

5. Suppose that a sequence of equally spaced points on the  $x$ -axis is defined by  $x_{n+1} = x_n + h$  where  $x_0$  is given. Suppose also that  $y = f(x)$  is the equation of a function and that  $y_n = f(x_n)$ ,  $n = 0, 1, 2, \dots$

- (a) Use Taylor's expansion to show that

$$\frac{y_{n+1} - y_n}{h} - f'(x_n) = \frac{1}{2} h f''(x_n)$$

and find a similar approximation for  $\frac{y_{n+1} - y_{n-1}}{2h} - f'(x_n)$ .

[6]

- (b) Taking a value of  $h = 0.1$ , copy and complete the table, showing your method clearly, for the solution of the differential equation  $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$  which passes through  $(0,0)$ . (You may assume that the values of the third derivative are negligible, but you should not ignore the second derivative.)

$n$	$x_n$	$y_n$
0	0	0
2		
4		
6		
8		
10		

[6]

- (c) Explain briefly how you could improve on your approximation for the value of  $y$  when  $x = 1$ .

[2]