OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS LEVEL

Y410/01

FURTHER MATHEMATICS B (MEI)

Core Pure

Question Paper

MONDAY 14 MAY 2018: Afternoon

TIME ALLOWED: 1 hour 15 minutes plus your additional time allowance

MODIFIED ENLARGED 36pt

YOU MUST HAVE:
Printed Answer Booklet sent with the standard paper or any suitable paper provided by the centre. The Printed Answer Booklet may be enlarged by the centre.
Formulae Further Mathematics B (MEI) sent with the standard paper

YOU MAY USE:
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CE/SW) 168882/1

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INSTRUCTIONS

Use black ink. HB pencil may be used for graphs and diagrams only.

Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number, or write them on the paper provided.

Answer ALL the questions.

IF YOU USE THE PRINTED ANSWER BOOKLET, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

The total number of marks for this paper is 60.

The marks for each question are shown in brackets [ ].

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.

Answer ALL the questions.

1 **The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined as follows:**

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 3 \end{pmatrix}.$$

Calculate all possible products formed from two of these three matrices. [4]

2 **Find, to the nearest degree, the angle between the vectors $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$. [3]**

3 **Find real numbers a and b such that $(a - 3\mathrm{i})(5 - \mathrm{i}) = b - 17\mathrm{i}$. [5]**

4 **Find a cubic equation with real coefficients, two of whose roots are $2 - \mathrm{i}$ and 3. [5]**

5 **A transformation of the x-y plane is represented by the matrix $\begin{pmatrix} \cos\theta & 2\sin\theta \\ 2\sin\theta & -\cos\theta \end{pmatrix}$, where θ is a positive acute angle.**

(i) Write down the image of the point (2, 3) under this transformation. [2]

(ii) You are given that this image is the point (a, 0). Find the value of a. [5]

6 **Find the invariant line of the transformation of the x-y plane represented by the matrix $\begin{pmatrix} 2 & 0 \\ 4 & -1 \end{pmatrix}$. [4]**

7 (i) Express $\dfrac{1}{2r-1}-\dfrac{1}{2r+1}$ as a single fraction. [2]

(ii) Find how many terms of the series

$$\frac{2}{1\times3}+\frac{2}{3\times5}+\frac{2}{5\times7}+\dots+\frac{2}{(2r-1)(2r+1)}+\dots$$

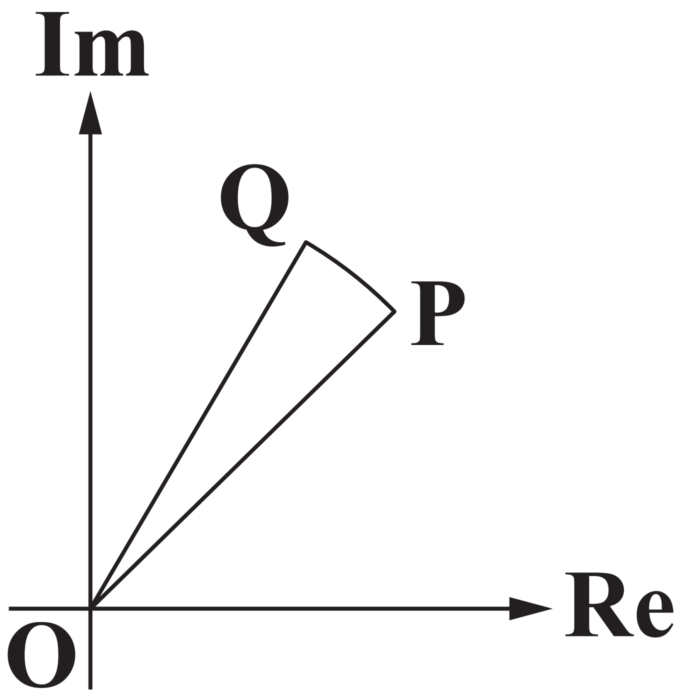
are needed for the sum to exceed 0.999 999. [7]

8 Prove by induction that $\begin{pmatrix}1 & 1\\ 0 & 2\end{pmatrix}^n=\begin{pmatrix}1 & 2^n-1\\ 0 & 2^n\end{pmatrix}$ for all positive integers n. [6]

9 Fig. 9 shows a sketch of the region OPQ of the Argand diagram defined by

$$\left\{z : |z| \leqslant 4\sqrt{2}\right\} \cap \left\{z : \tfrac{1}{4}\pi \leqslant \arg z \leqslant \tfrac{1}{3}\pi\right\}.$$

FIG. 9



(i) Find, in modulus-argument form, the complex number represented by the point P. [2]

(ii) Find, in the form $a + \mathrm{i}b$, where a and b are exact real numbers, the complex number represented by the point Q. [3]

(iii) In this question you must show detailed reasoning.

Determine whether the points representing the complex numbers:
$3 + 5\mathrm{i}$;
$5.5(\cos 0.8 + \mathrm{i}\sin 0.8)$;
lie within this region. [4]

10 Three planes have equations

$$\begin{aligned} -x + 2y + z &= 0 \\ 2x - y - z &= 0 \\ x + y &= a \end{aligned}$$

where a is a constant.

(i) Investigate the arrangement of the planes:
when $a = 0$;
when $a \neq 0$. [6]

(ii) Chris claims that the position vectors $-\mathbf{i}+2\mathbf{j}+\mathbf{k}$, $2\mathbf{i}-\mathbf{j}-\mathbf{k}$ and $\mathbf{i}+\mathbf{j}$ lie in a plane. Determine whether or not Chris is correct. [2]

END OF QUESTION PAPER