OCR RECOGNISING ACHIEVEMENT

Monday 23 January 2012 – Morning

A2 GCE MATHEMATICS

4730 Mechanics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4730
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

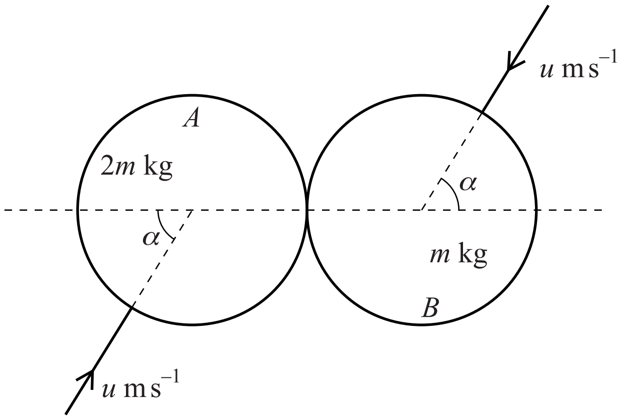
1 A particle P of mass 0.05 kg is moving on a smooth horizontal surface with speed $2\,\text{m s}^{-1}$, when it is struck by a horizontal blow in a direction perpendicular to its direction of motion. The magnitude of the impulse of the blow is I N s. The speed of P after the blow is $2.5\,\text{m s}^{-1}$.

(i) Find the value of I. **[4]**

Immediately before the blow P is moving parallel to a smooth vertical wall. After the blow P hits the wall and rebounds from the wall with speed $\sqrt{5}\,\text{m s}^{-1}$.

(ii) Find the coefficient of restitution between P and the wall. **[4]**

2



Two uniform smooth spheres A and B, of equal radius, have masses $2m$ kg and m kg respectively. They are moving in opposite directions on a horizontal surface and they collide. Immediately before the collision, each sphere has speed $u\,\text{m s}^{-1}$ in a direction making an angle α with the line of centres (see diagram). The coefficient of restitution between A and B is 0.5.

(i) Show that the speed of B is unchanged as a result of the collision. **[5]**

(ii) Find the direction of motion of each of the spheres after the collision. **[3]**

3 A particle P of mass 0.3 kg is projected horizontally with speed $u\,\text{m s}^{-1}$ from a fixed point O on a smooth horizontal surface. At time t s after projection P is x m from O and is moving with speed $v\,\text{m s}^{-1}$. There is a force of magnitude $1.2v^3$ N resisting the motion of P.

(i) Find an expression for $\frac{\mathrm{d}v}{\mathrm{d}x}$ in terms of v and hence show that $v = \frac{u}{4ux + 1}$. **[5]**

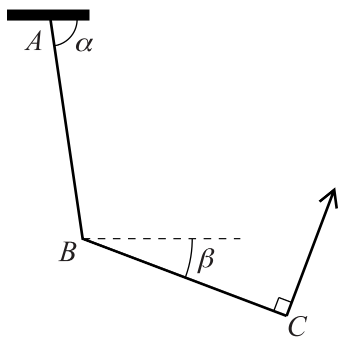
(ii) Given that $x = 2$ when $t = 9$ find the value of u. **[4]**

4 One end of a light elastic string, of natural length 0.75 m and modulus of elasticity 44.1 N, is attached to a fixed point O. A particle P of mass 1.8 kg is attached to the other end of the string. P is released from rest at O and falls vertically. Assuming there is no air resistance, find

(i) the extension of the string when P is at its lowest position, **[4]**

(ii) the acceleration of P at its lowest position. **[4]**

5



Two uniform rods AB and BC, each of length $2L$ m and of weight 84.5 N, are freely jointed at B, and AB is freely jointed to a fixed point at A. The rods are held in equilibrium in a vertical plane by a light string attached at C and perpendicular to BC. The rods AB and BC make angles α and β to the horizontal, respectively (see diagram). It is given that $\cos\beta = \frac{12}{13}$.

(i) Find the tension in the string. **[3]**

(ii) Hence show that the force acting on BC at B has horizontal component of magnitude 15 N and vertical component of magnitude 48.5 N, and state the direction of the component in each case. **[4]**

(iii) Find α. **[4]**

6 A particle P starts from rest at a point A and moves in a straight line with simple harmonic motion. At time t s after the motion starts, P's displacement from a point O on the line is x m towards A. The particle P returns to A for the first time when $t = 0.4\pi$. The maximum speed of P is $4\,\text{m}\,\text{s}^{-1}$ and occurs when P passes through O.

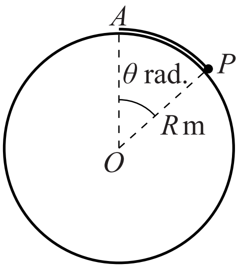
(i) Find the distance OA. **[4]**

(ii) Find the value of x and the velocity of P when $t = 1$. **[4]**

(iii) Find the number of occasions in the interval $0 < t < 1$ at which P's speed is the same as that when $t = 1$, and find the corresponding values of x and t. **[5]**

[Question 7 is printed overleaf.]

7



One end of a light elastic string, of natural length $\frac{2}{3}R$ m and with modulus of elasticity $1.2mg$ N, is attached to the highest point A of a smooth fixed sphere with centre O and radius R m. A particle P of mass m kg is attached to the other end of the string and is in contact with the surface of the sphere, where the angle AOP is equal to θ radians (see diagram).

(i) Given that P is in equilibrium at the point where $\theta = \alpha$, show that $1.8\alpha - \sin\alpha - 1.2 = 0$. Hence show that $\alpha = 1.18$ correct to 3 significant figures. **[7]**

P is now released from rest at the point of the surface of the sphere where $\theta = \frac{2}{3}$, and starts to move downwards on the surface. For an instant when $\theta = \alpha$,

(ii) state the direction of the acceleration of P, **[1]**

(iii) find the magnitude of the acceleration of P. **[7]**