OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS

4724 Core Mathematics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4724
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

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1 When the polynomial f(x) is divided by $x^2 + 1$, the quotient is $x^2 + 4x + 2$ and the remainder is $x - 1$. Find f(x), simplifying your answer. **[3]**

2 **(i)** Find, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line l through the points (4, 2, 7) and (5, –4, –1). **[3]**

(ii) Find the acute angle between the line l and a line in the direction of the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. **[4]**

3 The equation of a curve C is $(x + 3)(y + 4) = x^2 + y^2$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. **[4]**

(ii) The line $2y = x + 3$ meets C at two points. What can be said about the tangents to C at these points? Justify your answer. **[2]**

(iii) Find the equation of the tangent at the point (6, 0), giving your answer in the form $ax + by = c$, where a, b and c are integers. **[2]**

4 **(i)** Expand $\left(1 - 4x\right)^{\frac{1}{4}}$ in ascending powers of x, up to and including the term in x^3. **[5]**

(ii) The term of lowest degree in the expansion of

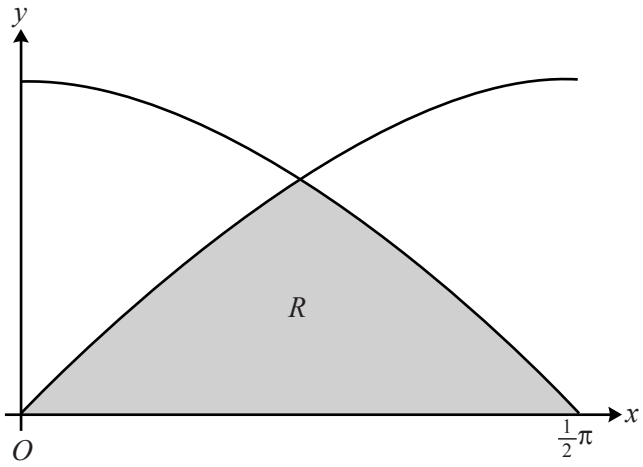
$$\left(1 + ax\right)\left(1 + bx^2\right)^7 - \left(1 - 4x\right)^{\frac{1}{4}}$$

in ascending powers of x is the term in x^3. Find the values of the constants a and b. **[4]**

5 Use the substitution $u = \cos x$ to find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \cos^2 x \, \mathrm{d}x \,. \quad \textbf{[6]}$$

6



The diagram shows the curves $y = \cos x$ and $y = \sin x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The region R is bounded by the curves and the x-axis. Find the volume of the solid of revolution formed when R is rotated completely about the x-axis, giving your answer in terms of π. **[7]**

7 The equation of a straight line l is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

O is the origin.

(i) Find the position vector of the point P on l such that OP is perpendicular to l. **[3]**

(ii) A point Q on l is such that the length of OQ is 3 units. Find the two possible position vectors of Q. **[3]**

8 A curve is defined by the parametric equations

$$x = \sin^2 \theta, \quad y = 4 \sin \theta - \sin^3 \theta,$$

where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{4 - 3\sin^2 \theta}{2 \sin \theta}$. **[3]**

(ii) Find the coordinates of the point on the curve at which the gradient is 2. **[3]**

(iii) Show that the curve has no stationary points. **[2]**

(iv) Find a cartesian equation of the curve, giving your answer in the form $y^2 = \mathrm{f}(x)$. **[2]**

[Questions 9 and 10 are printed overleaf.]

9 Find the exact value of $\int_0^1 (x^2 + 1)e^{2x}\,dx$. **[7]**

10 **(i)** Write down the derivative of $\sqrt{y^2 + 1}$ with respect to y. **[1]**

(ii) Given that $\frac{dy}{dx} = \frac{(x - 1)\sqrt{y^2 + 1}}{xy}$ and that $y = \sqrt{e^2 - 2e}$ when $x = e$,

find a relationship between x and y. **[8]**