\pm)\underline{\mathbf{1.85}}$	A1		Dependent on at least M1 M0 AWRT (1.8510) Ignore sign (p-value = 0.0642)
	Evidence, at 5% level, that $\boldsymbol{\mu_B = \mu_G}$ **or** **No evidence**, at 5% level, that $\boldsymbol{\mu_B \neq \mu_G}$	AF1	6	F on CV & z-value; consistent signs Definitive conclusion $\Rightarrow$ AF0 F on 5% & p-value; consistent areas
		Total	6	

Q	Solution	Marks	Total	Comments
3 **(a)**		B1 B1 B1	3	Shape; 3×3 branches Labels; C, V, L **and** ≥ 1 F, M, A Percentages or equivalent for C, V, L **and** ≥ 1 F, M, A
(b) **(i)**	$P((C \cup L) \cap M) = P(C \cap M) + P(L \cap M)$ $= (0.65 \times 0.55) + (0.15 \times 0.65)$ $= 0.3575 + 0.0975 = \underline{\mathbf{0.455 \text{ or } 91/200}}$	M1 A1	(2)	 CAO
(ii)	$P(L \mid A) = P(L \cap A) \div P(A)$ $= \dfrac{0.15 \times 0.25}{(0.65 \times 0.15) + (0.20 \times 0.20) + (0.15 \times 0.25)}$ $= \dfrac{0.0375}{0.0975 + 0.04 + 0.0375} = \dfrac{0.0375}{0.1750} = \underline{\mathbf{0.214}}$	M1 M1 A1	(3)	Numerator Denominator AWRT (0.21429) CAO (3/14)
(iii)	$P(F' \mid C') = P(F' \cap C') \div P(C')$ $= \dfrac{0.2 \times (0.45 + 0.20) + 0.15(0.65 + 0.25)}{0.35}$ $= \dfrac{0.13 + 0.135}{0.35} = \dfrac{0.265}{0.35} = \underline{\mathbf{0.757}}$	M1 M1 A1	(3)	Numerator Denominator AWRT (0.75714) CAO (53/70)
			8	
(c)	Prob $= P(C \mid F) \times P(V \mid F) \times P(L \mid F) \times 3! =$ $\dfrac{(0.65 \times 0.30) \times (0.20 \times 0.35) \times (0.15 \times 0.10)}{\left[(0.65 \times 0.30) + (0.20 \times 0.35) + (0.15 \times 0.10)\right]^3} \times 6$ $= \dfrac{(0.195 \times 0.07 \times 0.015) \text{ or } (0.00020475)}{0.28^3} \times 6$ $= \underline{\mathbf{0.056}}$	M1 M1 M1 A1	4	Numerator Denominator $\times$ 3! or 6 AWRT (0.05596) CAO (351/6272)
		Total	**15**	

Q	Solution	Marks	Total	Comments
4 **(a)**	$98\% \;\Rightarrow\; z = \underline{\mathbf{2.32 \text{ to } 2.33}}$	B1		AWFW (2.3263)
	CI for $\mu_E - \mu_G$: $(\bar{e} - \bar{g}) \pm z\sqrt{\dfrac{s_E^2}{n_E} + \dfrac{s_G^2}{n_G}}$	M1 m1		General form used Correct form used for SD Accept pooling
	$(42.6 - 39.7) \pm 2.3263\sqrt{\dfrac{6.2^2}{50} + \dfrac{5.3^2}{50}}$	AF1		F on z Pooling gives $2.3263\sqrt{1.3306}$
	$\underline{\mathbf{2.9 \pm 2.7 \text{ or } (0.2,\ 5.6)}}$	A1	5	AWRT
(b) **(i)**	**Random**	B1	1	CAO
(ii)	**Large samples** (both > 25 or 30) so can apply **Central Limit Theorem**	B1 Bdep1	2	Dependent on B1
		Total	**8**	

Q	Solution	Marks	Total	Comments
5 **(a)(i)**	Distribution of X is **symmetrical** around 4	B1		Accept calculation
	$E(X^2) = 0.2^2 \times 0.05 + \ldots\ldots + 6^2 \times 0.05$	M1		Must show method for $E(X^2)$
	$= 0.20 + 2.25 + 6.40 + 6.25 + 1.80 = \underline{\mathbf{16.9}}$	A1		CAO
	$\text{Var}(X) = E(X^2) - 4^2 = 16.9 - 16 = \underline{\mathbf{0.9}}$	B1	4	**AG**; must show method for $\text{Var}(X)$
(ii)	$\text{Cov(X, Y)} = 14.4 - 4 \times 3.7$ $= \underline{\mathbf{-0.4}}$	M1 A1		
	$\rho_{XY} = \dfrac{-0.4}{\sqrt{0.9 \times 0.61}} = \underline{\mathbf{-0.54}}$	M1 AF1	4	Expression AWRT (–0.53985) F on $\text{Cov}(X, Y)$
(b) **(i)&(ii)**	$E(T) = \underline{\mathbf{7.7}}$ $\quad E(D) = \underline{\mathbf{0.3}}$	B1		CAO; both
	$\text{Var}(T) = 0.9 + 0.61 + 2 \times (-0.4)$	M1		Use of either $\text{Var}(X \pm Y) =$ $\text{Var}(X) + \text{Var}(Y) \pm 2\text{Cov}(X, Y)$
	$= \underline{\mathbf{0.71}}$	A1		CAO
	$\text{Var}(D) = 0.9 + 0.61 - 2 \times (-0.4) = \underline{\mathbf{2.31}}$	A1	4	CAO
		Total	**12**	

Q	Solution	Marks	Total	Comments
6 (a)	$\text{Var}\left(\bar{X}_A - \bar{X}_B\right) = \frac{18.8}{n} + \frac{18.8}{n}$	M1		Award for $\frac{18.8}{n}$ or $\frac{(2)\sigma^2}{n}$
	$= \underline{\mathbf{37.6/\mathit{n}}}$	A1	**2**	OE
(b)	$99\% \Rightarrow z = \underline{\mathbf{2.57 \text{ to } 2.58}}$	B1		AWFW (2.5758)
	Require: $2 \times z \times \sqrt{\frac{37.6}{n}} \le 5$	M1		Award if "no 2", incorrect z-value, $\sqrt{\frac{18.8}{n}}$ or $\sqrt{\frac{(2)\sigma^2}{n}}$ or $\sqrt{\frac{c}{n}}$ from (a)
	$2 \times 2.5758 \times \sqrt{\frac{37.6}{n}} \le 5$	A1		Fully correct expression
	$n \ge \frac{4 \times 2.5758^2 \times 37.6}{25}$	m1		Attempt at solving equation involving $\sqrt{n}$ for n or $\sqrt{n}$
	$n = \underline{\mathbf{40}}$	A1	**5**	CAO
Note	Accept equalities or strict inequalities throughout			
		Total	**7**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$\underline{X \sim \text{Po}(\lambda)}$ $\text{E}(X) = \sum_{x=0}^{\infty} x \times \frac{e^{-\lambda}\lambda^x}{x!}$	M1		Used; ignore limits until A1 Accept a list of ≥ 3 terms summed
	$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!}$	M1		Factor of (at least) λ Division of $x!$ by x
	$= \lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda e^{-\lambda} e^{\lambda} = \underline{\boldsymbol{\lambda}}$ $\quad (y = x - 1)$	A1	3	**AG**; fully correct convincing solution with valid reason for (= λ)
(ii)	$\text{Var}(X) = \text{E}(X^2) - \lambda^2 = (\lambda^2 + \lambda) - \lambda^2 = \underline{\boldsymbol{\lambda}}$	B1	1	**AG**; fully correct convincing solution
(b)(i)	H_0: $\lambda = 10$ H_1: $\lambda > 10$	B1		Both; here or in (b)(ii)(A) and only mark available here if not exact test
	$\text{P}(X \geq 15 \mid \lambda = 10) = \mathbf{1 - (0.9165 \text{ or } 0.9513)}$	M1		
	$= \underline{\mathbf{0.083 \text{ to } 0.084}}$	A1		AWFW (0.0835)
	Calculated p-value > 0.05 (5%)	m1		Comparison with 0.05
	No evidence, at 5% level, that $\boldsymbol{\lambda} > \mathbf{10}$	AF1	5	OE; F on p-value Definitive conclusion $\Rightarrow$ AF0
(ii)(A)	5% $\Rightarrow$ CV for z = $\underline{\mathbf{1.64 \text{ to } 1.65}}$	B1		AWFW; seen anywhere (1.6449)
	$z = \frac{241(-0.5) - 200}{\sqrt{200 \text{ or } 241}} = \underline{\mathbf{2.86 \text{ to } 2.9}}$	M1 A1		OE; allow (+0.5) AWFW
	Evidence, at 5% level, that $\boldsymbol{\lambda} > \mathbf{10}$	AF1	4	OE; F on z-value & CV Definitive conclusion $\Rightarrow$ AF0
(B)	$\frac{\text{CV}(-0.5) - 200}{\sqrt{200 \text{ or } 241}} = 1.6449$	M1 AF1		OE; allow (+0.5) but must be for **total** number of faults F on {(CV for z) & (z-statistic)} in (A)
	CV for X = $\underline{\mathbf{223 \text{ to } 224}}$	A1	3	AWFW
(C)	P(Type II error) = P(accept H_0 | H_0 false)	B1		OE; stated or used
	$\text{P}(X < \text{CV} \mid \lambda = 12) =$ $\text{P}\left(Z < \frac{(222 \text{ to } 224) - 240}{\sqrt{240 \text{ or } 200}}\right) =$	M1		OE; FT on CV from (B)
	$\text{P}(Z < -1.1 \text{ to } -1.03) = 1 - \text{P}(Z < 1.03 \text{ to } 1.1)$	m1		Area change
	$= 1 - (0.848 \text{ to } 0.865) = \underline{\mathbf{0.13 \text{ to } 0.16}}$	A1	4	AWFW
		Total	**20**	