

**ADVANCED SUBSIDIARY GCE UNIT
MEI STATISTICS**

G243/01

Statistics 3 (Z3)

MONDAY 18 JUNE 2007

Morning

Time: 1 hour 30 minutes

Additional Materials:
Answer booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.

ADVICE TO CANDIDATES

- Read each question carefully and make sure you know what you have to do before starting your answer.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

Section A (45 marks)

- 1 Scientists at a horticultural research laboratory are comparing two types, A and B, of a particular flower in terms of the intensity of its colour. The intensity is rated on a continuous scale. 10 flowers of each type are planted in an experimental field. A paired arrangement is used, with a type A and a type B flower planted near to one another in each of ten positions scattered around the field.

- (i) Suggest why a paired arrangement has been used. [2]
- (ii) The intensity ratings are as follows.

Position in field	I	II	III	IV	V	VI	VII	VIII	IX	X
Type A	35.3	36.2	33.2	28.1	37.9	30.9	38.8	29.3	30.6	38.8
Type B	31.6	34.8	35.8	28.7	37.5	31.9	38.6	27.2	30.7	38.1

Stating an appropriate assumption about underlying Normality, use a t test to examine, at the 10% level of significance, whether it appears that the mean ratings for the two types are the same. State carefully your null and alternative hypotheses and your conclusion. Name a test which could be used if no assumption of underlying Normality could reasonably be made. [13]

- 2 Traffic engineers are studying the correlation between traffic flow on a busy main road and air pollution at a nearby air quality monitoring station. Traffic flow is recorded automatically by sensors and reported each hour as the average flow in vehicles per hour for the preceding hour. The air quality monitoring station provides, each hour, an overall pollution reading in a suitable unit (higher readings indicate more pollution). Data for a random sample of 15 hours are as follows.

Traffic flow x	1815	2206	1835	1918	2420	2315	1796	2850
Pollution y	3.5	8.3	5.0	4.8	20.0	18.0	3.6	32.0

Traffic flow x	2635	2588	2040	2368	2170	2285	2120
Pollution y	24.2	24.4	9.5	16.2	10.6	13.8	12.0

- (i) Draw a scatter diagram to illustrate these data, and comment on it briefly. [4]
- (ii) Calculate the value of the product moment correlation coefficient. [2]
- (iii) What assumption about the underlying population is required for it to be valid to carry out the usual test based on the product moment correlation coefficient? Discuss briefly whether it appears reasonable, from the scatter diagram, to suppose that this assumption holds. [2]
- (iv) Supposing that the assumption in part (iii) does hold, test at the 1% level of significance whether it is reasonable to suppose that there is a positive correlation between traffic flow and air pollution. State carefully the null hypothesis you are testing. [5]
- (v) The local newspaper carries a headline “Heavy traffic causes air pollution”. Comment briefly on this headline in the light of your results. [2]

- 3 Environmental scientists are investigating the heights of tides at two seaside towns as part of a study to see whether either appears more vulnerable to flooding than the other. A common baseline level is established and, at each town, an automatic measuring device gives a reading for the height of each tide above this level. Results for a random sample of 10 tides at town A and a separate random sample of 12 tides at town B are as follows, the units being metres.

Town A	1.4	4.4	2.1	0.7	5.4	3.7	2.6	0.2	0.9	3.8		
Town B	2.7	0.6	5.9	4.0	3.6	5.6	4.2	5.5	5.1	4.6	5.7	3.9

- (i) It is desired to see whether, on average, the heights at the two towns are the same. However, it is not thought appropriate to assume Normality of the underlying populations. State null and alternative hypotheses that can be investigated and carry out a suitable test at the 5% level of significance. [10]
- (ii) Discuss whether it appears sensible to have used two separate random samples
- (A) if the towns are within a few miles of each other on the same stretch of coastline,
- (B) if one of the towns is on the east coast of the country and the other on the west. [5]

[Question 4 is printed overleaf.]

Section B (27 marks)

- 4** Market researchers are advising a cosmetics company whether a product should be introduced into a region where it is not currently widely available. The company wants to know how attractive the product is likely to be to the population.

(i) Discuss issues that arise in defining ‘the population’ in this context. [4]

As part of the work, the researchers decide to carry out a survey of female sixth-form students attending schools or colleges in the region. There are many such schools and colleges and a large number of such students.

(ii) Describe briefly how cluster sampling might be carried out in this situation. [2]

(iii) Why might cluster sampling be a better way to proceed than simple random sampling over all the schools and colleges? [2]

(iv) Explain why cluster sampling would be unwise if some of the students attend schools with sixth forms, some attend sixth-form colleges, and others attend further education colleges. What sampling method would be more appropriate? [3]

A researcher forms a testing panel consisting of a random sample of 7 female students from one sixth-form college and a random sample of 10 from another.

(v) Each member of the panel is asked to rate the overall attractiveness of the product on a numerical scale. The ratings are summarised as follows.

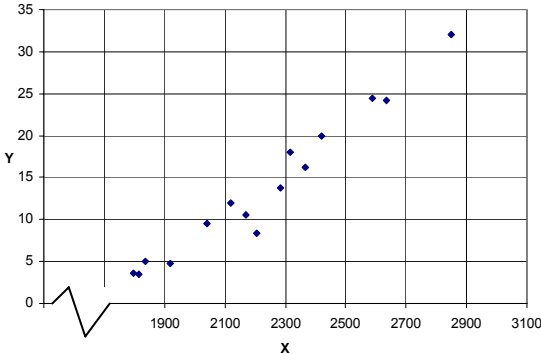
College A	$n = 7$	Sample mean = 68.6	Sample variance = 9.28
College B	$n = 10$	Sample mean = 64.2	Sample variance = 12.16

Assuming underlying Normality, test at the 5% level of significance whether the mean rating for the population in college A may be taken as the same as that in college B. What else do you need to assume when carrying out this test? [11]

(vi) The sample of 7 female students from college A was in fact selected by systematic sampling. Altogether there are 420 female students in the college. Describe how the systematic sampling could have been undertaken. Discuss whether systematic sampling in this situation might reasonably be assumed to be equivalent to simple random sampling. [5]

**Mark Scheme G243
June 2007**

<p>Q1 (i)</p>	<p>There may be differences in ‘fertility’ in different parts of the field – any such differences should affect the pairs equally.</p>	<p>E2</p>	<p>(2, 1, 0)</p>	<p>2</p>
<p>(ii)</p>	<p>Population of <u>differences</u> in ratings is Normally distributed. $H_0: \mu_D = 0$ (or $\mu_A = \mu_B$ etc) $H_1: \mu_D \neq 0$ (or $\mu_A \neq \mu_B$ etc)</p> <p>Where μ_D is the population mean for differences.</p> <p>MUST be paired comparison t test. Use of differences. Differences are: 3.7, 1.4, -2.6, -0.6, 0.4, -1.0, 0.2, 2.1, -0.1, 0.7 $\bar{d} = 0.42$ $s_{n-1} = 1.736$ Test statistic is $\frac{0.42 - 0}{1.736/\sqrt{10}} = 0.765$</p> <p>Refer to t_9 Double-tailed 10% critical value is 1.833 Not significant Seems mean ratings may be assumed equal.</p> <p>Wilcoxon signed rank test for paired samples.</p>	<p>B1 B1 B1 B1 M1 A1 M1 A1 M1 A1 E1 E1 B1</p>	<p>Do NOT allow \bar{D} or similar unless it is clearly and explicitly stated to be a <u>population</u> mean. Hypotheses in words only must include “population”.</p> <p>For adequate verbal definition. Allow absence of “population” here if correct notation μ has been used.</p> <p>For both. [$s_n = 1.647$ <u>NOT</u> allowed.]</p> <p>A1 is c.a.o. but FT from here if M1 awarded (but no marks from here on if not paired t test). Use of $0 - \bar{d}$ scores M1 A0, but FT.</p> <p>Allow c’s \bar{d} and/or s_{n-1}. Allow alternatives: $0 + (c's\ 1.833) \times \frac{1.736}{\sqrt{10}}$ (= 1.006) for subsequent comparison with \bar{d} , <u>OR</u> $\bar{d} - (c's\ 1.833) \times \frac{1.736}{\sqrt{10}}$ (= $\bar{d} - 1.006$) for subsequent comparison with 0.</p> <p>No FT if wrong. No FT if wrong.</p>	<p>13</p>

<p>Q2 (i)</p>	 <p>Looks strongly linear.</p>	<p>G1 Axes, including labels. “x” and “y” suffice as they are defined in the question.</p> <p>G1 Clear “break” in the x-axis.</p> <p>G1 All points correct (allow one error).</p> <p>B1 Comments re bivariate Normality may be rewarded in part (iii).</p>	<p>4</p>
<p>(ii)</p>	<p>$r = 0.9774$</p> <p>If used: $\sum x = 33361$ $\sum x^2 = 75604869$ $S_{xx} = 1407780.9$ $\sum y = 205.9$ $\sum y^2 = 3870.63$ $S_{yy} = 1044.3093$ $\sum xy = 495410.1$ $S_{xy} = 37474.773$</p>	<p>M1 A1</p> <p>Regard as implicit from correct answer.</p>	<p>2</p>
<p>(iii)</p>	<p>Bivariate Normality. Yes – (long thin !) “cigar” shape.</p>	<p>M1 E1</p>	<p>2</p>
<p>(iv)</p>	<p>H_0 is $\rho=0$ where ρ is the correlation coefficient for the underlying bivariate population. From tables, upper 1% point for $n=15$ is 0.5923 Significant. Seems there is a positive correlation between traffic flow and air pollution.</p>	<p>B1 B1 A1 E1 E1</p> <p>No FT if wrong.</p>	<p>5</p>
<p>(v)</p>	<p>For any sensible comments, <u>such as</u>:</p> <ul style="list-style-type: none"> – correlation does not necessarily imply causation. – there could be another confounding factor – this might be a “false positive”. 	<p>E1, E1</p>	<p>2</p>

<p>Q3 (i)</p>	<p>H_0: the medians of the two populations are the same. H_1: the medians of the two populations are different. [Or more formal statements.] Wilcoxon rank sum test (or Mann-Whitney form thereof). Ranks are: A 5 15 6 3 18 10 7 1 4 11 B 8 2 22 13 9 20 14 19 17 16 21 12</p> <p>Rank sum for smaller sample is 80.</p> <p>Refer to (10, 12) table. Two-tail 5% critical value is 84 [or 29 for M-W]. Significant. Seems median heights are different.</p>	<p>B1 B1 M1 M1 A1 B1 M1 A1 E1 E1</p>	<p>Allow 1 for “medians”, but require “population” for second mark.</p> <p>Combined ranking. All ranks correct (FT if M1 earned). [Or M-W statistic = $1+6+1+1+8+3+1+0+1+3=25$] No FT from here if wrong. No FT from here if wrong.</p>	<p>10</p>
<p>(ii)</p>	<p>A No, seems to need “pairing”. B Not clear-cut, perhaps no strong reason for “pairing”, but large-scale weather systems affecting whole country might be important.</p>	<p>E1 E1 E1 E1 E1</p>	<p>Or other sensible comments.</p>	<p>5</p>

Q4 (i)	Need to define the region geographically. Age and/or sex sub-classifications? May be problems of people who go out of/come into the region to buy goods. Frequency of purchase sub-classifications?	E1 E1 E1 E1	Or other sensible comments	4
(ii)	Elect a (simple) random sample of schools/colleges. Select sample (or complete enumeration) in each selected school/college.	E1 E1		2
(iii)	Comments to effect of administrative convenience, less cost.	E1 E1		2
(iv)	Each cluster should be representative of entire population. The chosen clusters might all represent only a sub-population. Stratified sampling.	E1 E1 B1		3
(v)	Pooled s^2 is $\frac{(6 \times 9.28) + (9 \times 12.16)}{15} = \frac{165.12}{15} = 11.008$ Test statistic is $\frac{68.6 - 64.2}{\sqrt{11.008} \sqrt{\frac{1}{7} + \frac{1}{10}}}$ $= \frac{4.4}{1.635} = 2.691$ Refer to t_{15} . Double-tailed 5% point is 2.131. Significant. Seems that population mean ratings are different. Must assume <u>population</u> variances are the same.	M1 A1 M1 M1 M1 A1 M1 A1 E1 E1 B1	For any reasonable attempt at pooling If correct For numerator For $\sqrt{11.008}$ (or candidate's value) For $\sqrt{\frac{1}{7} + \frac{1}{10}}$ FT from here if all M marks earned. Allow alternatives as in question 1. No FT if wrong No FT if wrong	11
(vi)	(From e.g. college roll) select one of the first 60 at random. and then every 60 th . Equivalent to simple random sampling, with some reason consistent within candidate's answer. Discussion	M1 M1 M1 E2	E0,1 or 2. Allow E1 for comment re each student has equal chance of selection. Allow E2 for comment re no cycles in list of names.	5