General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS MPC3
Unit Pure Core 3

Monday 19 January 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 3 (enclosed).

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

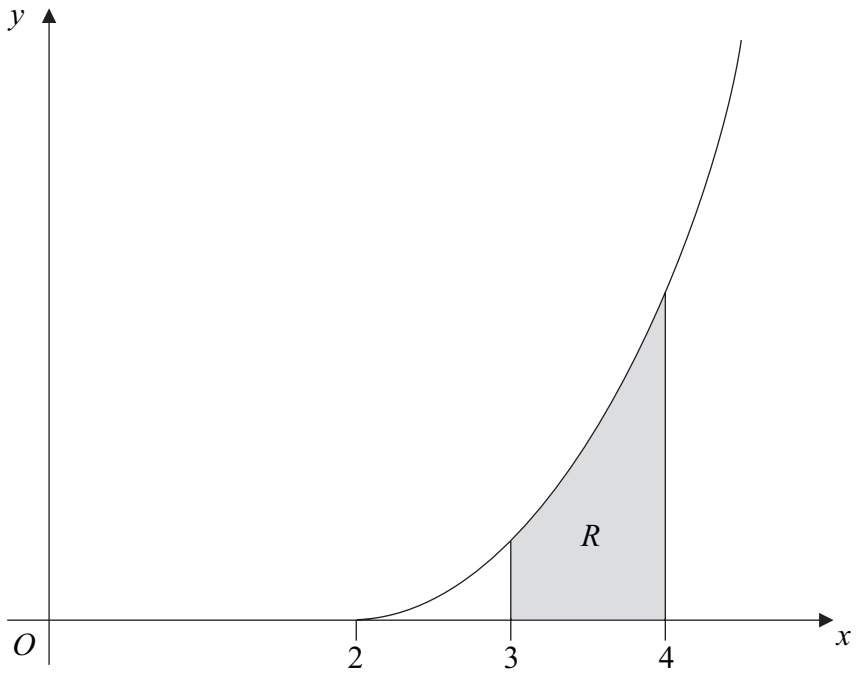
Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 Use Simpson's rule with 5 ordinates (4 strips) to find an approximation to $\int_1^9 \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

2 The diagram shows the curve with equation $y = \sqrt{(x-2)^5}$ for $x \geqslant 2$.



The shaded region R is bounded by the curve $y = \sqrt{(x-2)^5}$, the x-axis and the lines $x = 3$ and $x = 4$.

Find the exact value of the volume of the solid formed when the region R is rotated through 360° about the x-axis. *(4 marks)*

3 [**Figure 1**, printed on the insert, is provided for use in this question.]

The curve with equation $y = x^3 + 5x - 4$ intersects the x-axis at the point A, where $x = \alpha$.

(a) Show that α lies between 0.5 and 1. *(2 marks)*

(b) Show that the equation $x^3 + 5x - 4 = 0$ can be rearranged into the form

$$x = \frac{1}{5}(4 - x^3)$$ *(1 mark)*

(c) Use the iteration $x_{n+1} = \frac{1}{5}(4 - {x_n}^3)$ with $x_1 = 0.5$ to find x_3, giving your answer to three decimal places. *(2 marks)*

(d) The sketch on **Figure 1** shows parts of the graphs of $y = \frac{1}{5}(4 - x^3)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

4 (a) Solve the equation $\sec x = \frac{3}{2}$, giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(2 marks)*

(b) By using a suitable trigonometrical identity, solve the equation

$$2\tan^2 x = 10 - 5\sec x$$

giving all values of x to the nearest degree in the interval $0° < x < 360°$. *(6 marks)*

Turn over for the next question

5 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = 2 - x^4 \qquad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{x-4} \qquad \text{for real values of } x,\ x \neq 4$$

(a) State the range of f. *(2 marks)*

(b) Explain why the function f does not have an inverse. *(1 mark)*

(c) (i) Write down an expression for $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = -14$. *(3 marks)*

6 A curve has equation $y = \mathrm{e}^{2x}(x^2 - 4x - 2)$.

(a) Find the value of the x-coordinate of each of the stationary points of the curve. *(6 marks)*

(b) (i) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *(2 marks)*

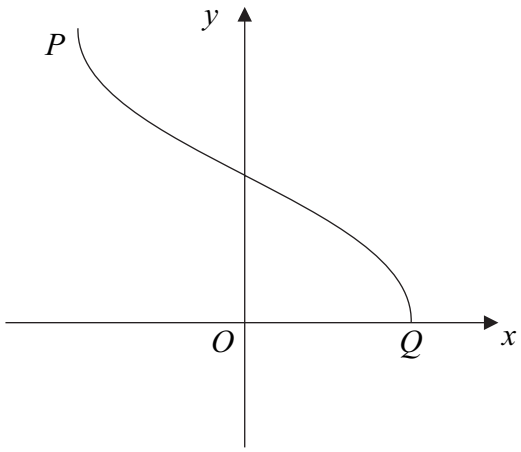
(ii) Determine the nature of each of the stationary points of the curve. *(2 marks)*

7 (a) Given that $3\mathrm{e}^x = 4$, find the exact value of x. *(2 marks)*

(b) (i) By substituting $y = \mathrm{e}^x$, show that the equation $3\mathrm{e}^x + 20\mathrm{e}^{-x} = 19$ can be written as $3y^2 - 19y + 20 = 0$. *(1 mark)*

(ii) Hence solve the equation $3\mathrm{e}^x + 20\mathrm{e}^{-x} = 19$, giving your answers as exact values. *(3 marks)*

8 The sketch shows the graph of $y = \cos^{-1} x$.



(a) Write down the coordinates of P and Q, the end points of the graph. *(2 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos^{-1} x$ onto the graph of $y = 2\cos^{-1}(x - 1)$. *(4 marks)*

(c) Sketch the graph of $y = 2\cos^{-1}(x - 1)$. *(2 marks)*

(d) (i) Write the equation $y = 2\cos^{-1}(x - 1)$ in the form $x = \mathrm{f}(y)$. *(2 marks)*

(ii) Hence find the value of $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ when $y = 2$. *(3 marks)*

9 (a) Given that $y = \dfrac{4x}{4x - 3}$, use the quotient rule to show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{k}{(4x - 3)^2}$, where k is an integer. *(2 marks)*

(b) (i) Given that $y = x\ln(4x - 3)$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = x\ln(4x - 3)$ at the point where $x = 1$. *(3 marks)*

(c) (i) Use the substitution $u = 4x - 3$ to find $\displaystyle\int \frac{4x}{4x - 3}\,\mathrm{d}x$, giving your answer in terms of x. *(4 marks)*

(ii) By using integration by parts, or otherwise, find $\displaystyle\int \ln(4x - 3)\,\mathrm{d}x$. *(4 marks)*

END OF QUESTIONS

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