OXFORD CAMBRIDGE AND RSA EXAMINATIONS

A2 GCE

4723

MATHEMATICS

Core Mathematics 3

QUESTION PAPER

FRIDAY 20 JANUARY 2012: Afternoon
DURATION: 1 hour 30 minutes

SUITABLE FOR VISUALLY IMPAIRED CANDIDATES

Candidates answer on the Printed Answer Book, or any suitable paper provided by the Centre. The Printed Answer Book may be enlarged by the Centre.

OCR SUPPLIED MATERIALS:

Printed answer book 4723
List of Formulae (MF1)
Insert for question 5

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (NF/SW) 12MEP01010 55323/1
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INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **YOU ARE REMINDED OF THE NEED FOR CLEAR PRESENTATION IN YOUR ANSWERS.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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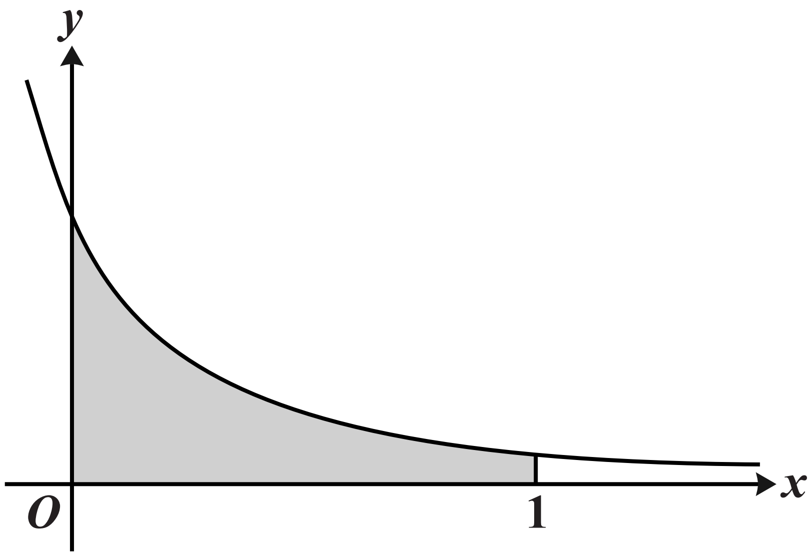
1 **Show that** $\int_{\sqrt{2}}^{\sqrt{6}} \frac{2}{x}\, dx = \ln 3$. **[3]**

2 **Look at the following diagram.**



The diagram shows part of the curve $y = \frac{6}{(2x+1)^2}$. **The shaded region is bounded by the curve and the lines** $x = 0$, $x = 1$ **and** $y = 0$. **Find the exact volume of the solid produced when this shaded region is rotated completely about the** x**-axis. [5]**

3 **Find the equation of the normal to the curve** $y = \frac{x^2+4}{x+2}$ **at the point** $(1, \frac{5}{3})$, **giving your answer in the form** $ax + by + c = 0$, **where** a, b **and** c **are integers. [7]**

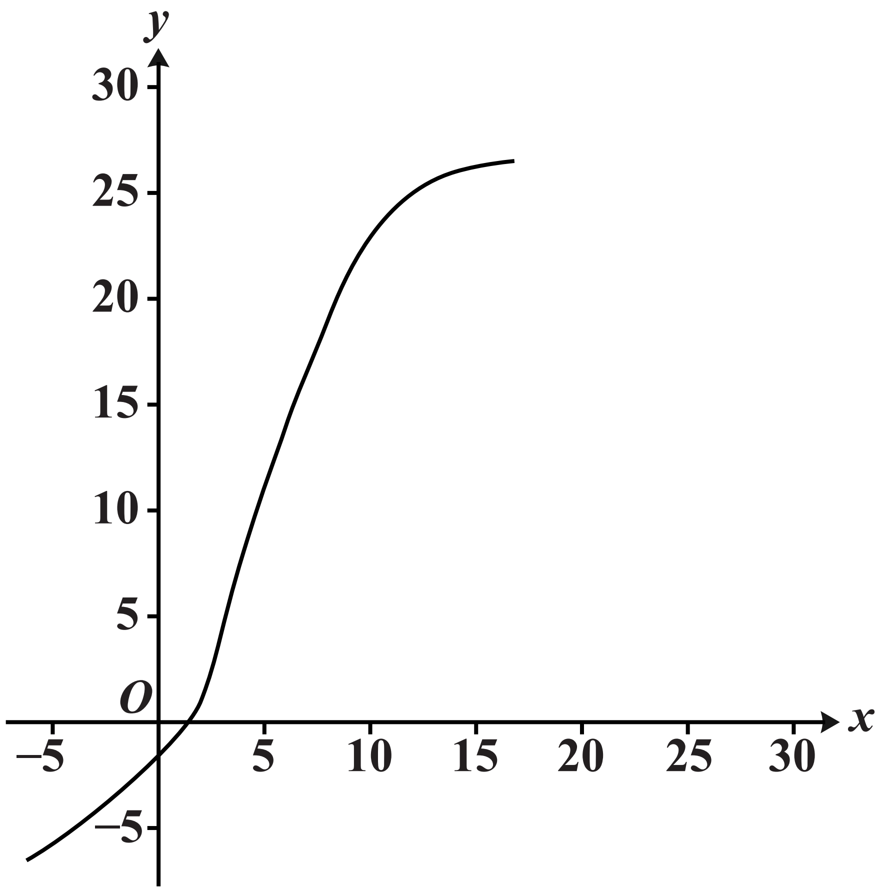
4 **The acute angles α and β are such that**

$$2\cot\alpha = 1 \quad \text{and} \quad 24 + \sec^2\beta = 10\tan\beta.$$

(i) State the value of $\tan\alpha$ and determine the value of $\tan\beta$. **[4]**

(ii) Hence find the exact value of $\tan(\alpha + \beta)$. **[3]**

5 **Look at the following diagram.**



It is given that f is a one-one function defined for all real values. The diagram above shows the curve with equation $y = \mathrm{f}(x)$. The coordinates of certain points on the curve are shown in the following table.

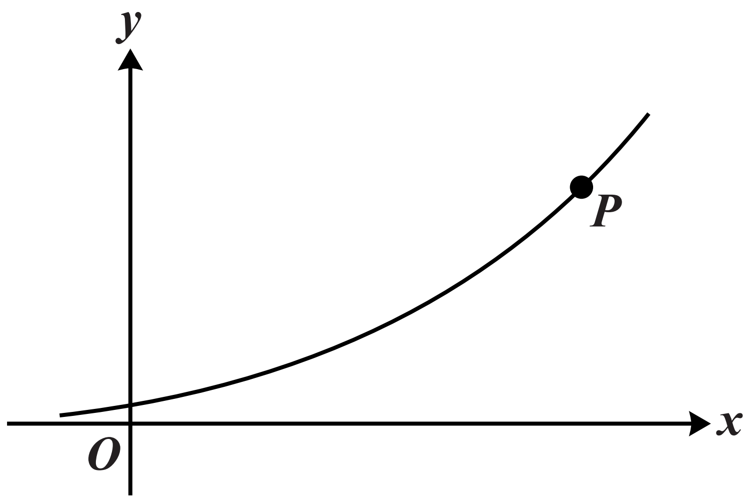
x	2	4	6	8	10	12	14
y	1	8	14	19	23	25	26

(i) State the value of ff(6) and the value of $\mathrm{f}^{-1}(8)$. [2]

(ii) On the copy of the diagram provided, sketch the curve $y = \mathrm{f}^{-1}(x)$, indicating how the curves $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$ are related. [2]

(iii) Use Simpson's rule with 6 strips to find an approximation to $\int_2^{14} f(x)\,dx$. [4]

6 Look at the following diagram.



The diagram above shows the curve with equation $x = \ln(y^3 + 2y)$. At the point P on the curve, the gradient is 4 and it is given that P is close to the point with coordinates (7.5, 12).

(i) Find $\frac{dx}{dy}$ in terms of y. [2]

(ii) Show that the y-coordinate of P satisfies the equation $y = \frac{12y^2 + 8}{y^2 + 2}$. [3]

(iii) By first using an iterative process based on the equation in part (ii), find the coordinates of P, giving each coordinate correct to 3 decimal places. [5]

7 (i) Substance A is decaying exponentially and its mass is recorded at regular intervals. At time t years, the mass, M grams, of substance A is given by

$$M = 40e^{-0.132t}.$$

(a) Find the time taken for the mass of substance A to decrease to 25% of its value when $t = 0$. [3]

(b) Find the rate at which the mass of substance A is decreasing when $t = 5$. [3]

(ii) Substance B is also decaying exponentially. Initially its mass was 40 grams and, two years later, its mass is 31.4 grams. Find the mass of substance B after a further year. [3]

8 (i) Express $\cos 4\theta$ in terms of $\sin 2\theta$ and hence show that $\cos 4\theta$ can be expressed in the form $1 - k\sin^2\theta\cos^2\theta$, where k is a constant to be determined. [3]

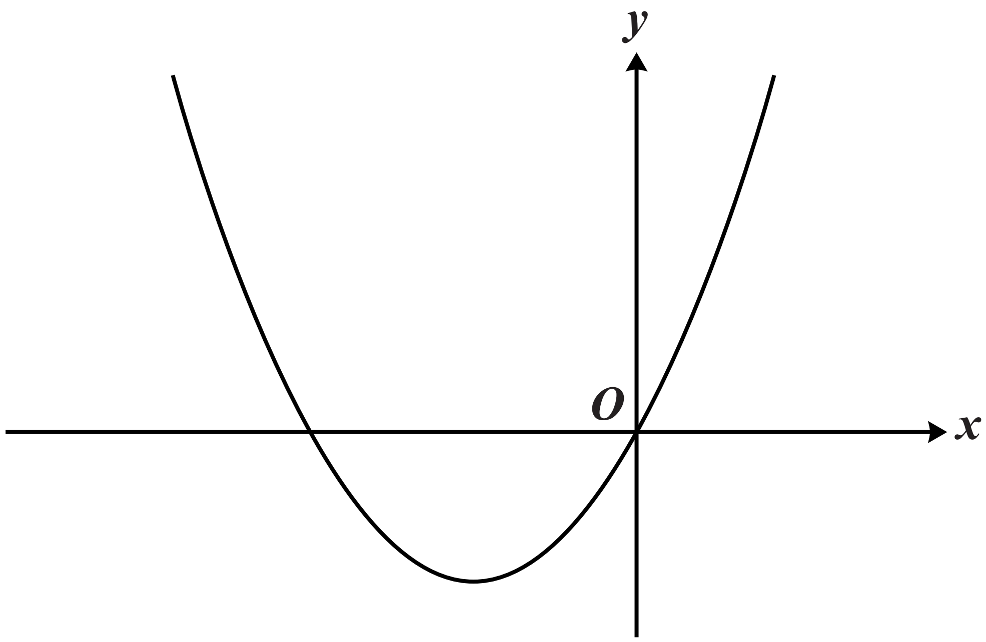
(ii) Hence find the exact value of $\sin^2(\frac{1}{24}\pi)\cos^2(\frac{1}{24}\pi)$. [2]

(iii) By expressing $2\cos^2 2\theta - \frac{8}{3}\sin^2\theta\cos^2\theta$ in terms of $\cos 4\theta$, find the greatest and least possible values of

$$2\cos^2 2\theta - \frac{8}{3}\sin^2\theta\cos^2\theta$$

as θ varies. [5]

9 Look at the following diagram.



The function f is defined for all real values of x by
$f(x) = k(x^2 + 4x)$,

where k is a positive constant. The diagram shows the curve with equation $y = f(x)$.

(i) The curve $y = x^2$ can be transformed to the curve $y = f(x)$ by the following sequence of transformations:
a translation parallel to the x-axis,
a translation parallel to the y-axis,
a stretch.
Give details, in terms of k where appropriate, of these transformations. [5]

(ii) Find the range of f in terms of k. [2]

(iii) It is given that there are three distinct values of x which satisfy the equation $|f(x)| = 20$. Find the value of k and determine exactly the three values of x which satisfy the equation in this case. [6]

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THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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