RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced GCE **A2 7890 – 2**

Advanced Subsidiary GCE **AS 3890 – 2**

OCR Report to Centres

January 2012

1565602308

3890-2/7890-2/R/12J

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This report on the examination provides information on the performance of candidates which it is hoped will be useful to teachers in their preparation of candidates for future examinations. It is intended to be constructive and informative and to promote better understanding of the specification content, of the operation of the scheme of assessment and of the application of assessment criteria.

Reports should be read in conjunction with the published question papers and mark schemes for the examination.

OCR will not enter into any discussion or correspondence in connection with this report.

Any enquiries about publications should be addressed to:

OCR Publications
PO Box 5050
Annesley
NOTTINGHAM
NG15 0DL

Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

CONTENTS

Advanced GCE Mathematics (7890)

Advanced GCE Pure Mathematics (7891)

Advanced GCE Further Mathematics (7892)

Advanced Subsidiary GCE Mathematics (3890)

Advanced Subsidiary GCE Pure Mathematics (3891)

Advanced Subsidiary GCE Further Mathematics (3892)

OCR REPORT TO CENTRES

Overview – Pure Mathematics

At this series for the first time, all seven units used Printed Answer Books where candidates wrote their solutions. In general, candidates have coped admirably with this development and there have been relatively few cases where candidates' use of the answer books has created difficulties in the marking process. There were some instances where a candidate's answer to a particular question was written in the wrong space, usually following a question where the candidate was unable to make any response. Examiners are usually sympathetic in such cases but it is very much in the candidates' interests to place answers in the designated spaces. Most candidates found that the allotted spaces for their solutions were adequate; indeed, it is to be hoped that the appearance of the allotted space available for a solution serves as a useful discipline for candidates, encouraging them to give a little prior thought to their solutions. This is an examination technique which it would be advantageous for some candidates to practise.

Inevitably, there were a few occasions where candidates needed extra paper. In the vast majority of such cases, only one side of the extra paper was used by the candidate. Accordingly, centres should provide single sheets of paper in such cases and not whole booklets. Graph paper should not be issued either; solutions are often not very clear on such paper after the scanning process.

Precision is a vital element of mathematics. Candidates are well aware of the need for precision when it comes to numerical and algebraic work, even if their solutions do not always retain a necessary degree of accuracy. The corresponding report for June 2011 pointed out the need for that precision to be present as well in the use of notation and terminology. It was again apparent in this examining series that many candidates adopt a rather casual approach to definitions and notation. To mention one instance, it was clear that many candidates do not have a clear idea of the distinctions between integers, rational numbers and real numbers. Other instances will be apparent from consideration of the reports on the individual units which follow.

4721 Core Mathematics 1

General Comments

Candidates' overall performance in this paper was much improved from January 2011 with a large number of candidates scoring very high marks, although full marks was comparatively rare. Most candidates attempted nearly all of the paper, with only 7iii and the last two parts of 10 having significant numbers of omissions. It is possible that a few candidates did not have sufficient time to finish.

Relatively few candidates used additional sheets, indicating again that sufficient room was available in the answer booklet for the solutions to most questions. The exceptions were the few questions which were most likely to be subject to "restarts", including questions 5 and 8. Some candidates also used graph paper for question 2, which was entirely unnecessary.

Many candidates lost marks by not appreciating the links between different parts of questions, particularly between 7ii and 7iii and also 9i and 9ii; centres should encourage candidates to consider possible links both when forming and reviewing their solutions. Centres should also advise candidates to set their solutions out clearly. Those that did so were almost always more successful; where a large amount of number work is unclear and not obviously related to the question it is difficult to award credit. As ever, sketches are helpful in determining an appropriate approach and centres should encourage candidates to include sketches in coordinate geometry questions. Centres should also allow candidates more opportunities to develop their understanding of the notion of proof as this would allow them to explain answers or justify results more clearly.

Comments on Individual Questions

1) This question proved a comfortable starter for many candidates. Almost all approached the question by trying to rationalise the denominator, multiplying both parts of the fraction by $3+\sqrt{3}$. Some candidates struggled with the arithmetic and/or simplification and $\sqrt{3}^{\,2}$ was sometimes given as 9, but most candidates secured all four marks.

2) (i) Graph transformation continues to be a relatively poorly understood aspect of the syllabus. Although most candidates recognised that the f(–x) notation meant the given graph should be reflected, a large number chose to reflect in the x- rather than the y-axis, thus losing the accuracy mark. Most candidates made their intended end-points of the line segments clear, but some did not and again lost a mark.

   (ii) This was generally more successful than part (i). Almost all candidates realised this would be a translation and only a small number erroneously translated the graph horizontally or performed stretches. Most candidates vertically translated the graph upwards as expected, with only a few issues about making the end-points clear as in part (i).

3) As with similar recent questions, many candidates appeared to be slightly confused by more than one format of an expression appearing in a question. Those candidates who expanded the brackets on the right-hand side were usually successful in comparing the coefficients and secured all four marks easily, with only occasional sign errors causing problems. Candidates who attempted to complete the square on the left-hand side were usually less successful. It would appear that candidates generally would benefit from more experience of comparing expressions and appreciating the nature of identities. A large number of candidates with correct working incorrectly identified p as 10 instead of –10.

4) (i) Very few candidates failed to secure the mark for this simple recall of the meaning of negative index notation.

(ii) Most candidates appreciated the meaning of "to the power three-quarters" and the vast majority of these were able to successfully evaluate the expression. There were some errors such as $\sqrt[4]{16} = 4$ and $2^3 = 6$, but these were quite rare.

(iii) Most candidates simplified both the numerator and denominator, but $\frac{10\sqrt{2}}{2\sqrt{2}} = 5\sqrt{2}$ was a very common error even amongst high-scoring candidates. A few rationalised the denominator by multiplying by $\frac{\sqrt{8}}{\sqrt{8}}$, but many who tried this approach were unable to simplify $\sqrt{1600}$.

5) The vast majority of candidates were able to recognise this as a disguised quadratic, but many were unable to find an appropriate substitution and made little or no progress because of wrong working. Some of those who substituted $x = y^{-2}$ found the resulting quadratic difficult to solve and there were many accuracy errors. The final marks were often lost because candidates either rooted or reciprocated; relatively few remembered to do both. Some of those who did do so included the root of a negative number as an acceptable answer. Candidates who started by multiplying throughout by y^4 and then substituting $x = y^2$ were often more successful as they usually remembered to square root at the end. A common error, however, was to ignore the negative square root in the final step thus again losing an accuracy mark. Centres need to remind candidates of the need to state clearly their substitution, both to avoid confusion in the early stages of the solution and as a reminder to fully reverse the process at the end.

6) (i) This question was very well done, with the vast majority of candidates securing all three marks. It was pleasing to see the negative power manipulated accurately; very few candidates rewrote $\frac{4}{x}$ as x^{-4}.

(ii) Most candidates recognised the notation and realised the need to differentiate their answer from part (i) and then substitute in $x = \frac{1}{2}$. This substitution proved problematic for many; often the correct expression $8\left(\frac{1}{2}\right)^{-3}$ was worked out to be 1. Some candidates did misinterpret the notation and substitute $x = \frac{1}{2}$ first; this earned no credit.

7) (i) Most candidates scored highly on this familiar question, multiplying out the brackets, differentiating and setting their derivative to 0. It was pleasing that factorisation was the most common approach to solving the quadratic even though the first term was $3x^2$. The most common method for determining the nature of the stationary point was using the second derivate and this was usually done very well. Although there were slips in accuracy in finding the y-value for $x = 1$, this was quite rare. Some candidates used $x = 4$, the correctly obtained value of the second derivative. A common error amongst those who did not score well was to equate the second, rather than the first, derivative to zero.

(ii) Most candidates recalled the meaning of the word discriminant and were able to calculate it accurately. Some erroneously used $\sqrt{b^2 - 4ac}$, whilst a few others found the derivative.

(iii) This proved to be one of the most taxing questions on the paper with many candidates making no attempt. Many others gave explanations that were incomplete, eg "it doesn't cross so it's always positive", or did not justify their assertions. Others just stated "it's an increasing function" or tested one or two points and concluded it must therefore be true for all. Reference to the negative discriminant found in the previous part of this question was rare.

8) The unfamiliar phrasing of this question proved challenging for many candidates and relatively few marks were earned. Even those candidates who realised they needed to set up and solve a pair of simultaneous equations based on the equation of the line and the distance formula struggled to do so accurately, although a number of fully correct solutions were seen. Trial and improvement alone, often looking for square numbers that totalled 180, was not a suitable method; indeed this often resulted in multiple additional "solutions" which were only rarely tested to see if they met the other given criteria. A more sophisticated approach based on a $(6,\ 12,\ 6\sqrt{5}\ )$ right-angled triangle was often more successful. These candidates almost invariably included a clear sketch of the situation; this was also of benefit to candidates who used any of the other various approaches available.

9) (i) Although the negative coefficient of the squared term caused problems for some in factorising, there were many high-scoring solutions to this question. Most candidates sketched an inverted "U" regardless of whether the roots had been found, and the vast majority was able to identify (0, 12) as the y-intercept. A common error was to see this also as the maximum point of the function; some candidates realised the asymmetry of the roots about the y-axis and correctly placed the maximum point in the second quadrant to earn the final mark.

(ii) Roughly half of the candidates scored both marks for this part of the question. Many candidates did not seem to realise the connection with their graph from part (i) of the question and restarted, often incorrectly choosing the "outside region" as the answer to the inequality.

(iii) This straightforward simultaneous equations question with simple arithmetic was very well answered on the whole. A few errors occurred in substituting the negative x-value when trying to evaluate y.

10) (i) The majority of candidates were able to secure all three marks using the given centre and radius to find the equation of the circle. The main sources of error were arithmetical slips.

(ii) Around half of the candidates scored all five marks verifying the equation of the tangent given in this question. The most common approach was to find the gradient of the radius, find the negative reciprocal and then find the equation of a straight line. Attempts at implicit differentiation were almost always unsuccessful. As the answer was given, there were a number of purely circular arguments starting from the given equation, finding the gradient and using it again with the given point and also some "fiddling" where incorrect working did not in fact lead to the purported final answer.

(iii) Most candidates were successful and took the direct route of substituting the given point into the equation of the line but a significant number attempted far more complex methods to earn the single mark available. Some candidates also substituted (–5, 8) again rather than the given point.

(iv) There were many neat solutions to this final question. Some candidates, however, did not seem to realise that the right angle in the triangle would be between the radius and the tangent and made little progress. Many attempts were difficult to follow as the space was often filled with number work with little or no explanation, although use of Pythagoras' theorem was apparently intended. Alternative methods, such as surrounding the triangle with a rectangle and subtracting the area of more obvious right-angled triangles, were often better set out and successful.

4722 Core Mathematics 2

General Comments

This paper appeared to give the candidates plenty of opportunities to display their skills, and the overall standard of performance was extremely good. Candidates are becoming increasingly proficient on routine questions, but a number struggle when asked to apply these skills in less familiar contexts. They should also ensure that they use the most efficient solution method, such as using standard radian measure formulae in question 1 and using the remainder theorem rather than long division in question 5.

Candidates should ensure that they show sufficient method to make their intentions clear. Whilst in most questions the correct answer will attract full marks, should an error occur little credit can be given if no explicit method is shown and additionally in some questions candidates are expected to show enough detail to demonstrate that they have used the requested method. Candidates should use correct mathematical notation in their solutions. Incorrect notation was penalised on the trigonometric proof, whereas in the questions on the trapezium rule and the binomial theorem a lack of brackets resulted in expressions subsequently being incorrectly used. Candidates should check the reasonableness of their answers; in particular this may help to identify where they have had their calculator in the incorrect mode for a question involving trigonometry.

Candidates should be aware that if they make more than one attempt at a question, it is only the last complete solution that will be marked. Clearly it is in the candidate's best interests if they identify which their final solution is. This is particularly important if they have used extra sheets of paper or erroneously answered a question in the response box for another question. Erasing previous work and writing over it can be hard to read once scanned, and candidates should ensure that their final answer is clear. This was particularly noticeable on the graph sketching question, where some candidates had made several attempts.

Comments on Individual Questions

1) (i) This proved to be a straightforward start to the paper and most candidates gained full marks, though a few simply found the arc length and neglected to add on the two radii to find the perimeter as requested. Whilst most could quote and then use the correct formula, some chose to convert the angle to degrees and then work with fractions of a circle, often with an ensuing lack of accuracy. A surprising minority did not seem comfortable in working with the major sector. Some split it into a semicircle and an acute-angled sector, and others worked with the minor sector. Whilst most who employed the latter approach did then go on to find the perimeter of the requested sector, others gave the perimeter of the minor sector as their answer.

(ii) This part of the question was also done very well with the majority of the candidates gaining full marks. There was the occasional omission of the $\frac{1}{2}$ from the formula, and others lost a mark due to more long-winded methods resulting in a loss of accuracy. A small minority worked with the minor sector, with some giving this as their final answer and others then going on to find the requested area.

2) (i) The trapezium rule was successfully attempted by most candidates, and a pleasing number of fully correct solutions were seen. Generally the y-values were found correctly, though some failed to use the correct value for h despite it being given in the question. A few lost accuracy marks due to truncating or prematurely approximating their y-values prior to applying the trapezium rule. The most successful candidates wrote out a correct, exact expression and then evaluated this in one step, and others made effective use of a table before using these values

in the trapezium rule. Only a few candidates attempted integration prior to applying the trapezium rule, and using x-values directly in the rule was quite rare. Some candidates mistakenly evaluated the trapezium rule between $x = 0$ and $x = 6$.

(ii) Candidates were told that the approximation was an under-estimate, and were asked to justify this. Many candidates struggled to provide a convincing reason, and explanations often lacked clarity and precision. Examiners expected a reason that focused either on the tops of the trapezia being below the curve or on the gap left between the top of each trapezium and the curve. Too many explanations simply stated that the trapezia were under the curve, or stated that there was area not covered without identifying where this area was. A clear sketch is also helpful, and candidates should not be afraid to exaggerate the curve in order to make the gaps between the curve and trapezia clear. However too many had trapezia where the top vertices were not on the curve, and it was quite common to see the tops of the four trapezia share a common gradient so that only y_0 and y_n actually sat on the curve. A sketch showing rectangles rather than trapezia was occasionally seen.

3) (i) Whilst some candidates initially embarked on a full expansion, the majority was able to identify the term required and make an attempt at this. The binomial coefficient was usually correct, though ${}^4C_3$ was sometimes seen, and the 4^3 was also nearly always correct. However, many candidates failed to raise the whole of the algebraic term to the correct power, resulting in $1280ax^3$. Whilst some candidates then proceeded to equate this to $160x^3$ and solve as if the brackets were present, the majority continued with their incorrect expression to obtain $a = \frac{1}{8}$. There was some poor algebra seen in the attempts to solve, usually involving an x^3 term appearing on one side of their equation and not the other, which was penalised. A few candidates tried to take out a factor of 4^6 before expanding; some were successful, but most did not multiply back through by this factor.

(ii) Many candidates gained both marks on this question, as full credit was given for correctly following through on an incorrect value of a. A few candidates did not appreciate that the first term was the constant and instead gave the x and x^2 terms, and others gave the first two terms in descending order.

4) (i) Nearly all candidates were able to correctly state the cosine rule and substitute values, but the subsequent evaluation was not always done correctly. The most common errors included forgetting to square root, using an incorrect order of operations when evaluating and having the calculator in radian mode.

(ii) Most candidates could make an attempt at either angle A or C, usually by using the sine rule but more cumbersome methods were also seen. Many candidates then struggled to convert this to the requested bearing. Whilst a few gave the bearing of A from C, many more either made no attempt at the bearing or simply tried to combine their angle with 180° or 360° in some way without any thought given as to which angle was required. The more successful candidates made effective use of a sketch diagram.

(iii) Many candidates were able to make a reasonable attempt at the required distance, though it was surprising how many used the sine rule in the right-angled triangle rather than basic trigonometric ratios. Candidates who used information given in the question usually obtained the correct answer, whereas those who used values that they had calculated in earlier parts of the question often ended up with an inaccurate final answer. Some candidates made incorrect assumptions about the shape of the triangle, often taking it to be isosceles or attempting to use Pythagoras' theorem. Others seemed unfamiliar with this style of question, and just compared the lengths of CA and CB.

5) (i) Candidates were familiar with the remainder theorem and the majority gained full marks on this question with just the odd slip seen from others when evaluating their expression. A few candidates attempted to use long division and, whilst it was usually correct, candidates should be encouraged to use the most efficient method to solve a given problem.

(ii) The vast majority of candidates could correctly identify $(x – 2)$ as the linear factor and then make a reasonable attempt at finding the quadratic factor. A variety of methods were used, including long division and coefficient matching, and a few simply used inspection. It was pleasing to see many candidates multiply out their two factors to check their answer. Whilst a few failed to gain the final mark through not writing their two factors as a product, there was a very pleasing number of fully correct solutions seen.

(iii) This final part of the question was found to be much more challenging, and there was a wide variation in the quality of answers. Some candidates simply stated the number of roots that they believed the equation had, but provided no evidence for this assertion which gained no credit. The more astute candidates realised that the most efficient method was to evaluate the discriminant, though the negative value of c caused problems for some. Most knew that this indicated that the quadratic would have two roots but some then neglected to include $x = 2$ as the third root. Other candidates decided to use the quadratic formula to attempt to find the actual roots. Whilst this was often done correctly, the conclusions stated revealed a lack of understanding of the meaning of 'real' roots, with a number of candidates discarding roots because they were negative and/or irrational.

6) (i) Virtually every candidate was able to gain both marks by stating the correct three values for the given sequence.

(ii) Most candidates appreciated that they were being asked to sum the first 20 terms of the sequence and gained a mark for attempting to do this. Most could quote the correct formula and substitute in values. Whilst the value of a was usually taken to be 80, the value of d was often used as 5 rather than –5, despite having listed the terms in the previous part of the question. Some candidates determined the value of u_{20} from the nth term definition and then used $\frac{1}{2}n(a + l)$, with this approach tending to be more successful.

(iii) This proved to be much more challenging for many candidates, some of whom struggled to make any progress. Some ignored the given information that it was a GP and treated the terms as an AP, with $d = –20$. Of those who used 80 and 60 in an attempt to determine the common ratio, some made slips such as a incorrect order of division and others concluded that $r^4 = 0.75$, but a number did obtain the third term as 45. Some gave this as their final answer and others then struggled with what to equate it to. Common errors included generating a pth term for the AP with $d = 5$, rather than use the initial definition, or equating 45 to the pth term of a GP. The more able candidates were able to work their way through the question, producing concise and accurate solutions.

(iv) This part was answered correctly by the majority of candidates, and many more gained a method mark for attempting to use the correct formula for the sum to infinity, though some used an incorrect value for a, usually 85.

7) (a) Candidates appreciated the need to expand the brackets before integration could be attempted and most did this correctly, though there were some errors in the signs and/or the powers. The integration attempt was nearly always correct, but some candidates failed to gain the final mark by omitting the constant of integration.

(b) The majority of candidates demonstrated their competence with integration, but devising a strategy to find the requested area required a lot more thought. The majority could integrate $6x^{1.5}$ correctly, though the coefficient was sometimes left unsimplified or was incorrectly simplified. Equally $8x^{-2}$ was also usually integrated correctly, but the –2 sometimes remained as –2 or disappeared altogether. The point of intersection with the x-axis was also usually found correctly, by those who realised the need to find it. Whilst most candidates demonstrated their knowledge of definite integration, selecting the correct limits to use proved much more difficult. Only a few candidates had a well-thought-out strategy for which limits to use with which curve. A common error was to find the sum or difference of the two functions before integration, which meant that it was then impossible to use appropriate limits. Even those who kept the two integrals separate sometimes seemed unsure of which limits to use. Numerical errors, such as $-4 - 4 = 0$, were also seen when candidates evaluated. Those who chose to integrate either one or both curves between the curve and y-axis mostly had limited success.

8) (a) Whilst candidates may be proficient in using logarithms to solve basic equations, this question revealed a lack of understanding that they cannot be applied to an equation term by term. Those who failed to move the –4 across as the initial step were unable to gain any credit, whereas those who were astute enough to do this tended to then gain full marks, barring the odd slip.

(b) Many candidates seem able to recall the generic laws of logarithms, and quote them in an abstract form, but are unable to apply them consistently and accurately to a problem of this nature. Many candidates rewrote the second equation as $\log 3x + \log y = 1$ and then attempted a subtraction between the two equations to eliminate $\log y$. Others simply removed the logarithms term by term to get $x + y = 3$ from the first equation. Some candidates did gain a mark for correctly stating $3x + y = 10$, and the method mark for combining two of the terms in the first equation was also sometimes gained, but for many candidates this was the full extent of their progress, and many more gained no marks at all. Even those who did obtain two correct equations sometimes struggled to then solve them, and it was quite common to see a single solution, invariably (3, 1), appear from inspection. Nevertheless, a number of candidates were able to demonstrate their proficiency with logarithms and produce fully correct solutions, though the final pairings were not always explicitly stated.

9) (i) Most candidates gained a mark for (0, 3), but drawing the correct graphs proved more challenging. Whilst many candidates sketched clearly incorrect graphs, it was disappointing that many of those who did understand the effect of the transformations did not take sufficient care when sketching their graphs. Common errors on the $\tan(\frac{1}{2}x)$ graph included having overlapping branches or large gaps between the branches, both of which could have been avoided by including the asymptotes, or the graph only existing for $-1 \le y \le 1$. The $3\cos(\frac{1}{2}x)$ graph sometimes lacked symmetry in the y-axis or showed no indication that the graph was intended to level off at the extremities.

(ii) When proving the given result, candidates tended to use the relevant trigonometric identities correctly but poor notation was a recurring issue. The most common error was $\tan(\frac{1}{2}x) = \frac{\sin}{\cos}(\frac{1}{2}x)$ but $\cos^2(\frac{1}{2}x) = (1 - \sin^2)(\frac{1}{2}x)$ was also seen. This was penalised by withholding the accuracy mark. Most candidates then recognised the equation as a quadratic in $\sin(\frac{1}{2}x)$ and made some attempt to solve it. Using the quadratic formula was most successful, completing the square less so due to the initial coefficient of 3 and some just had a half-hearted attempt at factorisation and then gave up. Once solutions for $\sin(\frac{1}{2}x)$ had been obtained (either correctly or otherwise), most candidates applied the correct method to solve for x, though there were some who divided their angle by 2, others who divided their root by 2 before applying inverse sine and some who thought that their roots were already the solutions for x. In addition, many answers were given in degrees only and the second angle was often found incorrectly or not attempted at all. It was also disappointing to see extra incorrect solutions within the given range appear, even when the graphs in part (i) had both been sketched correctly, and clearly showed the only points of intersection to be for positive x-values. Whilst most candidates gained at least some credit on this question, fully correct solutions were not common.

4723 Core Mathematics 3

General Comments

The first seven questions of this paper proved accessible to the vast majority of the candidates with the only request causing widespread difficulty being question 6(ii). There were few candidates recording very low marks; approximately 1% of the candidates recorded a total of 9 or fewer. It was pleasing to note the competent work of many candidates in question 7; although the requests here were reasonably straightforward, the topic involved is not one that has always been approached with confidence in the past. It was anticipated that the routine requests in questions 2 and 3 would have led to more success than was the case; most of the errors seemed to be the result of carelessness rather than a lack of knowledge. In particular, the algebraic skills of some candidates were not adequate for work at this level. For instance, on several occasions a process of ‘cancellation’ reducing the expression $\frac{x^2+4x-4}{x^2+4x+4}$ in question 3 to the value -1 was seen.

The last two questions did present more of a challenge to candidates and success was more limited. So it is encouraging to note that more than 1% of candidates recorded full marks on the paper and well over 3% recorded 70 marks or more out of 72. Solutions to question 8 suggested that, for many candidates, knowledge of trigonometry exists at a fairly superficial level. In particular, it seemed that many did not appreciate the nature of identities. The fundamental identity $\cos 2\theta \equiv \cos^2\theta - \sin^2\theta$ was known but many were unable to adapt this to write down $\cos 4\theta \equiv \cos^2 2\theta - \sin^2 2\theta$. Either there was no idea how to deal with $\cos 4\theta$ or the basic identity was adapted, with 4 replacing 2, to give $\cos 4\theta = \cos^4\theta - \sin^4\theta$. Study at this level does require candidates to be able to apply their mathematical knowledge and techniques to the solution of slightly unfamiliar problems; the amount of thought and analysis needed initially in question 9 was evidently lacking in the approach taken by many candidates.

Comments on Individual Questions

1) This question proved to be a straightforward introduction to the paper for candidates, about three quarters of whom duly recorded full marks. With the answer given in the question, candidates were required to show some detail and many did show clearly the use of the appropriate logarithm properties to give $\ln 3$. Candidates going directly from $2\ln\sqrt{6} - 2\ln\sqrt{2}$ to $\ln 3$ earned only the first mark. The same was true of those candidates who included a step involving a term $\frac{\ln 6}{\ln 2}$ and those whose approach was to compare the decimal approximations of $2\ln\sqrt{6} - 2\ln\sqrt{2}$ and $\ln 3$.

2) Although just over a half of candidates earned full marks on this question, a number of avoidable errors occurred in the solutions of other candidates. The factor π either never appeared or, after an initial appearance, was not present in the final answer. The process of squaring $(2x+1)^{-2}$ led in a few cases to $(2x+1)^4$ and sometimes the 6 was not squared. Further errors in the integration included expressions involving $(2x+1)^{-5}$ and some involving a natural logarithm. The vast majority of candidates did attempt evaluation using the two limits and attempted to give an exact value for the volume as requested.

3) Most candidates attempted to use the quotient rule to find the first derivative; many did so accurately but, for many other candidates, the absence of necessary brackets in the numerator or the faulty removal of them where they did appear meant that solutions started to go wrong at an early stage. With such errors, it was often the case that the derivative reduced to 1; there was no indication that candidates obtaining this were puzzled at finding that the derivative of the curve was constant.

Candidates who succeeded in obtaining the correct value $\frac{1}{9}$ for the derivative at the point concerned did not always go on to conclude their solutions correctly. Some went on to find the equation of the tangent and others used a gradient of 9 rather than –9. A few candidates lost the final mark by failing to give the answer in the form indicated in the question.

About two fifths of the candidates recorded full marks on what was expected to be a routine request.

4) (i) This question was generally answered well and most candidates had no difficulty in finding the two correct values. Many did proceed to give values for the two angles but these extra steps were ignored in the marking. A few candidates showed rather protracted attempts at finding the value of $\tan\alpha$ and, indeed, some concluded incorrectly with $\tan\alpha = \frac{1}{2}$. A few candidates failed in their attempts to find $\tan\beta$ by opting for the identity $\sec^2\beta \equiv \dfrac{1}{\cos^2\beta}$.

(ii) This part was also answered well with the appropriate angle sum identity being used efficiently. The principal errors occurred when candidates used angles to find the value of $\tan(\alpha+\beta)$ or when candidates revealed considerable uncertainties about their understanding of trigonometry by attempting to evaluate an expression such as $\dfrac{\tan 2 + \tan 5}{1 - \tan 2 \tan 5}$.

5) A number of candidates decided that the values given in the table were insufficient and they tried to find a formula for f(x). Using such a formula meant that it was only in part (ii) that they could record any marks. Most candidates did proceed without undue difficulty and about half of all candidates earned all eight marks.

Most candidates earned both marks in part (i) although there was some evidence of unfamiliarity with function notation as answers such as $\mathrm{ff}(6) = 196$ and $\mathrm{f}^{-1}(8) = \frac{1}{19}$ appeared not infrequently.

The vast majority of candidates earned the two available marks in part (ii). The sketch was usually fine although, in a few cases, it strayed into the fourth quadrant or was drawn only in the first quadrant. There were more instances of the second mark of part (ii) not being earned. A statement referring in some way to reflection in the line $y = x$ was needed; some candidates seemed to believe that it was enough just to draw the line on the diagram.

Most candidates had no difficulty with applying Simpson's rule. A few associated 2 and 4 with the wrong y-values or used the wrong value of h. Misuse of brackets led a few candidates to an answer of 300 via the calculation $\frac{2}{3}(1+26)+4(8+19+25)+2(14+23)$. There was occasional use of x-values rather than y-values and those candidates using y-values from an invented formula rather than from the table gained no credit.

6) (i) Most candidates answered this first part correctly although the incorrect answer $\dfrac{1}{y^3+2y}$ was not uncommon.

(ii) This part was not answered well and only a quarter of candidates recorded all three marks. One common approach was to substitute $y = 12$ into the right-hand side of the given equation and to claim the result proved when the outcome was a value close to 12. Many other candidates showed that they had no appreciation of the distinction between $\frac{dx}{dy}$ and $\frac{dy}{dx}$ by equating their expression from part (i) to 4 and trying in vain to reach the given result. Those candidates recognising that the reciprocal of their expression for $\frac{dx}{dy}$ had to be equated to 4 proceeded with due care and attention to detail to confirm the result.

(iii) The iteration process was usually carried out successfully with candidates showing the successive iterates in their solutions. A few candidates did no more in this part than just write down the two coordinates; they received no credit because there was no guarantee that they had used an iteration process rather than, say, an equation solving facility on their calculators. Two things prevented greater success in this part. Candidates were expected to show an appreciation of the nature of the question by starting their iteration with the value 12; many lost the first mark by starting instead with the value 7.5 or with some other value such as 0 or 1. The final mark was also lost in many instances because candidates either forgot to find the *x*-coordinate or, having found its value, did not give the value correct to 3 decimal places as required.

7) (i) Parts (a) and (b) were answered well with candidates showing commendable fluency in dealing with exponential functions. In part (b), there was immediate recognition that differentiation was required and an acceptable answer was usually reached without trouble. Success in part (a) was not quite so common, due, in many cases it seemed, to insufficient care in reading the question; as a result there were many attempts to solve either $40e^{-0.132t} = 30$ or $40e^{-0.132t} = \frac{1}{4}$ instead of the correct $40e^{-0.132t} = 10$.

(ii) It was pleasing that as many as three quarters of the candidates recorded all three marks on this more awkward part. Most proceeded to find a formula for the mass of substance *B* and to substitute $t = 3$ to find the requested mass. A few successfully adopted the alternative approach of dealing with the appropriate power of $\frac{31.4}{40}$. A minority of candidates earned no marks in this part. Either they thought that the decrease in the mass of substance *B* was a constant 4.3 grams per year or they tried to use a formula which still featured $e^{-0.132t}$.

8) (i) It was expected that most candidates, being familiar with the identity $\cos 2\theta \equiv 1 - 2\sin^2\theta$, would be able immediately to write down $\cos 4\theta = 1 - 2\sin^2 2\theta$ but, for all but a minority of candidates, this was not the case. Many embarked on lengthy procedures, often involving powers of $\sin\theta$ and $\cos\theta$, in the vain hope of reaching a satisfactory conclusion. A mark was available to those candidates stating or implying $\sin 2\theta = 2\sin\theta\cos\theta$. But it was then surprising to note how many candidates seemed to think that this led to the identity $\sin^2 2\theta = 2\sin^2\theta\cos^2\theta$ with the result that many reached a conclusion in which *k* was 4.

(ii) There was limited success in this part. Even many of those who had reached an expression of the required form in part (i) did not see how to exploit that result in part (ii). Often there was further involved trigonometry or candidates just resorted to their calculators to give a decimal approximation for the value.

(iii) Whilst a minority of candidates was able to use the identity from part (i) together with the identity $2\cos^2 2\theta \equiv 1 + \cos 4\theta$ to form the equivalent expression $\frac{2}{3} + \frac{4}{3}\cos 4\theta$ in just a few lines of working, many other candidates spent considerable effort, again usually involving powers of $\sin\theta$ and $\cos\theta$, without making any relevant progress. Some of the candidates with an expression in terms of $\cos 4\theta$ did proceed appropriately to offer the greatest and least values but others did not seem to understand the request and tried to solve an equation. As a question towards the end of the paper, question 8 was intended to involve more challenging requests. Nevertheless, only a small proportion of the candidates were able to record all ten marks and half were unable to record more than one mark.

9) (i) Part (i) required a little initial thought from candidates in order to devise an appropriate strategy. This could involve either completing the square or differentiating to find the coordinates of the minimum point. Candidates opting for either of these usually managed to complete this part successfully and often made significant progress with the later parts too. But for many candidates the only mark earned in this part was one mark for correct details of the stretch (although some failed to earn even this mark by omitting the direction of the stretch). For such candidates, their attempts to describe the transformations without any preparatory work usually involved translations of 4 or 4*x* or 4*k* units. The first two marks for the necessary preparatory work were sometimes earned by the solution to part (ii).

(ii) Candidates were a little more successful with this part. For many, there seemed to be little understanding of the term range and answers such as all real values or answers involving –2 were seen.

(iii) This part also required some initial thought. Those candidates who sketched the graph of $y = |\mathrm{f}(x)|$ and stopped to consider how a line parallel to the *x*-axis could meet the graph three times quickly realised that $4k = 20$ and then finding the three values of *x* was straightforward. However, for many candidates, the sight of an equation involving modulus signs immediately meant that both sides of the equation had to be squared and, in practically all such cases, there was no satisfactory conclusion. Many other candidates attempted to use the discriminant of either $k(x^2 + 4x) = 20$ or of $k(x^2 + 4x) = -20$ or, more usually, of both. These attempts tended to be somewhat haphazard and it seemed that conclusions, when reached, had occurred as much by accident as by design. The best solutions to this part showed that candidates had analysed the problem correctly and, in many cases, they added a few words of explanation; each step was taken with purpose and, as a result, concise and accurate solutions led to the award of all six marks.

4724 Core Mathematics 4

General Comments

Whilst there were some well presented scripts, in other cases it was difficult to decipher what was going on. Candidates should be reminded that if their work is unclear and difficult to follow, they run the risk of examiners misreading their responses which may result in marks being lost.

Comments on Individual Questions

1) The words 'quotient' and 'remainder' were fully understood by the majority of candidates although a number thought that $\mathrm{f}(x)$ was $(x^2+1)(x^2+4x+2)+\frac{x-1}{x^2+1}$. Generally the multiplication was satisfactory; some misreads were seen where $x+1$ or -1 were used instead of $x-1$.

2) A few candidates were careful in this question, particularly with respect to the signs of the coordinates of the point $(5,-4,-1)$. Candidates should note the form in which answers were requested; in part (i), the vector equation of the line was required and candidates presenting their answer as $\mathbf{r} = (4, 2, 7)+t(1,-6,-8)$ were not awarded the final mark.

3) (i) Most candidates dealt with the implicit differentiation work satisfactorily; some treated $(x+3)(y+4)$ as a product whilst others expanded $(x+3)(y+4)$ first. Mistakes most often occurred where, after rearrangement, candidates were required to differentiate the term $-xy$; the derivative of this was often given as $-x\frac{\mathrm{d}y}{\mathrm{d}x}+y$.

(ii) It was rare that the denominator of $\frac{\mathrm{d}y}{\mathrm{d}x}$ did not consist of $x-2y+3$ and most candidates recognised that the restriction $2y=x+3$ meant that the denominator was 0, so obtaining the first mark. However, the interpretation of this was not well done. What was required was something simple such as the tangents are vertical or are parallel to the y-axis or are of the form $x=k$. 'Gradient of tangent is asymptotic', 'gradient is infinity' and 'gradient of tangent cannot be found' are examples of responses which were not awarded the second mark.

(iii) This part was answered very well. Candidates who had made errors in part (i) could earn full marks in this part, provided the equation was given with integer coefficients.

4) (i) Candidates proceeded with care, and usually accuracy, in this part.

(ii) The wording 'The term of lowest degree … is the term in x^3' was not understood well by many of the candidates and, after the initial step of expanding $(1+bx^2)^7$, relatively few seemed to know what to do. Inequalities were frequently seen, comparing coefficients of x^3 in the two separate parts of the expression. There were many errors connected with the negative signs in the $-(1-4x)^{\frac{1}{4}}$ part of the expression. The first part of the expression was sometimes interpreted as $[(1+ax)(1+bx^2)]^7$.

5) This was one of the better answered questions. Candidates clearly understood the basic techniques although errors in some of the details often prevented full marks being earned. Negative signs were often dropped and the new limits, 1 and $\frac{1}{2}$, sometimes appeared in the wrong positions. Sometimes such errors 'cancelled out' but anything that was definitely wrong was penalised. Some candidates integrated $u^2(1-u^2)$ by parts.

6) Many candidates would have benefited from a few moments' thought before tackling this question but, too often, they appeared to have launched directly into their solutions. Being sure which graph is which and establishing their point of intersection were useful initial steps. It was also advantageous to note the symmetry in the diagram which meant that the result could be found by evaluating $2\pi \int_0^{\frac{1}{4}\pi} \sin^2 x \, dx$. It was encouraging to note so few candidates integrating $\sin^2 x$ as $\frac{1}{3}\sin^3 x$; in fact, most candidates' double angle work was exemplary, whether proceeding as above or by dealing with two integrals.

7) Many candidates showed little understanding of what was involved in this vector question. A simple diagram showing the origin, the point $(1, 0, 2)$ with a straight line passing through it plus two points *P* and *Q* marked on the line would have helped. A key point was that, for different values of *t*, the points *P* and *Q* could be represented by $(1+t, -t, 2)$ but this idea was frequently not used. The predictable mistakes in the calculations needed in part (i) – errors in dealing with negative signs and with multiplication by 0 – duly occurred in many cases. Most candidates with an acceptable approach to part (i) were able to deal successfully with part (ii).

8) (i) The expression for the derivative was given in this part so that parts (ii) and (iii) were accessible to all candidates. But, with the answer given, candidates' working was checked carefully to ensure that the given expression was genuinely reached. Accordingly, the many candidates who wrote down $\frac{dx}{d\theta} = 2\sin\theta$ and $\frac{dy}{d\theta} = 4 - 3\sin^2\theta$ followed by the required result did not receive credit. It was a pity that, because $\sin^2\theta$ was involved, some candidates automatically opted to convert to an expression involving $\cos 2\theta$.

(ii) The vast majority of candidates used the given gradient and produced the appropriate quadratic equation. The only occasional error to occur was the omission of the step to produce the coordinates.

(iii) Most candidates realised that $3\sin^2\theta = 4$ if there were to be any stationary points. This observation was rewarded but, in general, attempts to explain why this equation could not be solved were poor. 'Maths Error' was a common response whilst $\sin\theta = \frac{2}{\sqrt{3}} > 1$ (with no mention of $\sin\theta = -\frac{2}{\sqrt{3}}$) was also unacceptable.

(iv) Only a few candidates attempted this part first and tried to make use of it in part (i). Most candidates were able to produce a cartesian equation such as $y = 4\sqrt{x} - (\sqrt{x})^3$, but very few were able to square satisfactorily and a common answer was $y^2 = 16x - x^3$.

9) Almost all candidates realised that this question was testing the topic of integration by parts although a few failed at the second stage of the integration when $\int x\mathrm{e}^{2x}\,\mathrm{d}x$ became $\frac{1}{2}x^2\mathrm{e}^{2x}$ or something similar. Manipulation of the negative signs at the second stage did go wrong for some candidates; care and the judicious use of different styles of brackets would have prevented this. The use of the limits was performed well and very few stated that the result of substituting $x = 0$ would be 0.

10) (i) This use of the chain rule was done very well. The appearance of $\frac{\mathrm{d}y}{\mathrm{d}x}$ in $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(y^2+1)^{-\frac{1}{2}}\,.\,2y$ was tolerated but it was not in $\frac{1}{2}(y^2+1)^{-\frac{1}{2}}\,2y\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Many candidates did not recognise this question as assessing the solution for a differential equation in which the variables are to be separated and they were unable to obtain any marks. Those who did attempt to separate the variables performed the operation well, duly using the answer from part (i) appropriately. The right-hand side $\frac{x-1}{x}$ proved more difficult with some using integration by parts (which then needed the integral of $\ln x$ at the second stage). The substitution of the boundary condition was satisfactory, although it was surprising how many candidates failed to recognise that $\left(\sqrt{\mathrm{e}^2-2\mathrm{e}}\right)^2+1$ was equal to $(\mathrm{e}-1)^2$.

4725 Further Pure Mathematics 1

General Comments

Most candidates attempted all the questions and there was no evidence of candidates being short of time. The presentation of answers was generally better than in previous sessions, which was pleasing. Candidates seemed well prepared for this paper, and correct solutions were seen to all questions, with a good proportion of candidates scoring very high marks. The space provided in the printed answer booklet was usually sufficient and only a few candidates needed additional answer paper.

Comments on Individual Questions

1) Most candidates answered this question correctly, the most common errors being using $a^2 - 25 = 169$ when finding a and using $\tan(\frac{5}{12})$ rather than $\tan^{-1}(\frac{5}{12})$.

2) A correct pair of simultaneous equations was generally found and solved correctly. Some candidates tried to multiply $p\mathbf{A}$ and $q\mathbf{B}$ while others tried to multiply the given equation by the inverse matrices.

3) The method of finding a quadratic equation in x^2 or y^2 was well understood and only arithmetic errors were the main loss of marks. However, some candidates only gave one square root while others gave four, rather than the correct pair.

4) The standard formulae were used well, and a good proportion of candidates found the correct factorised answer. Some expanded to obtain a quartic, and often failed to factorise correctly, rather than using a common factor at the first stage of simplification. Some candidates attempted to multiply two standard results as their first step of working.

5) (a) Most knew or found the correct matrix. The most common error was to give the matrix for reflection in the line $y = x$, while a few candidates gave the unit matrix.

(b) Some candidates did not recognise the stretch, and many did not give a satisfactory description that all points (not on x-axis) are transformed, using expressions like "in the y-axis" which are more suitable for a reflection.

6) Most candidates realised that the required loci were a circle and a half line. Common errors were not indicating clearly the coordinates of the centre, locating the centre in an incorrect quadrant and not showing the circle passing through the origin. Often the half-line did not pass through the centre of the circle or it started at the centre.

7) (i) Most candidates showed sufficient working to justify the given answer.

(ii) A correct matrix was generally stated, the only error being in not recognising $3^n - 1$.

(iii) Many did not show sufficient working to justify the base case, with the same matrix being written down twice. Similarly, insufficient working was often shown when establishing the case for $n = k + 1$. It is hoped that centres will emphasise to candidates that, in an induction proof, clear detail is required as well as clear explanation of the process.

8) (i) The given result was generally established correctly.

(ii) The method of differences was well understood and most showed clearly the cancelling process, with only a few errors at this stage.

(iii) Many did not understand that the sum to infinity was 1, which usually meant that they had difficulty finding the required sum. Many clearly wrote down the sum to infinity as the answer to the required sum.

9) (i) Most candidates knew how to find the determinant, and most used the top row. The most common reason for loss of marks was sign errors.

(ii) Most solved their det **X** = 0 correctly, but some thought that an inequality needed to be solved.

(iii) Most knew the processes required to find the inverse matrix. Sign errors or an incorrect element were the most common mistakes, but many used their quadratic expression from (ii), $a^2 + 9a - 10$, instead of the correct determinant.

10) (i) This part was answered accurately by most candidates; some sign errors occurred and some did not divide the coefficients by 3 when finding the values of the symmetric functions.

(ii) A high level of algebraic skill was demonstrated by a good number of candidates resulting in the correct values for a and b being found. Again, sign errors resulted in many marks being lost. Some used a substitution correctly, but a significant number who tried this method substituted in the wrong cubic equation. Those who used a mixture of the two methods usually managed to produce good solutions despite having to do rather more work than was needed.

4726 Further Pure Mathematics 2

General Comments

Most candidates were able to access all questions. The least well attempted was question 9; this might indicate a paper that was rather longer than usual.

Three points are worthy of note:

- Candidates often did not answer questions well that were of the "show that..." type with the answer given. It is the responsibility of the candidate to demonstrate conclusively that the answer would have been attained even if the answer had not been given and quite often examiners were not convinced.
- Algebraic manipulation was at times poor.
- Marks were lost through not reading the question properly and therefore not answering the question correctly.

These points are highlighted in the individual comments below.

Comments on Individual Questions

1) Many saw this question as an easy starter, but a large number not only made errors but took a long route to get to the answer. $f'(x)$ is $3\tan 3x$ but many left their answer as $f'(x)=\dfrac{-3\sin 3x}{\cos 3x}$, sometimes without the 3 (or placing it in the denominator) and without the negative sign. It was then necessary to find the second derivative using the quotient rule and with extra negative signs and 3s, this proved too much for many. Those that wrote $f'(x) = 3\tan 3x$ found $f''(x)=9\sec^2 3x$ and hence the result quite quickly.

2) Insecurity with basic algebra was evident here with a large number unable to complete the square. In addition to various incorrect constants given, some wrote $4x^2-4x+5=(2x-2)^2+...$ Those who obtained an inverse tangent often omitted constants, most notably a 2, giving an incorrect multiple of π .

3) In this question it was the arithmetic that let down many candidates. Those that were aware that the fraction was not proper either added a constant to their partial fractions or divided out before finding the partial fractions and did the first part well. However, when it came to solving the simultaneous equations to find their constants, there were many errors. Consequently, the number of candidates with full marks was low.

4) (i) A mark was lost by many candidates for not doing anything more than explaining about the total area of rectangles which was the expression given. A few failed to realise that the lack of any explanation resulted in an answer that was no more than writing down the statement given on the paper.

(ii) The problem with this part was a failure to understand the implication of the demand to use n rectangles and the fact that in part (i) there were only n – 1 rectangles.

(iii) Those who gave their answer in part (ii) using n – 1 rectangles usually obtained an incorrect upper bound.

(iv) A few did not read the question properly and continued to work with $n = 4$. A few others failed to think carefully about their answer, rounding down from a decimal value or just leaving a decimal value for N.

5) (i) Some candidates were careless with their algebra, particularly the negative signs, being content to write one line and then the result as given on the paper. Some candidates used an ingenious method to achieve the result, by adding $2x^3$ to both sides of the equation $x^3 = k$. This certainly produced a valid iterative formula and also matched that given but was not shown to be the Newton-Raphson formula, as required.

(ii) There were a few good sketches. However, many did not indicate an awareness of zero gradient at the intercept on the y-axis and many did not state the intercept on the axes as required. It was possible to take a point on the curve in the fourth quadrant to show that the tangent at that point meets the x-axis further away from the root than the point chosen. Many missed out this part; others drew tangents where this did not happen. A few seemed to draw the line $y = x$.

(iii) The demand here was to express the value of α exactly, and many failed to do so either by missing it out or giving an approximation. More than one candidate wrote "this is the exact value of α, correct to 5 decimal places".

(iv) Here, again, marks were lost through not reading the question properly or making assumptions. A number of candidates gave their results to 3 significant figures only, others calculated $x_n - \alpha$ rather than $\alpha - x_n$. Additionally, many wrote $\frac{e_2{}^3}{e_1{}^2} = 0.00013 \approx e_3$ which is incorrect, indicating that this value had not in fact been calculated.

6) (i) This standard bookwork question was also a question where a mark was often lost. It was necessary to justify the negative sign and many candidates failed to do this.

(ii) This was another question where marks were lost because of poor algebra. The derivation of the derivative required the chain rule; failure to deal with this properly left the middle part of the question rather more easy and the last part unattainable. For instance, a significant number of candidates were unable to obtain $\sqrt{1-\left(1-x^2\right)^2} = x\sqrt{2-x^2}$. Any correct expression other than the most simple form inside the square root meant that finding the second derivative was made much more complicated.

7) (i) In both of the standard methods for obtaining this result, some justification of the sign was necessary as a square root is taken. As with Q6, many candidates failed to do so and lost a mark.

(ii) The derivation of the equation in x by exponentiating was, in general, done well. The process of squaring this equation, however, resulted in some poor algebra. It was common to see $\left(\sqrt{x^2+1} - 2\sqrt{x^2-1}\right)^2 = \left(x^2+1\right) \pm 4\left(x^2-1\right)$ and even $4\left(x^2-1\right) = 4x^2 - 1$.
Once again, the final answer required a decision to be made about the possibility of a negative answer as well as the positive value.

8) (i) For those who could remember the double angle formulae from earlier units, or were able to use their formulae book effectively, this part caused no problems.

(ii) However, this part was very poorly done. The major difficulty appeared to be a lack of understanding of what area the integral $\frac{1}{2}\int_0^{\alpha} r^2 \, d\theta$ actually represented, and as a result a significant number of candidates started off with incorrect integrals (and limits). Even those who appeared to understand did not always get the limits correct. A few instances of the type $A = \frac{1}{2}\int \left(r_1 - r_2\right)^2 \, d\theta$ were seen.

Quite often the correct integrals were subtracted rather than added. For many candidates, finding the values of $\cos 2\alpha$ and $\sin 2\alpha$, given $\tan\alpha = \frac{1}{2}$, proved challenging. As a result, many solutions were confused and various approaches to reaching the given answer were unconvincing.

9) (i) The progression from the initial definition to the given answer was often unconvincing. Candidates needed to demonstrate that they had shown the given statement to be true and included all algebraic steps.

(ii) There were many correct answers to this part. The greatest error was to try to integrate I_n by parts with $\frac{dv}{dx} = 1$ or $\frac{dv}{dx} = \tanh x$. Candidates might have taken note of the fact that the reduction formula involved the reduction in the value of n by 2 which might have led them away from attempting these approaches.

(iii) This part was usually done well. A common error was an incorrect sign of *a.*

(iv) This part was not attempted by the majority of candidates. The method of differences seemed unfamiliar to many who did attempt it; others failed to deal with the negative sign adequately.

4727 Further Pure Mathematics 3

General Comments

Once again, there was a small entry for the January sitting of this paper. Overall the paper was found to be slightly more demanding than the paper last January, with some parts of questions being answered correctly by only a relatively small number of candidates. Some candidates were clearly extremely poorly prepared for this paper, often lacking knowledge not just of this paper but also of supporting techniques from Core 3 and Core 4. As there were many centres where only one candidate had been entered, it is suspected that some poor scripts were the work of candidates who had prepared for the examination in isolation and who had not organised their time well. At the other extreme, there were many excellent answers from some candidates. There did not appear to be any problem with the length of the paper with all candidates appearing to have sufficient time. All scripts were legible though sometimes candidates appeared to make transcription errors due to difficulty in reading their own writing.

Comments on Individual Questions

1) This was a straightforward lead-in question with which most candidates were comfortable.

(i) A few candidates did not have any technique for substituting for $\frac{dy}{dx}$.

(ii) Some candidates omitted the constant of integration and thus betrayed their lack of comprehension of the nature of general solutions. A few were unable to accomplish separation of variables despite this being a Core 4 topic.

2) (i) Almost all candidates were able to access this question. Some chose to use the given formula for $\cos\theta$ from the specification; others demonstrated that the '$2\cos\theta$' could be derived from $(\cos\theta + i\sin\theta) + (\cos\theta - i\sin\theta)$. Either method was acceptable.

(ii) Few candidates were able to derive the four factors in the correct form; many candidates, however, were able to use part (i) to at least get four linear factors. Some chose not to use part (i) and made some progress solving the quartic by solving the quadratic in z^2 first.

3) Good answers were marked by a clarity of algebraically expressed reasoning. There was a distinct divide between those candidates who could handle work on groups and those who lacked sufficient knowledge of the topic. The latter treated elements of this question as real numbers under multiplication, and understood x^{-1} to be the reciprocal of x.

(i) A few candidates erroneously assumed that the group comprised of solely the three listed elements. Others mistakenly assumed commutativity both here and in the other parts of this question.

(ii) Good answers were clearly explained, often giving thorough demonstration of how $(xy)^n = x(yx)^{n-1}y$.

(iii) The few who had thoroughly answered part (ii) were usually able to pick up the marks here; others moved elements about as though the operation was automatically commutative.

4) (i) This was a standard question which gave some candidates few problems. Some candidates improvised good alternative methods using techniques from Core 4. A diagram certainly appeared to help some candidates here and in part (ii).

(ii) A few candidates used the ratio theorem (maybe taking advantage of its inclusion in the formulae booklet), but more commonly candidates found $\overrightarrow{OA}+\frac{3}{4}\overrightarrow{AM}$, or the equivalent using M instead of A.

5) This was another standard question which gave most candidates the opportunity to score good marks. The main problem, for some candidates was caused by difficulty with differentiating a product.

(i) This straightforward question was answered well apart from the odd careless mistake in solving a quadratic.

(ii) Most candidates knew what process was required, but were let down by their lack of basic calculus techniques.

(iii) As in part (ii), a lot of candidates were well aware of how they should go about finding the solution, and some could solve the equation efficiently.

6) (i) This part and the next were standard questions. Most candidates found **n** by means of the vector product rather than by eliminating λ and μ, although, of course, either method was valid. Since the question asked for the equation in vector form, candidates were unable to gain the fourth mark if they, instead, gave only the cartesian form.

(ii) This was straightforward for those who had been able to do part (i). Those who had not (and some who had) often equated the original forms for l and Π, but often the subsequent solution of simultaneous equations was beyond them.

(iii) This question was well answered by some candidates who went straight to the vector product required. Other candidates made life harder for themselves by either using two scalar products or by using the vector product of **c** with either **n** or (2, –1, –1) in an equation. Either of these approaches tended to leave room for later calculation errors.

7) Like the previous group question, there were some candidates who seemed not to be familiar with the basics. However for most candidates this was a good source of at least part marks.

(i) This part was answered well.

(ii) Several candidates left their answer to the inverse as an element outside the group, not allowing for the fact that the group uses modulo 6.

(iii) The most common mistake here was caused by poor language; some candidates seemed unaware of the distinction between factor and multiple, or even of which number was being divided by which as their sentence ‘4 cannot be divided exactly by 6’ indicated.

(iv) There were a few excellent answers to this part that demonstrated a full understanding of the topic and went far beyond the minimum required for 2 marks.

8) (i) This part was well answered by many candidates, but others were not thorough enough in their proof. The number of marks allocated to the question should have acted as a clue to the fact that writing a first line that gave tan 5θ in terms of powers of sine and cosine, followed a second line with the answer given would not be sufficient explanation. The examiner wished to see explicit use of de Moivre’s theorem and also to see the process whereby sines and cosines were converted to tangents.

(ii) Although a few candidates clearly thought that this part was meant to follow on from the result in part (i), it was still disappointing to see a significant number who could not tackle a fairly straightforward solution to a trigonometric equation. Sometimes this was down to careless numeric calculations, sometimes to lack of knowledge of how to find values apart from the principal one or even, occasionally, lack of knowledge that such values existed.

(iii) A few candidates confidently worked their way through this question and presented their work clearly and concisely. Since the question asked candidates to ‘show that ...’, it was insufficient to jump to the solutions without evidence as to how they were come by.

Overview – Mechanics

In each unit examined, there were many good scripts, presented by candidates who were well prepared for the papers they sat.

The authors of the question paper attempt to leave sufficient space, when designing the answer booklet, for candidates to include their own diagrams. By carefully annotating their diagrams candidates might avoid some errors. For example, in mechanics 1, Q6(i) and mechanics 2, Q5(i), candidates would sometimes omit the component of weight down an inclined plane.

It is hoped that sufficient space is provided in the booklets for the full solution to a question. Disorganised, erroneous or lengthy methods of solution may well require more space than has been given. In such cases extra sheets should be provided, and used on both sides. Surplus space is not given within the answer booklet as this might suggest that the required solution requires more lines of work than is in fact the case.

4728 Mechanics 1

General Comments

The standard of mathematics in the work presented by candidates was good. There is evidence of increasing use of calculators to solve equations, eg Q3(ii) and Q5(iv).

In some cases badly organised or untidy work led to needless errors in scripts. A second reason for possible underachievement lay in a failure to answer the question being asked, or in believing that the solution required more complex mathematics than was appropriate, as exemplified by Q1(i), Q3(ii), Q4(iii) and Q5(iv). Together these factors led candidates to lose more marks than having insufficient understanding of the mechanics being tested.

Comments on Individual Questions

1) (i) Almost all candidates obtained full marks, though in some solutions the direction of motion of P was wrong.

(ii) There was a large minority of scripts in which no marks were awarded. Solutions involved the (incorrect) use of $s = (u + v)t/2$, with u and v being selected for the initial and final speeds in a variety of ways. Some candidates added the two distances which had been found correctly.

2) (i) Though some scripts had cumbersome solutions, often working via $u = 3$ m s^{-1}, a large majority of candidates found the correct value.

(ii) Most candidates appreciated the need to find the mass of the object, and found the frictional force. Some solutions ended there. Others used a negative force in conjunction with the 180 N, disregarding the more appropriate use of the magnitude of the frictional force. Candidates who worked correctly and gave their final answer as 0.013, mistakenly thinking this to be three significant figures, were not penalised.

3) (i) The first part of this question showed a predominance of solutions which found velocity by integration of deceleration. The printed answer was then conjured up by a variety of incorrect methods. However, the printed answer was used in subsequent parts of this question, and candidates could obtain full credit for those.

(ii) Candidates obtained the correct solution in a variety of ways. In some cases no choice of the more appropriate root was made.

(iii) Fully correct answers were obtained more frequently in (iii) than (i).

4) (i) Correct values were usually given, though some scripts suggested an unwillingness to give a negative answer for the value of a component in a specific direction.

(ii) Candidates encountered few problems in calculating the magnitude of the resultant. More problematic was finding the direction of the resultant, with a significant proportion of candidates not targeting the obtuse angle required.

(iii) The majority of candidates gained neither mark. Often no attempt was made. Wrong methods suggested that the diagram printed in the question paper was regarded as showing forces in a vertical plane. Candidates gaining a single mark usually gave 43 N as their correct answer; a minimum value of 3 N was common.

5) (i) Most candidates gained full marks. The commonest source of error lay in the calculation of the distance the athlete travelled while decelerating. The setting up of the equation in T was in general done well.

(ii) Nearly all candidates obtained full marks.

(iii) The majority of candidates scored all three marks, with candidates alert to the need to equate their algebraic expression for acceleration to –1.75.

(iv) Though most candidates followed the route indicated in the question ("Verify..."), some tried to solve the equation for the time taken by the robot to reach the 100 m line. Lacking the knowledge to solve this, they usually lost one, or both, marks.

6) (i) Most candidates found correctly the magnitude of the frictional force, and went on to obtain all six marks. The most common error was the omission of the component of weight when deriving the Newton's Second Law equation for the motion of the particle. It was also apparent that some candidates were reluctant to set up the equation with only negative forces included.

(ii) Almost half of the scripts showed the same acceleration being used for the motion up and down the slope. This was regarded as a fundamental error, and heavily penalised.

(iii) Correct answers were rare. Not only had candidates to avoid using the same acceleration throughout, but the vector nature of momentum was often overlooked.

7) (ia) Often this was answered correctly; erroneous methods usually entailed the inclusion of an irrelevant mass or weight. Mis-reading the tension as 2.25 N was also seen.

(ib) The motion of the particle Q is determined by three forces. The equation for its motion frequently omitted one of these.

(ii) Approximately one-third of candidates gave a fully correct solution, with a similar proportion gaining a majority of marks by correctly using their wrong values calculated earlier.

(iii) Candidates were often able to gain some credit for finding the distance travelled by R and its speed at the instant P strikes the ground. Valid calculation of the subsequent acceleration of R was seen, but many candidates assumed that it would be $-g$.

4729 Mechanics 2

General Comments

A large number of well-prepared, high-scoring candidates were entered for this paper. Only a small minority of candidates were unprepared for the demands of the paper. Candidates who used clear, well-drawn force diagrams frequently demonstrated a good level of understanding.

Comments on Individual Questions

1) This question proved accessible to the majority of candidates. Full marks were commonly seen by examiners. The usual cause of a lost mark was failure to describe the direction fully, but there has been an improvement in this recently.

2) (i) The inclusion of the formula for the volume of a sphere was intended to assist candidates, but a significant number used this rather than the volume of a hemisphere in their solutions. Examiners often saw the centres of mass of the two objects being taken as on the same side of their common face. Those who took moments about an axis other than through the common face often failed to realise an adjustment was required at the end.

(ii) A few good attempts were seen to this question. Often the relevance of the previous part was overlooked. The important idea that the weight of the object would create a clockwise moment about the point of contact was required, which implied that the centre of gravity was within the hemisphere.

3) (i) The majority of candidates knew that they needed to take moments about *A*. The common mistakes seen were using a component of the 1.6 m with the normal reaction at *P*, or mixing cos and sin in the moment of the weight.

(ii) Those who resolved in two directions usually scored full marks. Those who tried to take moments frequently omitted one or more of the forces. There was also confusion in a significant number of cases about the directions of the forces acting at *A*. Some candidates believed that the normal reaction is equal to weight.

4) (i)(a) This part was generally well done. Errors seen were in the calculation of the radius of the motion and in not resolving the tension when considering the horizontal motion.

(i)(b) Many correct solutions were seen to this part.

(ii) Candidates were expected to infer that the normal reaction was zero and so the value of the tension would alter compared to the value found earlier. Some persevered with their earlier tension. The request was for a speed, but a significant number gave angular speed as a final answer.

5) (i) This question was often fully correct. Occasionally the component of weight acting down the plane was omitted.

(ii) The intended approach was for the use of energy. Those who used this approach often omitted the work done by the engine. Finding the change in PE and then also including the weight component when calculating the work done against resistance was seen a number of times. Some candidates tried to use constant acceleration.

6) (i) Most candidates were able to find the speed after the bounce successfully. However a significant number made an error with the required impulse, the most common seen was with the lack of appreciation of the vector nature of momentum resulting in a wrong sign for momentum after the impact with the ground.

(ii) Many candidates found the given answer correctly.

(iii) Some candidates failed to see the connection to the previous part and set about calculations to answer the request. Others assumed a wrong relationship, usually giving 0.8e and 0.4e as answers.

(iv) This question was found to be the most difficult on the paper. Many candidates did not see the total time as the sum of an infinite geometric progression. Those who did consider a geometric progression should have used the formula for the sum to infinity, contained in the formula book for the specification. Some made errors with the first term. Only a few candidates found the time to the first bounce.

7) (i) Generally well answered with the majority finding the required angle correctly. Not all remembered to find the distance between the points of impact, a few found the 2 relevant distances but failed to subtract.

(ii) This question was in general poorly done. Few candidates considered the horizontal motion to find q, and some thought the angle was the same as in part (i). A common error was to equate y_p and y_q rather than put their sum equal to 60 and even when the correct approach was used there were often sign errors. Some candidates thought the direction of motion of Q could be determined from its position at the time of collision.

4730 Mechanics 3

General Comments

A wide range of marks were recorded in this paper which discriminated well between candidates. Many candidates presented their work in a neat and orderly fashion, which made marking much more straightforward. However, a considerable number of scripts were written untidily, and it was sometimes very hard to follow the thread of the mathematical argument being presented on these scripts.

There is always a variety of methods used on some of the questions on this unit. Many candidates used extremely efficient methods to solve the questions, while others used inefficient methods, or just wrote out the same stage several times. There is nothing wrong with using an additional sheet when an answer has gone wrong, and a candidate wishes to have another attempt, but normally a candidate's answer to a question should not need to go onto any additional paper.

Comments on Individual Questions

1) (i) There were very many efficient and correct solutions to this part. There were also quite a lot of answers where candidates went a long way round, for example finding an angle, though these too generally ended up with full marks. A minority of candidates did not understand the vector nature of momentum and impulse, and suggested that $I = 2.5m – 2m$; they received no marks for this.

(ii) Again, there were some very efficient answers to this part, where candidates realised that the component of velocity parallel to the plane was 2 ms^{-1} both before and after the collision with the wall, and quickly worked out the component of the velocity perpendicular to the wall before and after the impact, often by inspection. This gave the coefficient of restitution very easily. Quite a number of candidates were successful after having been the long way round, finding the angle between the path of *P* and the wall both before and after the impact, though some made errors along the way. Those candidates who thought the coefficient of restitution was to be found by dividing $\sqrt{5}$ by 2.5 scored no marks.

2) (i) Very nearly all candidates realised that they needed to use Newton's experimental law and the law of conservation of momentum – though a small minority wrongly used energy. A significant number of candidates lost marks through not having the velocities after the collision consistent in their two equations. Some candidates stopped after finding the velocity of *B* along the line of centres after the collision; they needed to either point out that both components of the velocity had the same magnitude as before the collision, or else use Pythagoras' rule, to show the speed of *B* was still u m s^{-1}.

(ii) Many candidates failed to give full answers to this part, often not completely defining the direction of *A* or *B* or both. For example, saying that *A* moved at right angles to the line of centres is not sufficient. Many candidates sensibly and easily got over this problem with a simple diagram.

3) (i) This question was generally well done, though some candidates failed to include the minus sign at the beginning, and others made slips after the integration in the algebra needed to establish the given answer.

(ii) It was expected that candidates would begin this part with the equation given at the end of part (i), write v as $\mathrm{d}x/\mathrm{d}t$, separate variables and integrate. Those who did this found it very easy to score full marks. However, quite a lot of candidates started the question again, with $–1.2v^3 = 0.3\ \mathrm{d}v/\mathrm{d}t$, and integrated this equation to find an expression involving t and v. They then had some tricky algebra to do as well as the substitutions for x and t to arrive at the correct answer. Although a significant number using this method were successful, many made errors or gave up along the way.

4) (i) This part was completed successfully by many candidates. Some candidates did a different question and found the extension of the string when P was hanging in equilibrium. A small number of candidates, although doing the question set, split the motion up unnecessarily and found the velocity of P at either a point 0.75 m below O, or at the point where P would hang in equilibrium, or both. Sometimes they were successful. A further group of candidates did not work, as had been expected, with the extension of the string, but with the total length, using '$x + 0.75$' rather than 'x'. Those who realised what they were doing were usually successful.

(ii) The majority of candidates did this correctly, though many candidates failed to make clear the direction of the acceleration of P.

5) (i) This part was generally done successfully, with almost all candidates realising that the most efficient method was to take moments about B for the equilibrium of BC.

(ii) Where candidates are asked to show a given answer they are expected to spell out the method and the calculation that is needed to do this. A small number of candidates failed to do this, and lost marks. Other candidates failed to make clear the direction in which these components acted. However, almost all candidates realised that they needed to resolve horizontally and vertically for the equilibrium of BC.

(iii) Most candidates took moments about A for the equilibrium of AB; those taking moments for the whole body about A generally got into difficulties with the trigonometry involved. Many candidates made errors in using sin or cos, or became confused whether lengths should be L or $2L$. Even so, a fair number of candidates gave fully correct solutions.

6) (i) This proved a very straightforward part for most candidates, though some became confused about the period for the motion, often doubling the value of 0.4π and then arriving at a distance of 1.6 m for OA.

(ii) A few candidates wrongly used $x = a\sin nt$, and a few other candidates must have had their calculators in degree mode when entering the value of 5 radians. Quite a number of candidates then used the formula $v^2 = n^2(a^2 - x^2)$; this only really provides the speed and not the velocity, so these candidates did not score full marks unless they also specified correctly the direction of the motion of P at this time.

(iii) Despite the fact that this question caused difficulty for many candidates there were quite a number of well presented, correct and logical answers. There were also a lot of answers that gained a mark or two for candidates who were only able to find some information about one or two of the required occasions. A small number of candidates found only the value of t or else only the value of x for one or more of these occasions.

7) (i) Although a considerable number of candidates used the energy in the string in error, many candidates established the given equation successfully. Some candidates stopped at this point; others who had not established the equation gained marks by working on the given equation in α. Many candidates showed that $\alpha = 1.18$ by considering the value of $1.8\alpha - \sin\alpha - 1.2$ at $\alpha = 1.175$ and $\alpha = 1.185$, which was the method expected. Rather more candidates used the iteration $\alpha = (1.2 + \sin\alpha)/1.8$ to show the result. A minority tried a different iteration, giving α in terms of $\sin^{-1}$; these candidates were not successful.

(ii) Only a minority of candidates scored this mark, though toward O was probably the most common answer.

(iii) This proved a challenging end to the paper. Even so, many candidates realised that a method using energy was the best way of tackling the question. Candidates who set out their work clearly and logically have an advantage in this sort of challenging question, since they are less likely to get muddled or to miscopy things, or to make slips in algebra. A small number of candidates started an alternative valid method by using $F = ma$ tangentially, but few got as far as introducing $d^2\theta/dt^2$, and none tried to integrate the equation they arrived at.

Overview – Probability and Statistics

In order to learn what answers are acceptable to examiners, candidates should refer to the mark scheme.

Many candidates would benefit from careful teaching of how to use the formulae in MF1.

In answering questions that require written comments, candidates are advised against simply reproducing conditions given in textbooks; they should always decide which criteria are relevant in any given context. Particular attention is drawn to the comments concerning this in S2.

Most candidates now set out solutions to hypothesis tests correctly, with appropriate statements of the hypotheses and conclusions (in context and with acknowledgement of the uncertainty involved).

4732 Probability & Statistics 1

General Comments

There were some very good scripts, and a few candidates gained full marks. Many candidates showed a good understanding of a good proportion of the mathematics in this paper. There were several questions that required an interpretation to be given in words, and these were sometimes answered poorly.

A significant number of candidates lost marks by premature rounding (especially in 5(iii)(a) and 8(ii)(b)) or by giving answers to fewer than three significant figures without having previously given a longer version of their answer. It is important to note that although an intermediate answer may be rounded to three significant figures, this rounded version should not be used in subsequent working. The safest approach is to use exact figures (in fraction form) or the intermediate answer correct to several more significant figures.

Few candidates appeared to run out of time.

Use of statistical formulae and tables

The formula booklet, MF1, was useful in questions 2(ii), 4(a), 4(b) and 8(i) (for binomial tables), In question 2(ii) a few candidates quoted their own (usually incorrect) formulae for r, rather than using one from MF1. In question 2(iii)(a) almost all candidates (wisely) used the formula $b = \frac{S_{xy}}{S_{xx}}$ rather than the alternative version given in MF1. Some thought that, eg, $S_{xy} = \Sigma xy$. This year, almost all candidates used the more convenient version of the formula for r from MF1 (avoiding the less convenient version, $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\left\{\Sigma(x-\bar{x})^2\right\}\left\{\Sigma(y-\bar{y})^2\right\}}}$) with the result that the vast majority found the answer correctly.

In question 4, Σd^2 was occasionally misinterpreted as $(\Sigma d)^2$ and the formula was sometimes miscopied as $\frac{6 \times \sum d^2}{4(4^2-1)}$ or $\frac{1-6 \times \sum d^2}{4(4^2-1)}$.

Responses to question 8(i) gave evidence that many students prefer to use the binomial formula rather than the tables. This caused no problem in this case, but centres should be aware that questions are sometimes asked in which the use of the formula is laborious whereas the use of the tables is quick.

In question 5(iii)(a) candidates needed to use formulae for the mean and standard deviation of a frequency distribution that are not given in MF1. Some had clearly learnt the formulae by rote but did not understand them. For example class widths were often used instead of mid-points. Some misquoted the formulae.

Candidates would benefit from direct teaching on the proper use of MF1, particularly in view of the fact that text books give statistical formulae in a huge variety of versions. Much confusion could be avoided if candidates were taught to use exclusively the versions given in MF1 (with care in the case of b, the regression coefficient). They need to understand which formulae are the simplest to use, where they can be found in MF1 and also how to use them.

Comments on Individual Questions

1) (i) Almost all candidates answered this part correctly. A few used $\Sigma xp = 1$.

(ii) Almost all candidates answered this part correctly.

2) (i) Many correct answers were seen. The most common incorrect answer was: "x because sand content depends on depth". Another was: "y because x is controlled".

(ii) This question was well answered.

(iiia) This question was also well answered. A few candidates made a sign error in calculating a. A few used their answer for r from part (ii) for the value of b. Others used $\frac{S_{xy}}{S_{yy}}$ instead of $\frac{S_{xy}}{S_{xx}}$ for b, even though they knew that they were finding the regression line of y on x rather than vice versa. A few candidates calculated b and a correctly, but then substituted them into the wrong equation, eg $y = b + ax$.

(iiib) The majority of candidates stated that the first estimate was reliable because it was interpolated but the second was not, because extrapolated. But most omitted to state that the essential point that the value of r is close to –1 and shows strong correlation. A few thought that this value of r shows poor correlation. Some candidates referred to the small size of the sample, which is not relevant.
($|r| = 0.926$ is very much larger than the critical value for even a 0.5% significance test with $n = 9$, although candidates are NOT expected to know anything about this).

3) (i) Many candidates found $P(X = 2)$ only, or $P(X = 1)$ only.

(ii) Many candidates just found $P(X = 2)$ without considering the fact that two values of X are involved. Some found $P(X_1 = 2) \times P(X_2 \neq 2)$ but omitted to multiply by 2. A few found $2 \times 0.12 \times 0.88$.

(a) Most candidates answered this correctly. A few omitted "1 –" from the formula. In ranking the data, some candidates used A = 1, B = 2, C = 3 etc. Some candidates did not show the ranks, but proceeded directly to the values of d. This is a risky procedure.

(b) Most candidates stated that Σd^2 would be unchanged. Some stated that n increased, but they ignored $(n^2 - 1)$. A significant number stated that since the fraction $\frac{6 \times \Sigma d^2}{n(n^2-1)}$ decreased in size, the value of r_s also decreased. A few gave the vague response that since the agreement between the judges was now greater than before, r_s would increase. These scored 1 mark.

5) (ia) Most candidates answered this part correctly, with just a few thinking that the class width is 2, rather than 3.

(ib) Many candidates found the frequency density correctly, but failed to note that one unit of frequency density is represented by 2 cm on the vertical axis. A few who did take note of the scale used a class width of 5 instead of 6.

(ii) Even though the diagram gives a clear hint, many candidates missed the point that the variable is discrete. Those who appreciated that class boundaries were not integers generally gave correct answers, Either (0.5, 0) and (3.5, 6) or
(3.5, 6) and (6.5, 15) were accepted as correct. Many candidates gave the midpoints of the first two classes. Many others gave (0, 0) as one of their responses.

(iiia) The responses to this simple question were often poor. Many found at least one mid-point incorrectly. The usual errors were frequently seen, such as division by 5 instead of 21 or division by 5 <u>after</u> division by 21. Some candidates used class widths for x. Some failed to divide $\Sigma x^2 f$ by 21. Some divided by, eg 25. Some omitted the frequencies in one or both calculations. Some were unmoved by obtaining a negative result after subtracting their mean squared. Some omitted to square the mean before subtracting it. A few candidates unwisely attempted to use $\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}$, which involves unnecessarily long calculation. Some of these succeeded, but others could not handle the arithmetic, or omitted the frequencies. Many used their 3-significant-figure answer for the mean in calculating the standard deviation. These achieved an answer of 3.07 instead of 3.08 and so lost the final accuracy mark. Some candidates omitted to take the square root. A few candidates used the statistical functions on their calculators without showing any working. These gained full marks if their answers were correct, but if not, no marks could be awarded at all. Candidates using this method would be advised to perform the calculations twice as a check and also to write down at least the values of the mid-points, of Σxf and of $\Sigma x^2 f$.

(iiib) Most candidates understood the point, although many found it difficult to explain their understanding clearly.

6) (i) Unfortunately the first diagram was incorrect. Each bar should be one unit to the right. Despite this, many candidates chose this diagram because the probabilities are all decreasing. Because of the incorrect diagram for V, candidates who stated "Z because $P(Z = 0) = 0$" were allowed full credit. A common wrong answer was "W, because in the geometric distribution all the probabilities are equal".

(ii) Many candidates identified the correct diagram and gave full explanations. Some gave partial explanations, for example mentioning symmetry but without excluding W. Others gave answers such as "W, because $p = q$ so all the probabilities are equal".

7) (i) Almost all gave Geo(0.6), although a few gave B(n, 0.6). But many candidates failed to give one or both conditions in context (eg "The probability of success is constant" is insufficient) and so failed to gain one or both of the last two marks. For one condition a few candidates stated that "The probability of a voter being a woman is independent". Although in context, this is incorrect. Many wrote about repeated trials and/or two possible outcomes, all of which is irrelevant.

(ii) Most candidates answered this part correctly. A few reversed p and q.

(iii) Common errors were: 0.4^4, $1 - 0.4^3$, $0.4^3 \times 0.6$ and $1 - P(X = 1, 2, 3 \text{ or } 4)$

8) (i) Many candidates found $1 - P(X \leq 7)$ or $1 - P(X \leq 5)$, using the table . Some used the formula to find either $P(X = 7)$ or $P(X = 8)$ (but not both) or $P(X \geq 6)$. A few used the formula to find $1 - P(X \leq 6)$, a correct but very long method. Others found $0.5^8 + 0.5^8$, omitting the coefficient in one term.

8) (iia) Many candidates correctly found $P(X = 11)$, but some either doubled or squared this, presumably thinking that they needed to find the probability of 11 blue flowers AND 11 red flowers. Some subtracted $P(X = 11)$ from 1.

(iib) A few candidates appreciated that the answer to part (ii)(a) could be used to give a short method. Some of these just found $1 - 0.168$, without halving the result. But many others tried to find $P(X = 12, 13, 14, ..., 22)$ or $P(X = 0, 1, 2, \ldots, 10)$, often omitting one term or adding an extra term. Candidates should note that if they find themselves in the examination with a binomial calculation involving a large number of terms, this is almost certainly either incorrect or very inefficient. In the case of this question, there is a much more direct method. In other cases, it may be that the binomial tables will give a short method.

9) (ia) Many candidates found ${}^9C_4$ instead of ${}^9P_4$. A few found $\frac{9!}{4!}$. Some added several permutations or combinations.

(ib) Only a few used the elegant method of multiplying their answer to part (i)(a) by $\frac{5}{9}$. Some candidates found ${}^8C_3 \times 5$. Assorted strange methods such as ${}^9C_5 \times {}^5C_4$ or $1 + {}^8C_4$ were common. $5! \times 4!$ was often seen.

(iia) Candidates generally used either combinations or permutations or fractions. Some used a mixture of these, generally unsuccessfully. Partial methods were common, such as: $\frac{{}^5C_4 \times 4}{{}^9C_4}$, $\frac{{}^5C_4}{{}^9C_4}$, $\frac{5!}{{}^9P_4}$, $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{4}{6}$, $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{4}{6}\times 4$, $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}$ and $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{4}{6}$ + $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times\frac{2}{6}$.

(iib) Most candidates identified the two possible sets of digits and then used either combinations, permutations or fractions. The number "2" was often seen, but was generally divided by an incorrect denominator such as ${}^9P_4$ or multiplied by $\frac{1}{9}\times\frac{1}{8}\times\frac{1}{7}\times\frac{1}{6}$ without the necessary $\times 4!$.

Another common partial method was $\frac{4}{{}^9P_4}$ without the necessary $\times 2$.

4733 Probability & Statistics 2

General Comments

The general quality of scripts was once again high. It was pleasing to see so many correct answers, and many candidates were able to express themselves well where written explanations were required. Nevertheless, questions requiring written explanations also often expose limitations of understanding, especially on the part of candidates who seem to learn likely responses by rote. This is not a good way of preparing for A-level examinations.

Most candidates seemed to have taken note of the expectation that hypothesis tests should always include statements of the hypotheses, and also that conclusions should be stated both in context and with appropriate acknowledgement of the uncertainty involved. The wordings “There is significant evidence that…” or “There is insufficient evidence that…” are recommended.
Many candidates lose marks by answering the question they have seen before, instead of the question in front of them.
The issue of modelling assumptions for the Poisson distribution continues to be a general weakness. Once again it is emphasised that the statement “events must occur singly” is inadequate. Centres are strongly advised to focus neither on “singly” nor “randomly” but only on “independence” and “constant average rate”.

Comments on Individual Questions

1) A straightforward start for almost all.

2) The correct numerical answer was usually seen. Not all candidates gave both conditions for a Poisson approximation to be valid. If numerical conditions are given, they must be those stated in the Specification ($n > 50$, $np < 5$).

3) This was generally well done. Many found efficient methods, though as indicated in many previous Reports it is surprising that not all candidates are trained to use elimination to solve simultaneous equations. In this question of course it was not necessary to do so as $\mu = 60$ can be written down by symmetry. A few candidates stopped after finding μ and σ and thereby lost easy marks.

4) Many candidates found the right normal distribution, though there were problems with variance/standard deviation confusion. Many omitted the continuity correction; very few used the easier method of considering the *sum* of 50 values rather than the mean.

5) (i)(a) This synoptic question worried many candidates. This part uses a simple binomial distribution.

(i)(b) The least well done question on the whole paper. Some tried to use a binomial distribution. Many had forgotten the method of using ${}^nC_r$ (which is best) and tried to multiply probabilities, but omitted the factor of 15 or failed to change probabilities correctly, or both.

(ii) In this question there was much confusion between sample and population. A surprising number of candidates started off with the distribution B(1200, 0.03). Most of the others answered the calculation correctly, with a pleasing number of correct continuity corrections. But few realised, in the verbal rider at the end, that with a large population it made almost no difference whether sampling was carried out with or without replacement; this is an important point which this question was intended to bring out. Some candidates argued in terms of a large sample, which is completely wrong.

6) This question was well done, with many fully correct answers. As usual those who found $P(< 2)$ or $P(= 2)$ instead of $P(\leq 2)$ lost several marks; Centres are advised to note this.

7) (i)(a) Most answered this well, apart from those whose integration skills were inadequate.

(i)(b) Many found this straightforward. But a substantial number of candidates attempted to use the mid-interval value of 2.5, or other wrong methods, and some went onto autopilot and found the variance instead of the median. To score any marks it was essential here to include the correct limits in the integral.

(ii) Many made a good job of this, though some failed to appreciate that evaluating $\sqrt{y}$ at the upper (infinite) limit of the integral gave an infinite answer; perhaps mimicking questions on past papers, they thought that this gave 0.

8) (i) The simple and correct statement is that the location of bacteria must be independent, which means that the position of one does not affect any others.
Many candidates tried to answer this by choosing one of several possibilities that they seemed to have learnt parrot-fashion. Both "randomly" and "constant average rate" were given in the question, so only "independence" was left (as explained in the General Comments, the "singly condition" should not be considered). It is not sufficient to say that "independence" means that the bacteria must occur singly as it must also rule out the presence of one bacterium *increasing* the probability of another occurring.
Many candidates gave answers appropriate to questions that they had seen before, rather than to this one. This is not a question about "choosing" bacteria, nor is it about the number of bacteria in one part of the fluid affecting the number in a different part of the fluid.

(ii) Generally very well done.

(iii) Also generally very well done. Those whose calculators can give this answer directly are still well advised to give the formula, as a wrong answer (even if it is an error only in the third significant figure) can get no marks if insufficient working is seen.

(iv) This question was very well answered. The correct continuity correction was pleasingly common, as was the condition that $\lambda > 15$.

9) (i) The important aspect of this question is that μ is the *population* mean. Over the years many candidates have shown their inability to distinguish between sample and population means when doing hypothesis tests, and here it was necessary to include the word "population".

(ii) Most knew roughly what to do, though there were frequent instances of $\sqrt{0.87}$ instead of 0.87, or failure to double the answer so as to take account of this being a 2-tailed test.

(iii) This simple request was surprisingly poorly done. Some gave an explanation of the wrong type of error, or of a mere "mistake". Some failed to give an answer in context – some mention of 8 hours of sleep, or equivalent, was needed. And many confused what a Type I error is with the probability of making one.

(iv) A few candidates attempted to standardise 8 rather than 7.72 and 8.28. Some found the probability of obtaining a result in the critical region, and thus scored no marks. Some made numerical errors, omitting the $\sqrt{64}$ or, as before, confusing 0.87 with $\sqrt{0.87}$. An unexpectedly large number considered only $P(> 7.72)$; the upper tail turns out to have negligible area but this cannot be assumed without calculation. However, pleasingly many correct answers were seen.

4734 Probability & Statistics 3

General Comments

The paper was of a similar standard to January 2011 and several candidates scored very high marks. Several candidates lost up to three marks for making their conclusions to significance tests too assertive. Otherwise, the five significance tests were answered well, although some of the hypotheses were not stated in terms of the population parameters by some candidates.
Q5 proved to be the easiest question on the paper. No question was too difficult for most candidates, but Q3(ii) and Q6(iii) stretched almost all the candidates.

Comments on Individual Questions

1) (i) Most answered this question correctly, but some stated that the null hypothesis was that the factors were not independent.

(ii) Most scored one mark for stating 'decreased' together with '–0.5'. Few scored the other mark for the correct formula including Yates' correction.

(iii) Most earned the first two marks, and the third if (i) had been answered correctly.

2) (i) Almost all answered this question correctly.

(ii) Some candidates lost a mark for not defining the population proportions clearly.
Most used a pooled sample, but some did not, despite having been pointed in that direction by (i).

3) (i) Most answered this question correctly, but some used a t-value instead of z.

(ii) Some used an incorrect variance in this part, but most knew what to do.

4) (i) Most answered this correctly, but there were some who used $\text{Var}(Y) = a^2\text{Var}(X) + b$.

(ii) Almost all stated 'Normal', but some did not state the parameters.

(iii) As in Q3(ii), some used an incorrect variance, but otherwise knew what was needed.

5) (i) Most answered this question correctly, but some used numerical limits such as 0,1 and showed that both the integral between these limits and the given expression produced the same answer. This was not acceptable.

(ii) Almost all produced the correct expected frequencies and went on to carry out the test correctly. A few used an incorrect critical value.

6) (i) Most answered this question correctly, but a few forgot to calculate (upper quartile – lower quartile).

(ii) Many lost a mark for not stating E(Y) = 0. Otherwise this question was answered well by most candidates.

(iii) Some candidates answered this question correctly, but many did not know how to deal with the modulus.

7) (i) Many used z instead of t, but otherwise this question was well answered.

(ii) Mainly correct, but there were some confused answers.

(iii)(a) Most knew the assumption about equal variances. Some did not calculate the variance from the second sample. Almost all who did went on to say that the variances were close enough for the test to be carried out.

(iii)(b) Many answered this question well, but some did not pool the samples and there were also many who used z instead of t.

Overview – Decision Mathematics

Much good quality work was seen in this series and most candidates were able to complete the papers in the time available.

A few candidates found it straight forward to construct coherent answers to questions requiring reasoning or an explanation. However, when asked for an explanation, candidates frequently repeated the information given in the question. Many described a specific example when a general argument was needed, and, when a specific instance was required, some failed to give detailed features. Candidates should be aware of when a general argument is needed, and when a specific counter-example is required.

The handwriting of some candidates was very difficult to read, particularly the writing of numbers. Candidates should be aware that work which has been erased can sometimes still be seen after scanning. This can make marking very difficult, so candidates should either make it very obvious when work has been deleted or they should make a new start, particularly when correcting a diagram.

4736 Decision Mathematics 1

General Comments

This was a straightforward paper and most candidates were able to complete it in the time allowed. Some lost marks by not answering the questions exactly as they were asked. The handwriting of some candidates is difficult to read, particularly when they are writing numbers, some of these candidates could usefully have had their scripts transcribed to make them easier to interpret. The answer booklet had appropriate space for each answer and most candidates used the space well. A few candidates used additional sheets, but when they did so they usually indicated this in the appropriate space in their answer booklet.

Comments on Individual Questions

1) Answered well by most candidates. Most used shuttle sort correctly, with only an occasional candidate sorting into increasing order or using bubble sort. Candidates only needed to show the result at the end of each pass, some chose to write out all their working, often necessitating an additional sheet, and these did not always identify the ends of the passes clearly. A significant number of these candidates did not record passes in which no swaps had taken place. The question specifically asked candidates to write down the number of passes, comparisons and swaps used, several did not state that they had used 8 passes, even though it was apparent from their working that they had.

2) (i) Many correct minimum graphs, showing a tree with 5 arcs. A significant minority drew a minimum cycle with 6 arcs. Some candidates had erased earlier attempts which still showed through on the scanned script, often making it difficult to see which arcs were being offered as the answer.

(ii) Most candidates attempted the complete graph for the maximum case, although some omitted an arc somewhere. A few miscounted their 15 arcs, but many correct answers were seen.

(iii) Some candidates thought that this was asking the same question as part (ii) and others carefully explained why the maximum graph in part (ii) was not Eulerian. Many candidates stated that the vertices must have even order with a maximum of 4, some then insisted on having two vertices of order 2 for some reason that was not obvious, and others seemed to then be working with the complete graph on four vertices, in both cases resulting in a total of 10 arcs instead of the required maximum of 12.

(iv) Many candidates just wrote that the graph is semi-Eulerian because it has two odd nodes, rather than being precise and saying that it has exactly two odd nodes, or specifying that there are two odd nodes and all the others are even.

3) (i) Nearly all the candidates could use Dijkstra’s algorithm correctly. A few recorded extra temporary labels. Very few candidates needed the additional copy of the diagram, those who did usually said so in their main answer, which was helpful.

(ii) Some omitted to say that the total weight of the arcs shown is 38, and some just wrote down a route and added up its weight. The question asked candidates to apply the route inspection algorithm and to show their working.

(iii) Many candidates found the weight of an appropriate route, but in many cases it was not apparent that they had chosen the arcs to be repeated for the correct reason. Nodes *C* and *D* needed to become even and nodes *A* and *F* needed to become odd, this required considering the three possible pairings between these four arcs before selecting the shortest pairing. It was possible to exclude the longer pairings by reasoning rather than calculation, but this was rarely done even when candidates had written down the six individual weights

4) (i) Many candidates just said ‘red bags’ or even ‘red’ without specifying that *x* was the number of red bags. The identification of the variable for a discrete variable should indicate both that something is being counted and what it is that is, for a continuous variable candidates would be expected to identify what is being measured and give appropriate units.

(ii) Most candidates recognised that the given constraint came from the number of sweets available. Nearly all the candidates were able to write down similar constraints for the balloons and toys.

(iii) The question asked for other constraints and restrictions. Most candidates realised that the other constraints were the non-negativity constraints, but several omitted the restriction to integer values.

(iv) Many candidates realised that every bag made would need to be sold for the objective to be valid.

(v) The initial Simplex tableau was usually correct.

(vi) A few candidates showed working for their pivot choice in part (v), but most then repeated it here as well. Some candidates omitted the calculations to explain the choice of pivot row and some gave written descriptions for this part. All that was needed was to show the ratios 80/3, 40/5 and 30/5 and then to select the 30/5 as the least non-negative ratio and the 5 in the ‘balloons’ row of the *x* column as the pivot. The pivot operations were generally carried out correctly but several candidates did not show how the pivot row was calculated and some showed their calculations for the other rows in forms that were either inconsistent (using, say, r4 to mean both the original and the new row 4), were incomplete (for example, saying +r4 rather than r1 + r4) or, if not written alongside the rows, did not specify which row the calculation was to be used for.

(vii) Some candidates had achieved a correct tableau but were not able to read off the resulting values of *x*, *y* and *z*. A few candidates wrote down the values of *x*, *y* and *z* but did not then interpret them in the context of the problem and rather more gave the interpretation but did not write down the values of *x*, *y* and *z*, although they had been specifically asked to do this in the question.

(viii) A number of candidates tried to adjust their answer from part (vii), some did an additional iteration, which gave the right answer but was not necessary, and several candidates left this part blank. The tightest constraint comes from the balloons, there are at least 4 balloons in each party bag and there are only 40 balloons in total, so the only way to make 10 bags is for them all to be yellow.

5) (i) Almost all candidates were able to interpret the table correctly.

(ii) Most candidates listed the weights in increasing order but some did not show their working for Kruskal’s algorithm on this list, and some left out the printed arc *BC* = 103 from their working. A few forgot to write down the weight of the minimum spanning tree.

(iii) Most candidates were able to add weights of the two shortest arcs (*FB* = 50 and *FD* = 59) to the weight of their tree.

(iv) Most candidates were able to apply the nearest neighbour method correctly, a few omitted vertex *F* and some tried to include *D* too early. Some candidates forgot to close their route to form a cycle.

(v) Some candidates were put off by the amount of description for this part, but those who attempted it often achieved the first four marks for finding the paths and their weights, *FAD* = 435 and *BEC* = 278. The candidates then needed to join these, using two joining arcs, to form a cycle. Several candidates just used one joining arc and formed a string, and some just wrote down the first cycle they found, rather than trying out the possible cases (of which there were only two). To gain the final two marks, candidates needed to write out their final cycle as well as give its weight. There were a number of candidates who achieved full marks on this part.

6) (i) Usually done well, apart from the odd arithmetic slip or going beyond the stopping condition.

(ii) Several arithmetic errors, particularly with calculating $-13 - -20$ as -33 instead of 7, but even these candidates were able to show that the algorithm got stuck in a repeating cycle.

4737 Decision Mathematics 2

General Comments

Most candidates were able to complete the paper in the time allowed. The handwriting of some candidates is very difficult to read, some of these candidates could usefully have had their scripts transcribed to make them easier to interpret. Candidates need to check that they have answered the question that was asked and not some variation on it.

Comments on Individual Questions

1) (i) A more difficult bipartite graph than usual because of the use of a third variable (type of film). Most candidates were able to draw a correct graph, some omitted the arc *KV* but this did not affect the remainder of the question.

(ii) Most candidates were able to write down the alternating path *N-S-J-V* and use it to write down the incomplete matching with *M* left out. Some candidates gave a longer alternating path or went on to find the complete matching in this part.
Candidates who drew a diagram for the incomplete matching needed to also write it down, as asked in the question.

(iii) Some candidates appeared to have started again in this part, but most were able to write down the alternating path *W-K-U-M* (rather than some longer version) and write down the complete matching.

(iv) Most candidates who considered all five people were able to construct an appropriate piece of reasoning. Usually candidates paired off *NS* and *JW*, for which there was only one possibility, and then explained how this forced the pairings *JV* and *MU* and hence *LT*.

2) (i) Most candidates were able to draw appropriate networks using activity on arc.
The arcs should be labelled (*A*, *B*, *C*, etc.) and should be directed. There only needed to be one dummy activity on the network.

(ii) Candidates could usually carry out the forward and backward passes on this fairly simple network. Some candidates were unsure how to deal with the dummy activity, particularly on the backward pass, where the dummy came into play. The early event time at the end of the dummy was 210 (being the larger of 120 + 90 and 180 + 0) and the late event time at the start of the dummy was 210 (being the smaller of 340 – 30 and 210 – 0). Some candidates completed the passes but did not write down the minimum completion time (360 minutes) and the critical activities.

(iii) Most candidates were able to draw an appropriate resource histogram; some only used one worker for activity *I* in the last 20 minutes.

(iv) Activity *G* could be moved to start at any time up to 270 minutes, but the extra workers would be needed as soon as *C* was started and the latest that this could be was at time 150 minutes. *G* could not happen at the same time as *F* because this would have meant that there were four workers needed in total, but it could happen alongside *C* and *D* or alongside *E*.

3) (i) Many candidates gained full marks on this part. Only a tiny minority forgot to add a dummy column (to make a square matrix) or to convert to a minimisation problem (by subtracting all values from a sufficiently large constant, usually either 100 or 80, chosen so

that the resulting matrix had no negative entries).
For some candidates either the row reductions or the column reductions happened by default, when this happens it is useful if the candidates can note it, although there was no loss of marks for not doing so in this case. Some candidates did not augment, and tried to find a matching from the matrix they had obtained after only doing the row and column reductions. Some candidates reverted to pairing the biggest values instead of the entries with reduced cost zero. Most candidates were able to find both complete matchings and write them out in an appropriate abbreviated form.

(ii) Some candidates argued that Agatha's values were back to front at this point, rather than accepting that the butler's scores had changed because of the additional evidence. The cook should now be thought to be innocent, not from the previous matchings but by considering the effect of removing the butler and sapphire bracelet from the matrix and realising that the least values in the *P* and *R* columns correspond to *G* and *H*, respectively.

4) (i)(a) Most candidates were able to state that the capacity of this cut is 90.

(i)(b) Some candidates were unsure how to deal with the arc *ED* flowing backwards across the cut (it should be at its minimum possible, in this case 0) resulting in a capacity of 90 instead of 100.

(i)(c) The two results so far showed that the maximum possible flow <u>cannot be more</u> than 90 (but we do not yet know whether or not 90 is possible, in fact it is not).

(ii) Most candidates appreciated that a cut must separate the source from the sink.

(iii) Most candidates were able to explain the significance of these arcs 'two-way flow' etc. A few thought that they were connected with the labelling procedure.

(iv) Many candidates claimed that 30 <u>is</u> flowing from *C* into *CF*, rather than that 30 is the maximum that can flow through *BC* (and as there is no other way into *CF* this is also the maximum that can flow along *CF*).

(v)(a) Using the information from the previous part, it was now possible to construct a flow in which 60 flowed along arc *FT*. Some candidates showed a flow of 60 from *S* to *T* but did not have 60 going along *FT*.

(v)(b) Most candidates found the cut $\{S, A\}$, $\{B, C, D, E, F, T\}$. Some also explicitly referred to the flow of 60 that had already been found, but most just said max flow = min cut, which was not enough for both marks.

(vi) Several candidates realised that a flow of $60 + x$ was now achievable, although many of these just added x around *SCFT* to their flow from part (v)(a), resulting in the claim that the maximum possible was when $x = 15$ and that if x was greater than that then 75 was still the maximum flow. In fact the flow of $60 + x$ applies for x up to 30, and after that the maximum is 90. This can be seen by using the cut from part (v)(b).

5) (i) Most candidates gave correct networks, some had arcs within stages (joining (1;0) to (1;1) and (2;0) to (2;1) and some just drew the path (0;0)-(1;0)-(1;1)-(2;0)-(2;1)-(3;0).

(ii)(a) The action value 1 in the last row tells you that this row refers to the transition to state 1 in the stage above, the transition from (0;0) to (1;1). Several candidates described routes from *A* to *C* or *A*-*B*-*C* instead of purely using the table.

(ii)(b) The 45 comes from the weight of the transition from (0;0) to (1;1), which we deduce from the stage state values and the action value for this row, and the 35 is the suboptimal minimax for (1;1) in the row above. Many candidates realised where the 45 came from, although sometimes this was described using letter labels, but fewer related the 35 to a minimax value.

(iii) Most candidates knew that the stops were at *C* and *D*. The explanations were often missing or just of the 'I worked backwards through the table' variety. The connection between the action for the minimax and the state for the stage above needed to be seen, both for (0;0)-(1;1) and for (1;1)-(2;0).

(iv)(a) Most candidates realised that in this longer version Henry needed to reach at least *J* by Saturday night if he is to finish on Sunday.

(iv)(b) Several candidates found that on Friday Henry must have reached at least *G* and that the corresponding places for Thursday and Wednesday were *D* and *B*. Some claimed *D* and *C*, presumably referring back to their answer to part (iii).

(v) Many candidates only gave (stage; state) labels for the places with state label 0, others left out *F* (only allowing two possibilities for each night) or included *I* (which was not possible as an overnight stop).

(vi) The candidates who persisted with the question were usually able to make some sort of attempt at the dynamic programming tabulation, but only those with correct (stage; state) labels were able to give a complete solution. Even without the tabulation, some candidates realised that Henry needed to stay at *C*, *D*, *G* and *J* with a maximum ride on any one day of 45 miles.

6) (i) Some candidates did not show their working for finding the play-safe, the values 5, 3, 2, 1 at the bottom of the columns were enough. If Colin plays safe then the most he can win is 2 points, some candidates assumed that the question was asking for the minimum that he will lose.

(ii) Most candidates correctly identified that *W* is dominated by *Y*.

(iii) Many good solutions in which candidates explained or showed that row *W* had been removed, 1 added throughout the table (to remove the negatives) and column *N* selected.

(iv) Candidates needed to show the calculations for all four constraints and then show that m must be less than or equal to 1.6, and hence the optimum value is 1.6 leading to the given result.

(v) Most candidates showed or stated that Colin's winnings are the negatives of the values in the table, and most realised that row *X* was then being used (although some thought that it was column *P*). The –0.6 arises because Rowena's optimum solution is to win 0.6 so Colin's will be to lose 0.6.

(vi) Some very strange arithmetic was seen, in particular candidates who added the third and fourth equations to give either $p = 0.4$ or $2p = 1.6$, instead of $2p = 0.4$. Even the candidates who achieved $p = 0.2$, and who usually then stated that $q + t = 0.8$, did not always realise that $q = t = 0.4$ (from the first equation).

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

OCR Customer Contact Centre

Education and Learning
Telephone: 01223 553998
Facsimile: 01223 552627
Email: general.qualifications@ocr.org.uk

www.ocr.org.uk