GCE Examinations

Advanced Subsidiary / Advanced Level

Pure Mathematics
Module P5

Paper G

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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P5 Paper G – Marking Guide

1. *(a)* $y = e^{\arctan x}$ $\therefore$ $\frac{dy}{dx} = \frac{1}{1+x^2} \times e^{\arctan x} = \frac{e^{\arctan x}}{1+x^2}$ M1 A1

$\frac{d^2y}{dx^2} = \frac{1}{1+x^2} \times \frac{e^{\arctan x}}{1+x^2} - (1+x^2)^{-2} \times 2x \times e^{\arctan x} = \frac{e^{\arctan x}(1-2x)}{(1+x^2)^2}$ M1 A1

(b) pt. of inflexion $\therefore \frac{d^2y}{dx^2} = 0$ M1

$\therefore e^{\arctan x} = 0$ (no solns) or $x = \frac{1}{2}$ A1

$\therefore$ coordinates (0.5, 1.59) [y-coord 3sf] A1 **(7)**

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2. *(a)* let $y = \text{arcosh}\, x$ $\therefore$ $\cosh y = x$

$\therefore$ $\sinh y \frac{dy}{dx} = 1$ M1

$\frac{dy}{dx} = \frac{1}{\sqrt{\cosh^2 y - 1}} = \frac{1}{\sqrt{x^2-1}}$ M1 A1

(b) $u = \text{arcosh}\, x$, $u' = \frac{1}{\sqrt{x^2-1}}$; $v' = 1$, $v = x$ M1

$\int \text{arcosh}\, x \; dx = x \,\text{arcosh}\, x - \int \frac{x}{\sqrt{x^2-1}} \, dx$ A1

$= x\,\text{arcosh}\, x - \sqrt{x^2-1} + c$ M1 A1 **(7)**

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3. $t = \tan x$ $\therefore$ $\frac{dt}{dx} = \sec^2 x = (1 + t^2)$ M1 A1

$\int_0^{\frac{\pi}{4}} \frac{1}{1+\sin 2x} \, dx = \int_0^1 \frac{1}{1+\frac{2t}{1+t^2}} \times \frac{1}{1+t^2} \, dt$ M1 A1

$= \int_0^1 \frac{1}{1+t^2+2t} \, dt$ M1

$= \int_0^1 \frac{1}{(t+1)^2} \, dt$ A1

$= [-(t+1)^{-1}]_0^1$ A1

$= -\frac{1}{2} - (-1) = \frac{1}{2}$ A1 **(8)**

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4. *(a)* $4x^2 - 4x + 10 \equiv (2x - 1)^2 - 1 + 10 \equiv (2x - 1)^2 + 9$ M1 A1

$$\int \frac{1}{\sqrt{4x^2 - 4x + 10}}\,dx = \int \frac{1}{\sqrt{(2x-1)^2 + 9}}\,dx$$

$u = 2x - 1,\ \frac{du}{dx} = 2$ M1

$= \frac{1}{2}\int \frac{1}{\sqrt{u^2+9}}\,du$ A1

$= \frac{1}{2}\text{arsinh}\left(\frac{u}{3}\right) + c = \frac{1}{2}\text{arsinh}\left(\frac{2x-1}{3}\right) + c$ M1 A1

(b) $\int_{\frac{1}{2}}^{2} \frac{1}{\sqrt{4x^2 - 4x + 10}}\,dx = \left[\frac{1}{2}\text{arsinh}\left(\frac{2x-1}{3}\right)\right]_{\frac{1}{2}}^{2}$

$= \frac{1}{2}\text{arsinh}\,1 - \frac{1}{2}\text{arsinh}\,0$ M1

$= \frac{1}{2}\ln(1 + \sqrt{2})$ M1 A1 **(9)**

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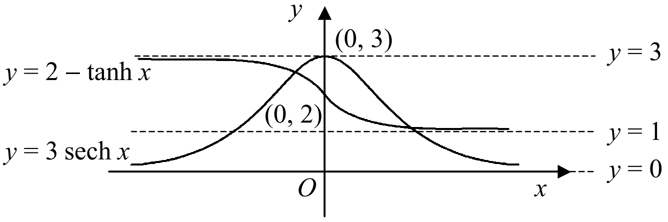
5. *(a)*



B3

B2

(b) $2 - \tanh x = 3\,\text{sech}\,x$

$2 - \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{6}{e^x + e^{-x}}$ M1

$2e^x + 2e^{-x} - e^x + e^{-x} = 6$

$e^x + 3e^{-x} - 6 = 0$ A1

$e^{2x} - 6e^x + 3 = 0$ M1

$e^x = \frac{6 \pm \sqrt{24}}{2} = 3 \pm \sqrt{6}$ M1 A1

$\therefore x = \ln(3 \pm \sqrt{6}) = -0.60$ or 1.70 (2dp) M1 A1 **(12)**

6.	*(a)*	$u = \sin^{n-1}x,\ u' = (n-1)\sin^{n-2}x\cos x;\ \ v' = \sin x,\ v = -\cos x$	M1 A1	
		$I_n = [-\cos x\sin^{n-1}x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} (n-1)\sin^{n-2}x\cos^2 x\ \mathrm{d}x$	A1	
		$I_n = 0 - 0 + (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}x(1-\sin^2 x)\ \mathrm{d}x$	M1 A1	
		$I_n = (n-1)\int_0^{\frac{\pi}{2}} \sin^{n-2}x\ \mathrm{d}x - (n-1)\int_0^{\frac{\pi}{2}} \sin^n x\ \mathrm{d}x$		
		$I_n = (n-1)I_{n-2} - (n-1)I_n$	M1	
		$nI_n = (n-1)I_{n-2}$		
		$I_n = \frac{n-1}{n} I_{n-2}$	A1	
	(b)	curve sym. about $x = \frac{\pi}{2}$ $\therefore V = \int_0^{\pi} \pi y^2\ \mathrm{d}x = 2\pi\int_0^{\frac{\pi}{2}} y^2\ \mathrm{d}x = 2\pi I_4$	M1 A1	
		$I_0 = \int_0^{\frac{\pi}{2}} \mathrm{d}x = [x]_0^{\frac{\pi}{2}} = \frac{\pi}{2}$	M1 A1	
		$I_2 = \frac{1}{2}I_0 = \frac{\pi}{4}$	M1	
		$I_4 = \frac{3}{4}I_2 = \frac{3\pi}{16}$	A1	
		$\therefore V = 4\pi \times \frac{3\pi}{16} = \frac{3}{4}\pi^2$	A1	**(14)**
7.	*(a)*	$x = a\cosh t,\ \frac{\mathrm{d}x}{\mathrm{d}t} = a\sinh t,\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = a\cosh t$	M1 A1	
		$y = 2a\sinh t,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 2a\cosh t,\ \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 2a\sinh t$	A1	
		at vertex $y = 0 \therefore t = 0 \therefore \frac{\mathrm{d}x}{\mathrm{d}t} = 0,\ \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = a,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 2a,\ \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 0$	A1	
		$\rho = \left\lvert \frac{(0+(2a)^2)^{\frac{3}{2}}}{0\times 0 - 2a\times a} \right\rvert = \left\lvert \frac{8a^3}{-2a^2} \right\rvert = 4a$	M1 A1	
	(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2a\cosh t}{a\sinh t} = \frac{2\cosh t}{\sinh t}$	M1 A1	
		eqn. is $y - 2a\sinh p = \frac{2\cosh p}{\sinh p}(x - a\cosh p)$	M1	
		$y\sinh p - 2a\sinh^2 p = 2x\cosh p - 2a\cosh^2 p$		
		giving $2x\cosh p - y\sinh p = 2a$	A1	
	(c)	asymptotes are "$y = \pm\frac{b}{a}x$" $\therefore y = \pm\frac{2a}{a}x = \pm 2x$	M1	
		$\therefore l_1$ is $y = 2x$; tangent at P is $2x\cosh p - y\sinh p = 2a$		
		$\therefore$ at Q $2x\cosh p - 2x\sinh p = 2a$	M1	
		giving $x = \frac{a}{\cosh p - \sinh p}$	A1	
		"$b^2 = a^2(e^2 - 1)$" $\therefore 4a^2 = a^2(e^2-1),\ e > 0 \therefore e = \sqrt{5} \therefore S$ is $(a\sqrt{5}, 0)$	M1 A1	
		$\therefore a\sqrt{5} = \frac{a}{\cosh p - \sinh p}$ $\therefore \cosh p - \sinh p = \frac{1}{\sqrt{5}}$	M1	
		$\frac{1}{2}(e^p + e^{-p}) - \frac{1}{2}(e^p - e^{-p}) = \frac{1}{\sqrt{5}}$	M1	
		$\therefore e^{-p} = \frac{1}{\sqrt{5}},\ -p = \ln 5^{-\frac{1}{2}},\ p = \frac{1}{2}\ln 5$	A1	**(18)**
			Total	**(75)**

Performance Record – P5 Paper G

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	diff. inv. trig.	diff., integr., inv. hyp. fns.	integr. $t = \tan x$	integr. std. forms	eqn. in hyp. fns.	reduction formula	hyperbola, rad. of curv., tangent	
Marks	7	7	8	9	12	14	18	75
Student								

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