

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4757/01

MATHEMATICS (MEI)

**Further Applications of Advanced
Mathematics (FP3)**

QUESTION PAPER

WEDNESDAY 3 JUNE 2015: Morning

**DURATION: 1 hour 30 minutes
plus your additional time allowance**

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4757/01

MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Read each question carefully. Make sure you know what you have to do before starting your answer.

Answer any THREE questions.

You are permitted to use a scientific or graphical calculator in this paper.

Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.

You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Option 1: Vectors

- 1 The point A has coordinates (2, 5, 4) and the line BC has equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 25 \\ 43 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 15 \\ 25 \end{pmatrix}.$$

You are given that $AB = AC = 15$.

- (i) Show that the coordinates of one of the points B and C are (4, 10, 18). Find the coordinates of the other point. These points are B and C respectively. [6]
- (ii) Find the equation of the plane ABC in cartesian form. [4]
- (iii) Show that the plane containing the line BC and perpendicular to the plane ABC has equation $5y - 3z + 4 = 0$. [4]

The point D has coordinates (1, 1, 3).

- (iv) Show that $|\overrightarrow{BC} \times \overrightarrow{AD}| = \sqrt{7667}$ and hence find the shortest distance between the lines BC and AD. [7]
- (v) Find the volume of the tetrahedron ABCD. [3]

Option 2: Multi-variable calculus

2 A surface has equation $z = 3x^2 - 12xy + 2y^3 + 60$.

(i) Show that the point A (8, 4, -4) is a stationary point on the surface. Find the coordinates of the other stationary point, B, on this surface. [5]

(ii) A point P with coordinates $(8 + h, 4 + k, p)$ lies on the surface.

(A) Show that $p = -4 + 3(h - 2k)^2 + 2k^2(6 + k)$. [3]

(B) Deduce that the stationary point A is a local minimum. [3]

(C) By considering sections of the surface near to B in each of the planes $x = 0$ and $y = 0$, investigate the nature of the stationary point B. [4]

(iii) The point Q with coordinates (1, 1, 53) lies on the surface.

Show that the equation of the tangent plane at Q is

$$6x + 6y + z = 65. \quad [4]$$

(iv) The tangent plane at the point R has equation $6x + 6y + z = \lambda$ where $\lambda \neq 65$.

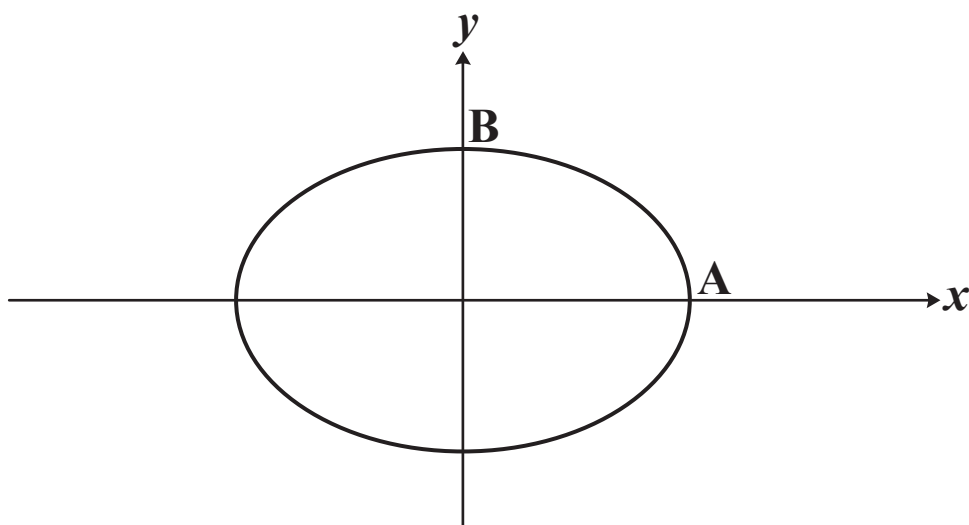
Find the coordinates of R. [5]

Option 3: Differential geometry

- 3 Fig. 3 shows an ellipse with parametric equations $x = a \cos \theta$, $y = b \sin \theta$, for $0 \leq \theta \leq 2\pi$, where $0 < b \leq a$.

The curve meets the positive x -axis at A and the positive y -axis at B.

Fig. 3



- (i) Show that the radius of curvature at A is $\frac{b^2}{a}$ and find the corresponding centre of curvature. [7]
- (ii) Write down the radius of curvature and the centre of curvature at B. [2]
- (iii) Find the relationship between a and b if the radius of curvature at B is equal to the radius of curvature at A. What does this mean geometrically? [1]

- (iv) Show that the arc length from A to B can be expressed as

$$b \int_0^{\frac{\pi}{2}} \sqrt{1 + \lambda^2 \sin^2 \theta} d\theta,$$

where λ^2 is to be determined in terms of a and b .

Evaluate this integral in the case $a = b$ and comment on your answer. [7]

- (v) Find the cartesian equation of the evolute of the ellipse. [7]

Option 4: Groups

- 4 **M is the set of all 2×2 matrices $m(a,b)$ where a and b are rational numbers and**

$$m(a,b) = \begin{pmatrix} a & b \\ 0 & \frac{1}{a} \end{pmatrix}, a \neq 0.$$

- (i) **Show that under matrix multiplication M is a group. You may assume associativity of matrix multiplication. [7]**

- (ii) **Determine whether the group is commutative. [3]**

The set N_k consists of all 2×2 matrices $m(k,b)$ where k is a fixed positive integer and b can take any integer value.

- (iii) **Prove that N_k is closed under matrix multiplication if and only if $k = 1$. [4]**

Now consider the set P consisting of the matrices $m(1,0)$, $m(1,1)$, $m(1,2)$ and $m(1,3)$. The elements of P are combined using matrix multiplication but with arithmetic carried out modulo 4.

- (iv) **Show that $(m(1,1))^2 = m(1,2)$. [2]**

- (v) **Construct the group combination table for P. [4]**

The group R consists of the set $\{e, a, b, c\}$ combined under the operation $*$. The identity element is e , and elements a , b and c are such that

$$a*a = b*b = c*c \quad \text{and} \quad a*c = c*a = b.$$

- (vi) **Determine whether R is isomorphic to P. [4]**

Option 5: Markov chains

This question requires the use of a calculator with the ability to handle matrices.

- 5** An inspector has three factories, A, B, C, to check. He spends each day in one of the factories. He chooses the factory to visit on a particular day according to the following rules.

If he is in A one day, then the next day he will never choose A but he is equally likely to choose B or C.

If he is in B one day, then the next day he is equally likely to choose A, B or C.

If he is in C one day, then the next day he will never choose A but he is equally likely to choose B or C.

- (i) Write down the transition matrix, P. [2]
- (ii) On Day 1 the inspector chooses A.
 - (A) Find the probability that he will choose A on Day 4. [3]
 - (B) Find the probability that the factory he chooses on Day 7 is the same factory that he chose on Day 2. [4]
- (iii) Find the equilibrium probabilities and explain what they mean. [4]

The inspector is not satisfied with the number of times he visits A so he changes the rules as follows.

If he is in A one day, then the next day he will choose A, B, C, with probabilities 0.8, 0.1, 0.1, respectively.

If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

- (iv) Write down the new transition matrix, Q , and find the new equilibrium probabilities. [3]**
- (v) On a particular day, the inspector visits factory A. Find the expected number of consecutive further days on which he will visit factory A. [3]**

Still not satisfied, the inspector changes the rules as follows.

If he is in A one day, then the next day he will choose A, B, C, with probabilities 1, 0, 0, respectively.

If he is in B or C one day, then the probabilities for choosing the factory the next day remain as before.

The new transition matrix is R .

- (vi) On Day 15 he visits C. Find the first subsequent day for which the probability that he visits B is less than 0.1. [3]**
- (vii) Show that in this situation there is an absorbing state, explaining what this means. [2]**

END OF QUESTION PAPER

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