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General Certificate of Education (A-level)
2011

Mathematics MS2B

(Specification 6360)

Statistics 2B

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Question	Solution	Marks	Total	Comments
1(a)	$\left.\begin{array}{l} Y \sim \mathrm{N}(\mu_Y, 4) \\ n = 16, \bar{y} = 450 \end{array}\right\}$ (known variance) $\Rightarrow$ use z			
	For 95% CI $\quad z_{crit} = 1.96$	B1		
	$\left.\begin{array}{l} 450 \pm 1.96 \times \dfrac{2}{\sqrt{16}} \\ 450 \pm 0.98 \end{array}\right\}$	M1		
	$(449, 451)$	A1	3	awrt
(b)(i)	$X \sim \mathrm{N}(\mu_X, \sigma^2)$ (unknown variance) $\Rightarrow$ use t_{n-1}			
	$\left.\begin{array}{l} n = 9 \quad \& \quad \bar{x} = \dfrac{4950}{9} = 550 \\ s_{n-1}^2 = \dfrac{334}{8} = 41.75 \quad (s_{n-1} = 6.461) \end{array}\right\}$	B1		both
	For 90% CI $\quad t_{crit} = 1.860$	B1		
	$550 \pm 1.860 \times \dfrac{\sqrt{41.75}}{\sqrt{9}}$	M1		$\left\{\begin{array}{l} \text{their } \bar{x} \pm t_8 \times \dfrac{\text{their } s_{n-1}}{\sqrt{9}} \\ \text{(must have a } t_8\text{-value)} \end{array}\right.$
	550 ± 4.0	A1ft		
	$(546, 554)$	A1	5	awrt
(ii)	545 not in 90% CI $\therefore$ Reject claim Evidence to suggest that **mean** content of a bottle of tomato sauce is not 545 grams and hence to reject Holly's claim at **10% level of significance**.	B1ft (dep on (b)(i)) E1ft (dep on (b)(i)) B1	3	**Alternative** $H_0: \mu_X = 545$ $H_1: \mu_X \neq 545$ $t = \dfrac{550 - 545}{\sqrt{41.75/9}} = 2.32$ $\left.\begin{array}{l} t_{\text{crit}} = 1.86 < 2.32 \\ \text{Reject } H_0 \end{array}\right\}$ **B1** Comment in context **E1** **10% significance level B1**
	Alternatives (such as): Claim justified at 1% level of significance			$t_{\text{crit}} = 3.355 > 2.32$ Accept H_0 1% significance level
	Total		**11**	

MS2B(cont)

<table>
<tr><th>Question</th><th>Solution</th><th>Marks</th><th>Total</th><th>Comments</th></tr>
<tr><td>2(a)</td><td>

	C	L	LD	OP	Tot
M	156	144	120	60	480
F	216	135	108	81	540
Tot	372	279	228	141	1020

</td><td>B1
B1</td><td>2</td><td>For each correct row</td></tr>
<tr><td>(b)</td><td>H_0: No association between gender and the way students vote

H_1: Association between gender and the way students vote</td><td>B1</td><td></td><td>For at least H_0 correct</td></tr>
<tr><td></td><td>

O_i	E_i	$(O_i - E_i)^2 / E_i$
156	175.06	2.075
216	196.94	1.844
144	131.29	1.230
135	147.71	1.093
120	107.29	1.505
108	120.71	1.337
60	66.35	0.608
81	74.65	0.541
		$X^2 = 10.233$

</td><td>M1
M1
M1
A1</td><td></td><td>Attempt at E_i
Attempt at $(O_i - E_i)^2 / E_i$
Attempt at $\sum\left(\frac{(O_i - E_i)^2}{E_i}\right)$
awfw 10.2 to 10.3
(A1 dependent on all 3 method marks)</td></tr>
<tr><td></td><td>$\nu = 3 \Rightarrow \chi^2_{crit} = 11.345$</td><td>B1
B1ft</td><td></td><td>ft on their v value</td></tr>
<tr><td></td><td>$X^2 < \chi^2_{crit}$ $\therefore$ accept H_0</td><td>A1</td><td></td><td></td></tr>
<tr><td></td><td>Accept claim at 1% level.
Evidence to suggest that the way students vote is independent of gender.</td><td>E1</td><td>9</td><td></td></tr>
<tr><td></td><td>Total</td><td></td><td>11</td><td></td></tr>
</table>

MS2B(cont)

Question	Solution	Marks	Total	Comments
3(a)(i)	$X \sim \mathrm{P_o}(0.6)$ $\mathrm{P}(X \le 1) = 0.8781$	B1	1	Awrt 0.878
(ii)	For matches : The number of run outs: $Y \sim \mathrm{P_0}(0.15)$ $\mathrm{P}(Y \ge 1) = 1 - \mathrm{P}(Y = 0)$ $\left.\begin{matrix} = 1 - e^{-0.15} \\ = 1 - 0.8607 \end{matrix}\right\}$	M1		must **use** $\mathrm{P_0}(0.15)$
	$= 0.1393$	A1		awrt 0.139
	$\mathrm{P}(X \le 1 \text{ and } Y \ge 1) = 0.8781 \times 0.1393$	M1		their (a)(i)$\times$their $\mathrm{P}(Y \ge 1)$
	$= 0.122$	A1	4	awrt
(b)	X and Y are independent. Number of catches and runouts independent	B1	1	
(c)(i)	For Season: $S \sim \mathrm{P_o}(9.6)$ $\mathrm{P}(S = 10) = \dfrac{e^{-9.6} \times 9.6^{10}}{10!}$	M1		Use of $\lambda = 9.6$ in correct Poisson expression
	$= 0.124$	A1	2	
(ii)	$T \sim \mathrm{P_o}(9.6 + 2.4) = \mathrm{P_o}(12)$	B1		$\mathrm{P_o}(12)$ used or seen
	$\mathrm{P}(T \ge 15) = 1 - \mathrm{P}(T \le 14)$ $= 1 - 0.7720$ $= 0.228$	B2,1	3	(1-0.8444 = 0.155 to 0.156) **B1**
	Total		**11**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
4(a)(i)	$\mathrm{E}(X)=\sum_x x\times\mathrm{P}(X=x)=2.8$	B1		
	$\mathrm{E}(X^2)=\sum_x x^2\times\mathrm{P}(X=x)=9$			
	$\mathrm{Var}(X)=9-2.8^2$	M1		$\left(\text{their }\mathrm{E}(X^2)-\text{their }\mathrm{E}^2(X)\right)$
	$=1.16$	A1	**3**	cao
(ii)	$\mathrm{E}(S)=3\times\mathrm{E}(X)=8.4$	B1ft		on their $\mathrm{E}(X)$
	$\mathrm{Var}(S)=3\times\mathrm{Var}(X)=3.48$	B1ft	**2**	$3\times$ their $\mathrm{Var}(X)$ from (i) >0
				NB There was a problem with part 4(a)(ii) which affected the marking of this part. Please see the Report on the Examination for details.
(b)	$\mathrm{E}(Y)=3.5$	B1		for E(Y)
	$\mathrm{E}(Y^2)=13$			
	$\mathrm{Var}(Y)=13-3.5^2=0.75$	M1		on their $\mathrm{E}(Y)$ and $\mathrm{E}(Y^2)$
		A1ft		Var (Y) > 0
	$\mathrm{E}(T)=3\times\mathrm{E}(Y)=10.5$	B1		cao
	$\mathrm{Var}(T)=3^2\times\mathrm{Var}(Y)$			
	$=9\times0.75$	M1		$9\times$their $\mathrm{Var}(Y)>0$
	$=6.75$	A1		cao
	Alternative:			

T	3	6	9	12	
T^2	9	36	81	144	
P	$1/20$	$2/20$	$3/20$	$14/20$	
$\mathrm{E}(T)$	$3/20$	$12/20$	$27/20$	$168/20$	10.5
$\mathrm{E}(T^2)$	$9/20$	$72/20$	$243/20$	$2016/20$	117

Solution	Marks	Total	Comments
($\mathrm{E}(T)$ row)	(M1A1)		
($\mathrm{E}(T^2)$ row)	(M1A1)		
$\mathrm{Var}(T)=\mathrm{E}(T^2)-\left[\mathrm{E}(T)\right]^2$	(M1)		(used)
$=117-10.5^2$			
$=6.75$	(A1)	**6**	

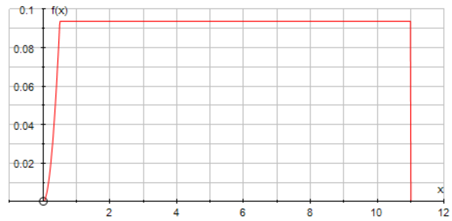
MS2B(cont)

Question	Solution	Marks	Total	Comments
4(c)(i)	$\mathrm{P}(X>1)=0.8$	B1	1	
(ii)	$\mathrm{P}(T=3)=\frac{1}{20}$ **and** $\mathrm{P}(T=3 \text{ or } 6)=\frac{3}{20}$ $\mathrm{P}(X+T\le 9 \text{ and } X>1)$ $=\mathrm{P}([2,3 \text{ or } 6],[3,3 \text{ or } 6],[4,3])$ $=0.1\times\frac{3}{20}+0.4\times\frac{3}{20}+0.3\times\frac{1}{20}$ $=0.015+0.06+0.015$ $=0.09$	B4	4	**Alternative:** $\mathrm{P}(T=3)=\frac{1}{20}$ and $\mathrm{P}(T=6)=\frac{2}{20}$ $\mathrm{P}(X+T\le 9 \text{ and } X>1)$ $=\mathrm{P}([(2,3,4),3],[(2,3),6])$ $=0.8\times\frac{1}{20}+0.5\times\frac{2}{20}$ $=0.04+0.05=0.09$ **sc** [any 4 correct p's from table B**2**] [0.12 **B3**] [0.096 **or** 0.072 **B2**] $\left\{\begin{array}{l}\mathrm{P}(T=3)=\frac{1}{20} \textbf{ and } \mathrm{P}(T=3 \text{ or } 6)=\frac{3}{20}\\ \mathrm{P}(T=3)=\frac{1}{20} \textbf{ and } \mathrm{P}(T=6)=\frac{2}{20}\end{array}\right.$ **B1**
(iii)	$\mathrm{P}(X+T\le 9 \mid X>1)=\frac{0.09}{0.80}$ $=\frac{9}{80}$ (0.1125)	M1 A1	 2	$\frac{\text{their (c)(ii)}}{0.80}$ $(0 < p < 1)$ cao
	Total		**18**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
5(a)(i)	$H_0: \mu = 165$ $H_1: \mu > 165$	B1	1	
(ii)	David (5%) / James (1%)			
	$z = \dfrac{167.1 - 165}{\sqrt{101.2/10}}$	M1		
	$= 2.09$	A1		awfw 2.08 to 2.09
	David: $z_{crit} = 1.6449$ $(t_{crit} = 1.660)$; James: $z_{crit} = 2.3263$ $(t_{crit} = 2.364)$	B1		(both)
	David: Reject H_0; James: Accept H_0	A1		(both) dependent on M1
	David: Evidence to suggest that the **mean** height of students in final year has **increased** at 5% level James: No evidence to suggest an **increase** in the **mean** height of final year students at 1% level	E1 E1	6	
(iii)	**Population not stated as being Normal / not known.** Heights of all students may not be Normal/ Known	B1	1	Large sample size of 100 indicates that the distribution of the sample mean is very likely to be Normal even though **the parent population not given as being Normal**. Hence $\bar{X} \sim N\left(\mu, s^2/n\right)$
(b)(i)	David: $\mu = 165$ $\therefore$ rejected H_0 when H_0 correct $\Rightarrow$ Type I error	M1 A1		
(ii)	James: $\mu = 165$ $\therefore$ accepted H_0 when H_0 correct $\Rightarrow$ No error	M1 A1	4	
	Total		**12**	

MS2B(cont)

Question	Solution	Marks	Total	Comments
6(a)		B3	3	B1 for concave curve from (0 , 1) to $\left(\frac{1}{2},\frac{3}{32}\right)$ B1 for horizontal straight line $f=\frac{3}{32}$ from $\left(\frac{1}{2},\frac{3}{32}\right)$ to $\left(\frac{1}{2},\frac{3}{32}\right)$ B1 for correct axes
(b)(i)	$\mathrm{P}\left(X\ge 8\frac{1}{3}\right)=\left[\frac{3}{32}\times\left(11-8\frac{1}{3}\right)\right]$	M1		Any correct method attempted in either part
	$=\frac{3}{32}\times\frac{8}{3}$			
	$=\frac{1}{4}$	A1		AG
(ii)	$\mathrm{P}(X\ge 3)=\frac{3}{32}\times(11-3)$			Any correct method attempted
	$=\frac{3}{4}$	A1	3	AG
(c)(i)	Interquartile Range = $5\frac{1}{3}$	B1		cao
(ii)	Median = $5\frac{2}{3}$	B2		cao
	Alternative : $\frac{1}{64}+\frac{3}{32}\left(m-\frac{1}{2}\right)=\frac{1}{2}$ $\Rightarrow 3\left(m-\frac{1}{2}\right)=15.5 \Rightarrow m=5\frac{2}{3}$		3	**sc** if B0 then: M1 for correct method seen $\frac{1}{2}\left(8\frac{1}{3}+3\right)$ **or** $\frac{1}{2}\times 11\frac{1}{3}$ **or** $\frac{3}{32}(11-m)=\frac{1}{2} \Rightarrow 11-5\frac{1}{3}$
(d)	$\mathrm{P}\left[(X<m)\cap(X\ge 3)\right]=\frac{1}{4}$	B1		$\left(\frac{3}{4}-\frac{1}{2}\right)$ **attempted**
	$\mathrm{P}(X<m \mid X\ge 3)=\frac{1/4}{3/4}=\frac{1}{3}$	M1		(their p)$/\frac{3}{4}$ for $0<p<1$
		A1	3	**cao**
				Alternative: (Ratio of relevant two areas) $\mathrm{P}(X<m \mid X\ge 3)=\frac{2\frac{2}{3}}{8}=\frac{1}{3}$ cao
	Total		**12**	
	TOTAL		**75**	