Relative position – key points

The set-up:

Two objects A and B arer moving in a set of axis

Their initial position (relative to the origin O) are the position vectors $\mathbf{a}_0$ *and* $\mathbf{b}_0$ *and* they have a speed $\mathbf{v}_A$ *and* $\mathbf{v}_B$.

- At any time t, the position vector of the point A is $r_A = \mathbf{a}_0 + t\mathbf{v}_A$
  the position vector of the point B is $r_B = \mathbf{b}_0 + t\mathbf{v}_B$
- The position of B RELATIVE to A is given by the vector $\overrightarrow{AB} = r_B - r_A$
  This position vector is noted ${}_Ar_B$
- The speed of B relative to A is the vector ${}_A\mathbf{v}_B = \mathbf{v}_B - \mathbf{v}_A$

- The two points will meet if it exists a time t for which $\overrightarrow{AB} = \mathbf{0}$

- The distance between the two point is given by $\left|\overrightarrow{AB}\right| = s.$
  The points A and B are the closest to each other when s is minimum
  which is equivalent to s^2 is minimum (We use s^2 to avoid working with square roots)

So, to work out when the two points are the CLOSEST to each other:

Find the value of t for which $\dfrac{d\left(s^2\right)}{dt} = 0$

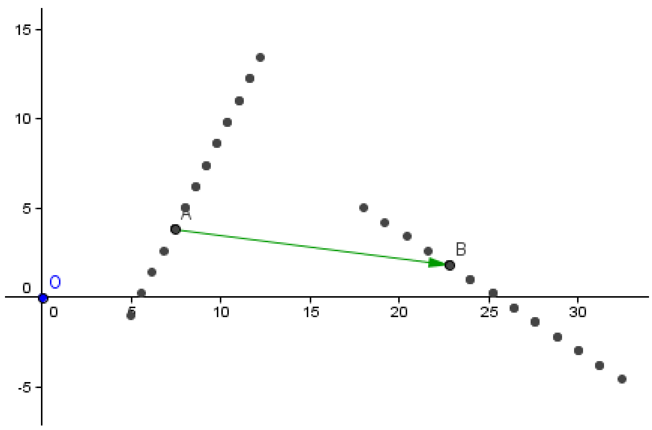
Example:

The unit vectors **i** and **j** are directed due east and due north respectively.

Two cyclists, Aazar and Ben, are cycling on straight horizontal roads with constant velocities of $(6\mathbf{i} + 12\mathbf{j})\,\text{km h}^{-1}$ and $(12\mathbf{i} - 8\mathbf{j})\,\text{km h}^{-1}$ respectively. Initially, Aazar and Ben have position vectors $(5\mathbf{i} - \mathbf{j})$ km and $(18\mathbf{i} + 5\mathbf{j})$ km respectively, relative to a fixed origin.

The position of B relative to A is : $\begin{pmatrix} 18-5 \\ 5--1 \end{pmatrix} + t\begin{pmatrix} 12-6 \\ -8-12 \end{pmatrix}$

$${}_Ar_B = \begin{pmatrix} 13 \\ 6 \end{pmatrix} + t\begin{pmatrix} 6 \\ -20 \end{pmatrix}$$



- Are they going to meet?

$${}_Ar_B = 0 \Leftrightarrow \begin{cases} 13 + 6t = 0 \\ 6 - 20t = 0 \end{cases} \Leftrightarrow \begin{cases} t = -13/6 \\ t = 6/20 \end{cases} \text{ Inconsistent.}$$

A and B do not meet.

- When are they the closest to each other?

$$s^2 = (13 + 6t)^2 + (6 - 20t)^2$$

$$and \quad \frac{ds^2}{dt} = 12(13 + 6t) - 40(6 - 20t) = -84 + 872t$$

$$\frac{ds^2}{dt} = 0 \quad for \quad t = \frac{21}{218} = 0.0963$$