

Answer **all** parts of the question

1. Consider the difference equation

$$r_{n+1} + Ar_n + Br_{n-1} = C \times D^n, \quad r_0 = E, \quad r_1 = F$$

- (a) Use mathematical induction to prove that if $A = -3$, $B = 2$, $C = 2$, $D = 3$, $E = 2$ and $F = 5$ then

$$r_n = 2 - 3 \times 2^n + 3^{n+1}$$
 is a solution to the difference equation. [8]
- (b) Find the solution to the equation if $A = -4$, $B = 4$, $C = 0$, $E = 3$ and $F = 10$. [6]
- (c) Find the solution to the equation if $A = -7$, $B = 12$, $C = 6$, $D = 1$, $E = 1$ and $F = 2$. [8]
- (d) Suppose that r_n represents the number of animals of some species alive in year n . If $B = C = 0$ and A is negative, what does A tell you about whether the population will grow, decay or remain stationary? [3]
- (e) Let $A = -1$, $B = 0$, $C = 1$, $D = 2$, $E = 1$ and $F = 2$.
 By solving the equation, interpret the difference equation in terms of Pascal's triangle. (That is, what aspect of Pascal's triangle is modelled by this difference equation?) [5]
- (f) Exhibit an iterative algorithm whose input is A, B, C, D, E, F and n (a non-negative integer) and whose output is r_n . [6]