Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise A, Question 1

Question:

Draw diagrams, as in Examples 1 and 2, to show the following angles. Mark in the acute angle that *OP* makes with the *x*-axis.

(a) $-80°$

(b) $100°$

(c) $200°$

(d) $165°$

(e) $-145°$

(f) $225°$

(g) $280°$

(h) $330°$

(i) $-160°$

(j) $-280°$

(k) $\frac{3\pi}{4}$

(l) $\frac{7\pi}{6}$

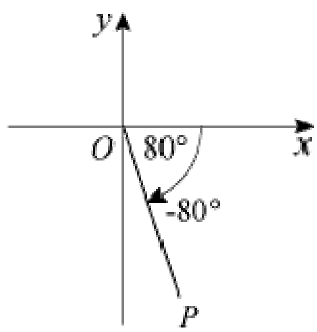
(m) $-\frac{5\pi}{3}$

(n) $-\frac{5\pi}{8}$

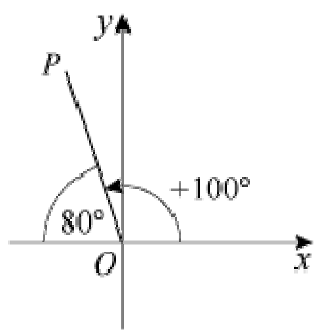
(o) $\frac{19\pi}{9}$

Solution:

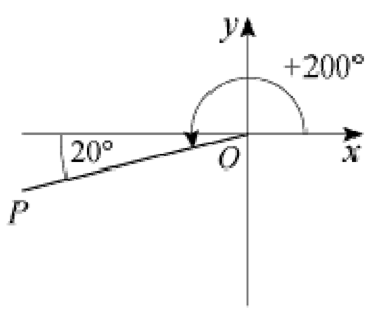
(a)



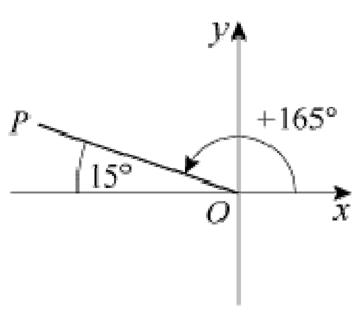
(b)



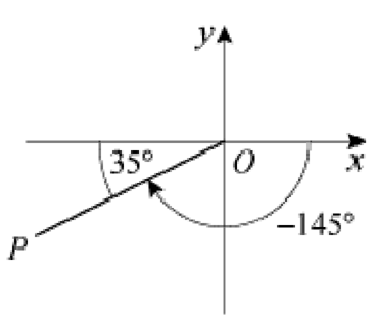
(c)



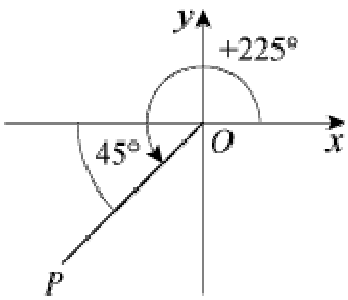
(d)



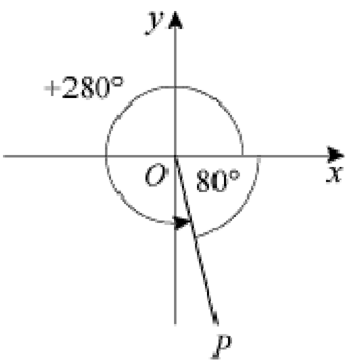
(e)



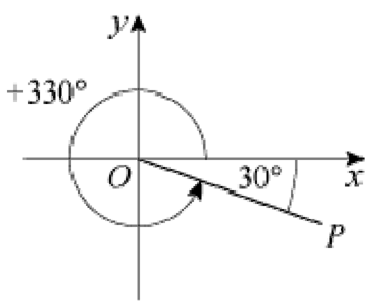
(f)



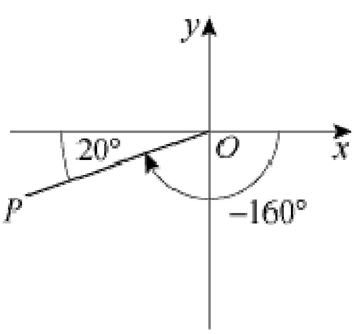
(g)



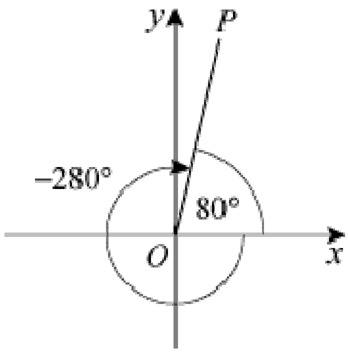
(h)



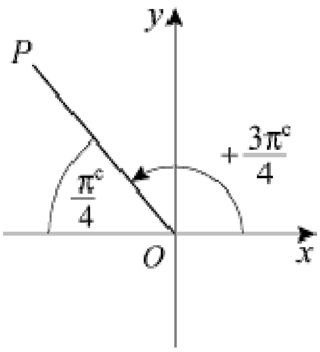
(i)



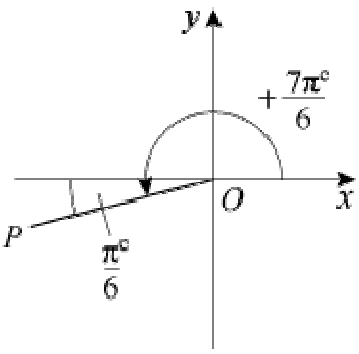
(j)



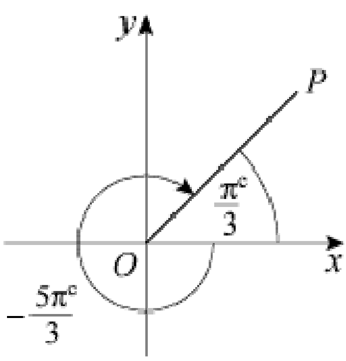
(k)


y
P
$+\frac{3\pi^c}{4}$
$\frac{\pi^c}{4}$
O
x

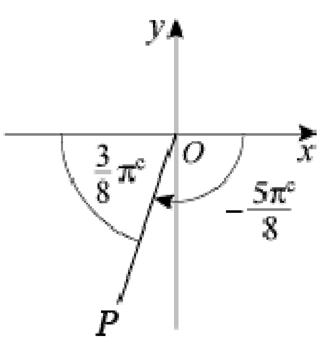
(l)


y
$+\frac{7\pi^c}{6}$
O
x
P
$\frac{\pi^c}{6}$

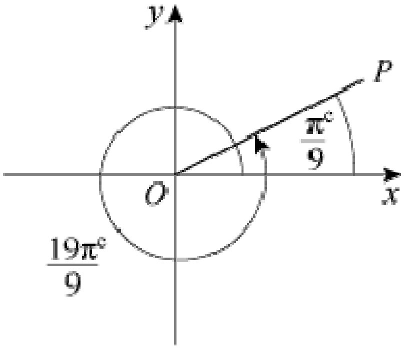
(m)


y
P
$\frac{\pi^c}{3}$
O
x
$-\frac{5\pi^c}{3}$

(n)


y
$\frac{3}{8}\pi^c$
O
x
$-\frac{5\pi^c}{8}$
P

(o)


y
P
$\frac{\pi^c}{9}$
O
x
$\frac{19\pi^c}{9}$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise A, Question 2

Question:

State the quadrant that *OP* lies in when the angle that *OP* makes with the positive *x*-axis is:

(a) 400°

(b) 115°

(c) $-210°$

(d) 255°

(e) $-100°$

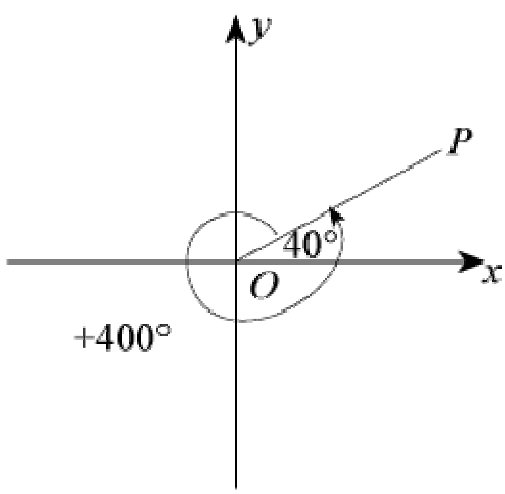
(f) $\frac{7\pi}{8}$

(g) $-\frac{11\pi}{6}$

(h) $\frac{13\pi}{7}$

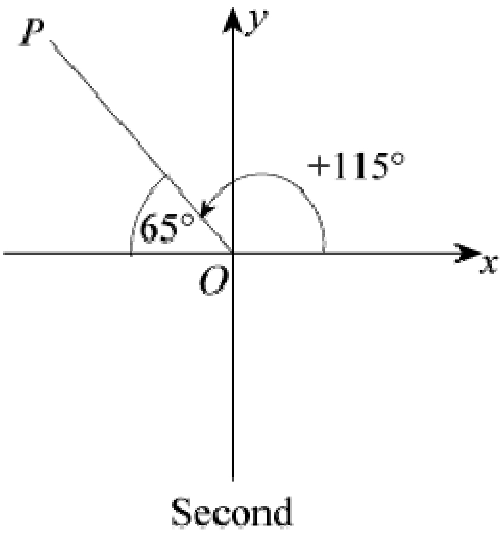
Solution:

(a)

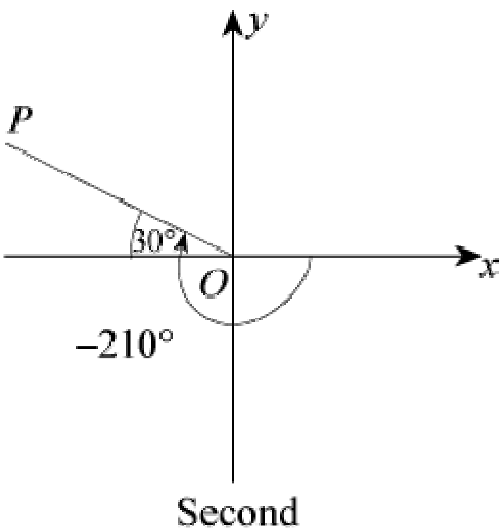


First

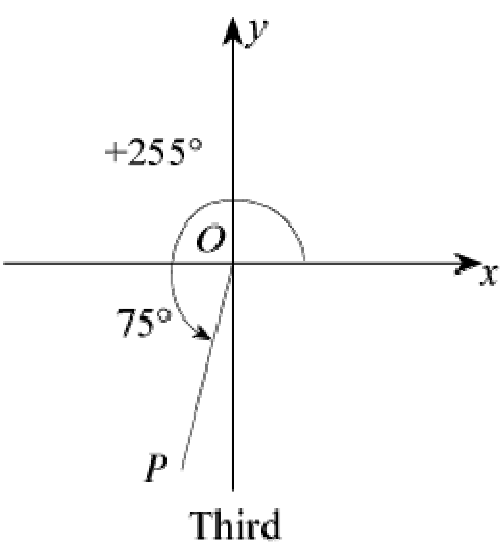
(b)



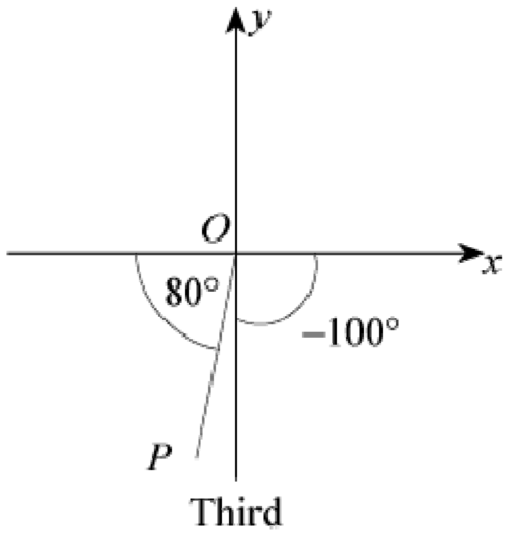
(c)



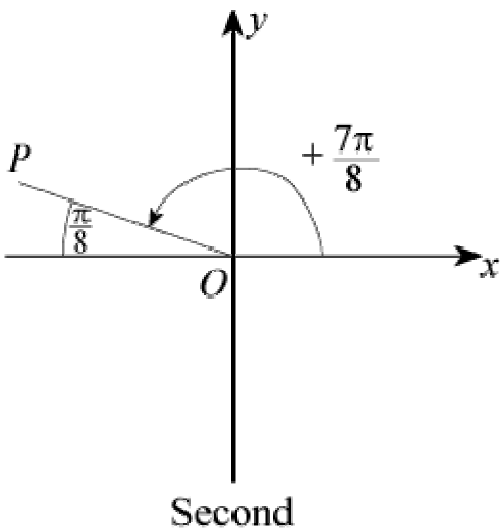
(d)



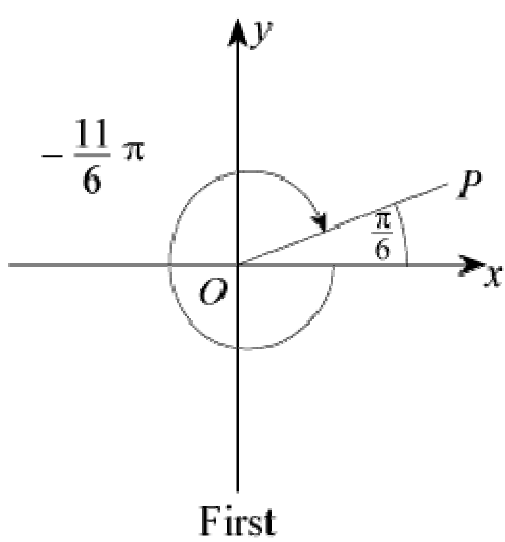
(e)


y
O
x
80°
−100°
P
Third

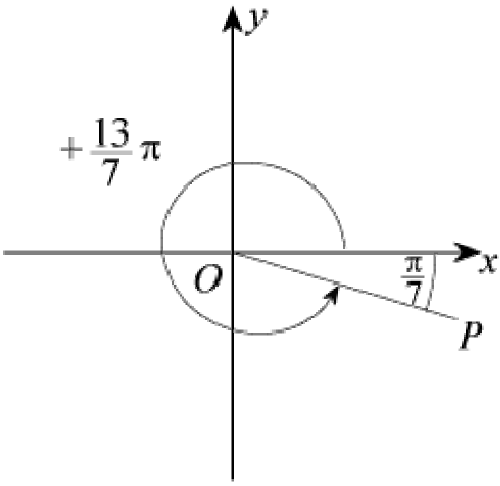
(f)


y
P
$+\frac{7\pi}{8}$
$\frac{\pi}{8}$
O
x
Second

(g)


y
$-\frac{11}{6}\pi$
P
$\frac{\pi}{6}$
O
x
First

(h)



Fourth

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise B, Question 1

Question:

(*Note:* do not use a calculator.)

Write down the values of:

(a) $\sin \left(-90 \right)^{\circ}$

(b) $\sin 450^{\circ}$

(c) $\sin 540^{\circ}$

(d) $\sin \left(-450 \right)^{\circ}$

(e) $\cos \left(-180 \right)^{\circ}$

(f) $\cos \left(-270 \right)^{\circ}$

(g) $\cos 270^{\circ}$

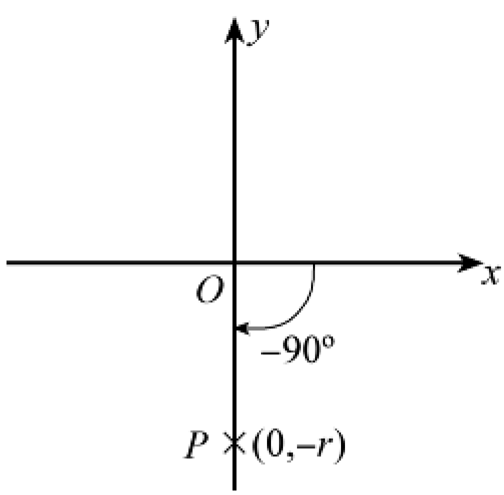
(h) $\cos 810^{\circ}$

(i) $\tan 360^{\circ}$

(j) $\tan \left(-180 \right)^{\circ}$

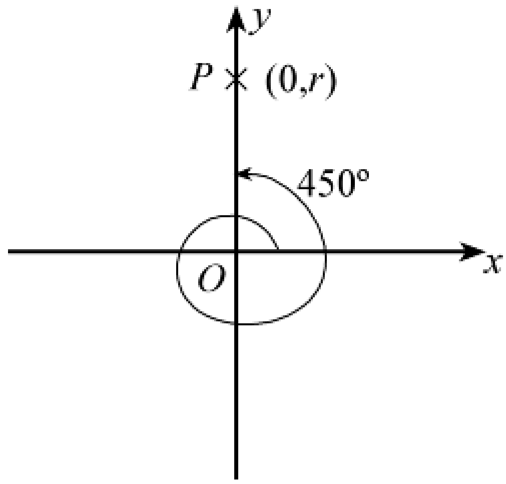
Solution:

(a)



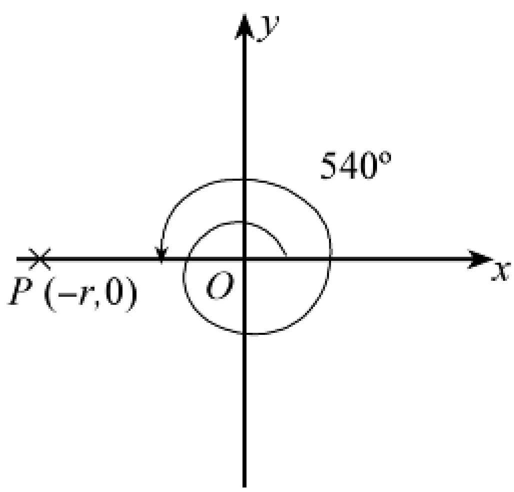
$$\sin \left(-90 \right)^{\circ} = \frac{-r}{r} = -1$$

(b)



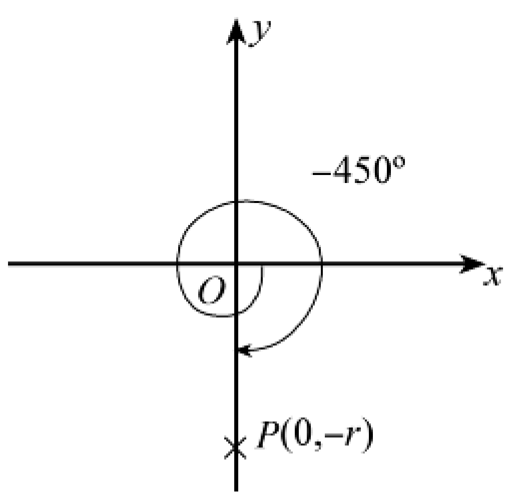
$\sin 450° = \frac{r}{r} = 1$

(c)



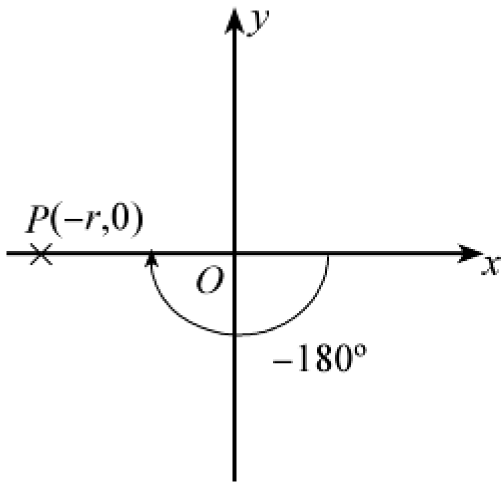
$\sin 540° = \frac{0}{r} = 0$

(d)



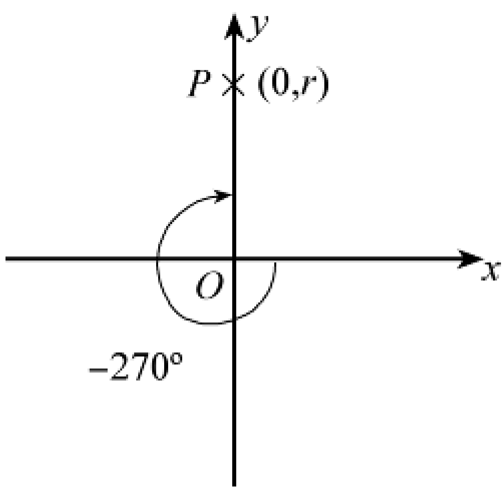
$$\sin\left(-450\right)° = \frac{-r}{r} = -1$$

(e)



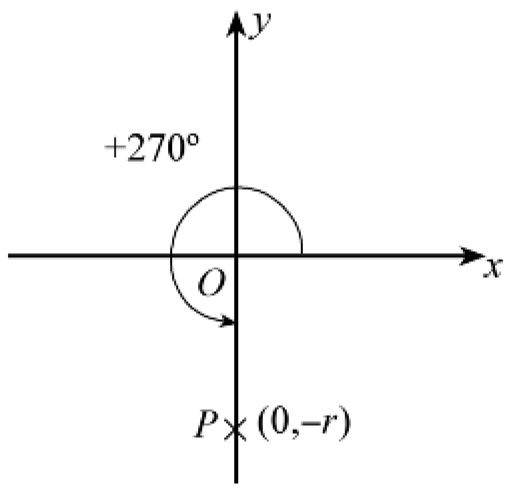
$$\cos\left(-180\right)° = \frac{-r}{r} = -1$$

(f)



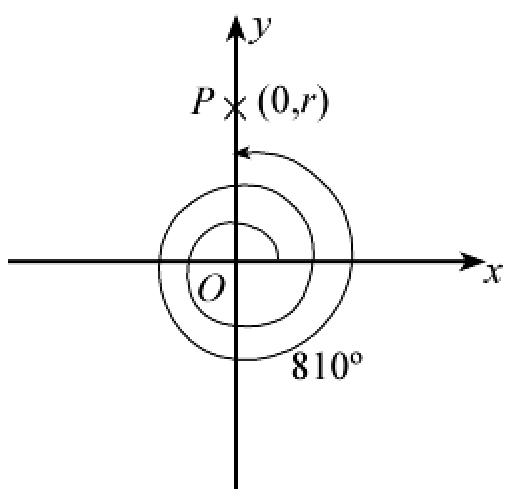
$$\cos\left(-270\right)° = \frac{0}{r} = 0$$

(g)



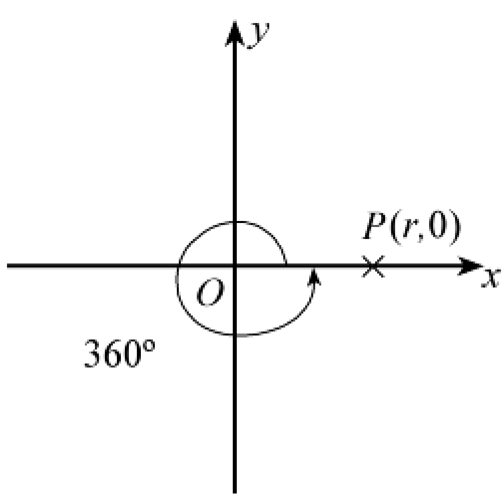
$\cos 270° = \frac{0}{r} = 0$

(h)



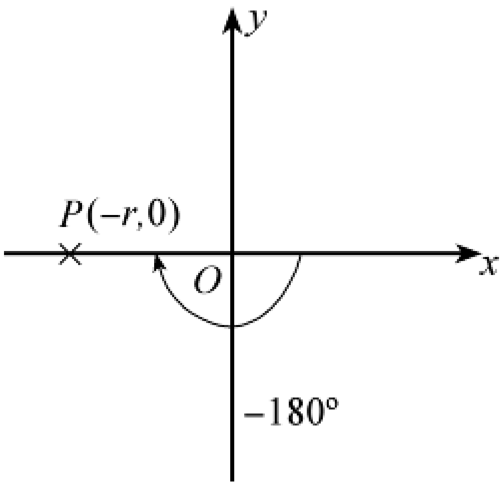
$\cos 810° = \frac{0}{r} = 0$

(i)



$\tan 360° = \frac{0}{r} = 0$

(j)



$\tan(-180)° = \frac{0}{-r} = 0$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise B, Question 2

Question:

(*Note:* do not use a calculator.)

Write down the values of the following, where the angles are in radians:

(a) $\sin \frac{3\pi}{2}$

(b) $\sin \left(- \frac{\pi}{2} \right)$

(c) $\sin 3\pi$

(d) $\sin \frac{7\pi}{2}$

(e) $\cos 0$

(f) $\cos \pi$

(g) $\cos \frac{3\pi}{2}$

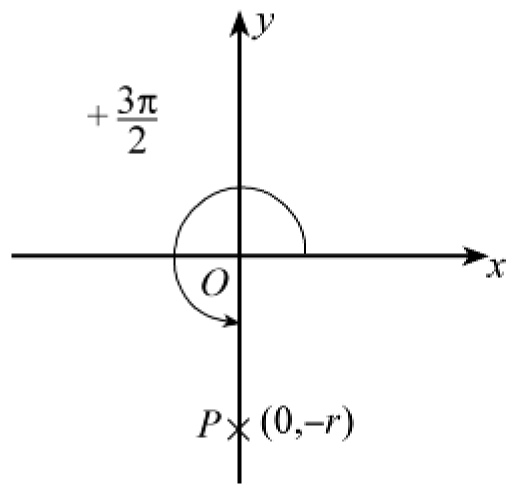
(h) $\cos \left(- \frac{3\pi}{2} \right)$

(i) $\tan \pi$

(j) $\tan (- 2\pi)$

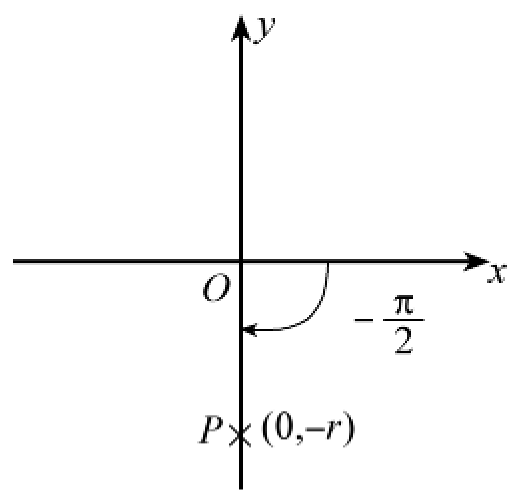
Solution:

(a)



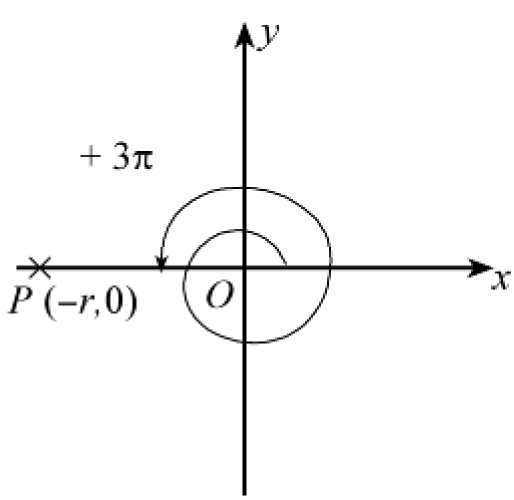
$\sin \frac{3\pi}{2} = \frac{-r}{r} = -1$

(b)



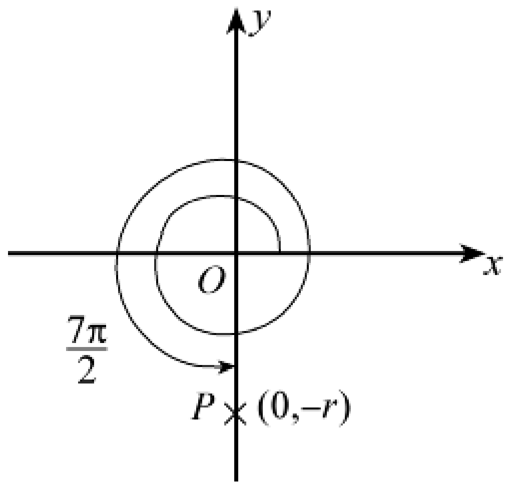
$\sin \left(\frac{-\pi}{2} \right) = \frac{-r}{r} = -1$

(c)



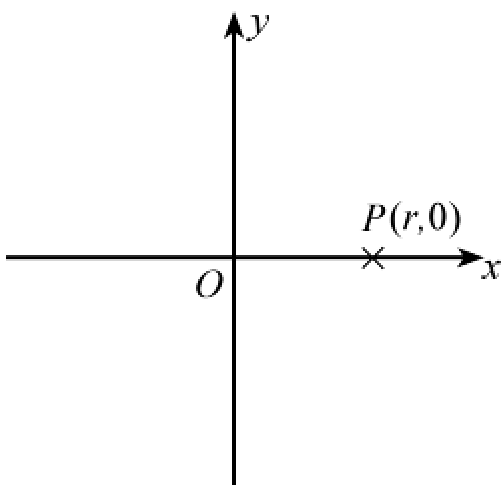
$\sin 3\pi = \frac{0}{r} = 0$

(d)



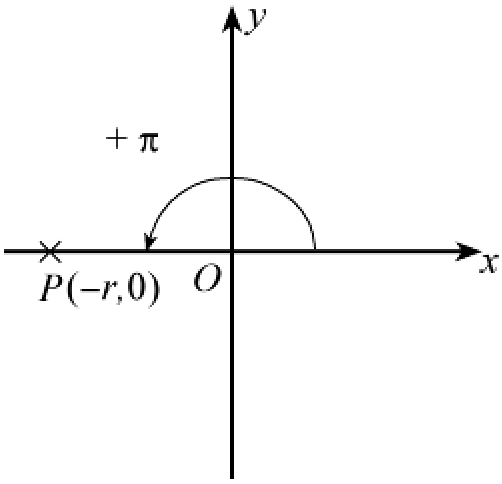
$\sin \frac{7\pi}{2} = \frac{-r}{r} = -1$

(e)



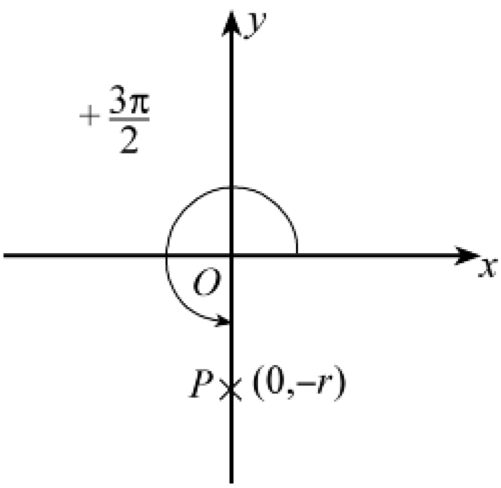
$\cos 0° = \frac{r}{r} = 1$

(f)



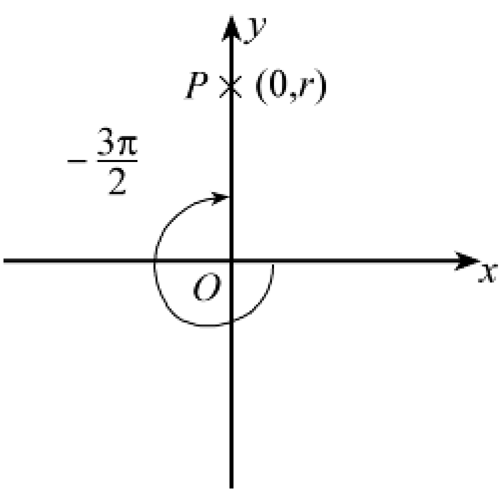
$\cos \pi = \frac{-r}{r} = -1$

(g)



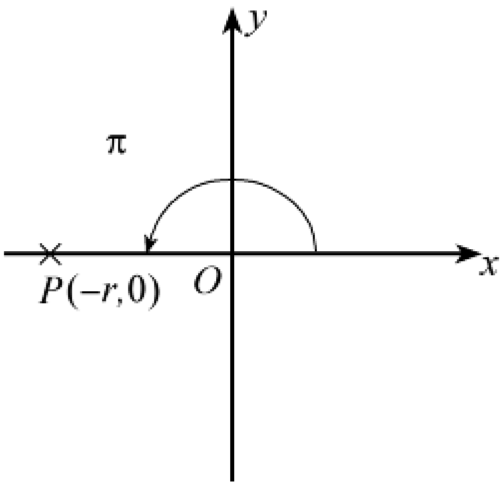
$\cos \frac{3\pi}{2} = \frac{0}{r} = 0$

(h)



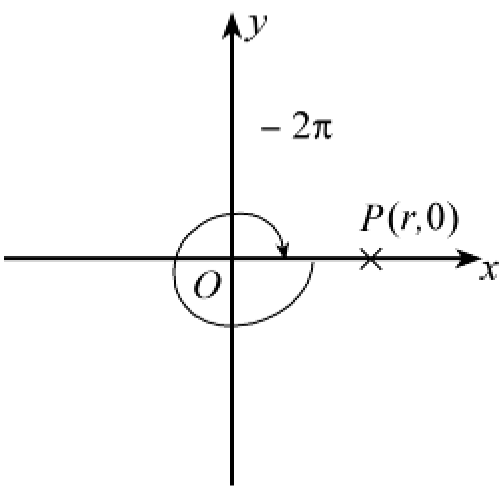
$\cos\left(-\frac{3\pi}{2}\right) = \frac{0}{r} = 0$

(i)



$\tan \pi = \frac{0}{-r} = 0$

(j)



$\tan\left(-2\pi\right) = \frac{0}{r} = 0$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise C, Question 1

Question:

(*Note:* Do not use a calculator.)

By drawing diagrams, as in Example 6, express the following in terms of trigonometric ratios of acute angles:

(a) $\sin 240°$

(b) $\sin(-80)°$

(c) $\sin(-200)°$

(d) $\sin 300°$

(e) $\sin 460°$

(f) $\cos 110°$

(g) $\cos 260°$

(h) $\cos(-50)°$

(i) $\cos(-200)°$

(j) $\cos 545°$

(k) $\tan 100°$

(l) $\tan 325°$

(m) $\tan(-30)°$

(n) $\tan(-175)°$

(o) $\tan 600°$

(p) $\sin \frac{7\pi}{6}$

(q) $\cos \frac{4\pi}{3}$

(r) $\cos\left(-\frac{3\pi}{4}\right)$

(s) $\tan \frac{7\pi}{5}$

(t) $\tan\left(-\frac{\pi}{3}\right)$

(u) $\sin\frac{15\pi}{16}$

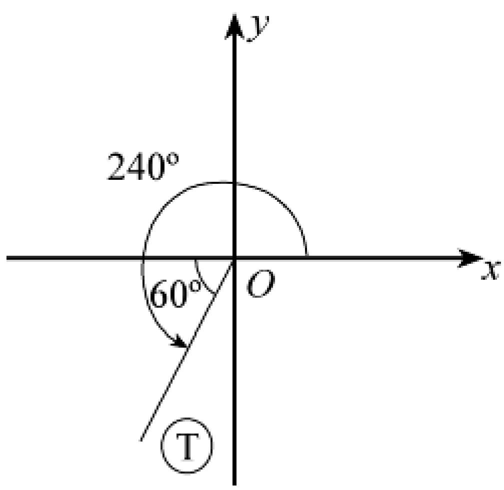
(v) $\cos\frac{8\pi}{5}$

(w) $\sin\left(-\frac{6\pi}{7}\right)$

(x) $\tan\frac{15\pi}{8}$

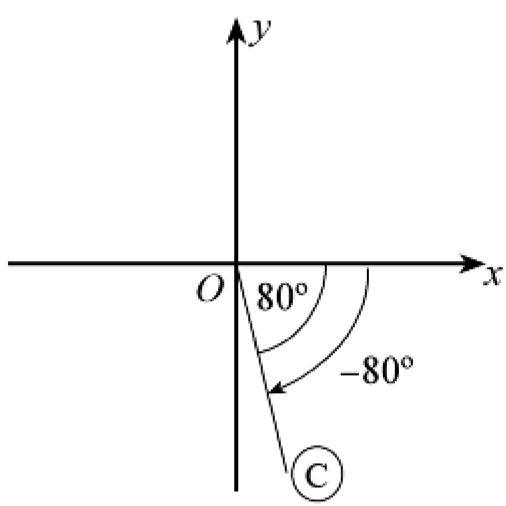
Solution:

(a)



60° is the acute angle.
In third quadrant sin is − ve.
So $\sin 240° = -\sin 60°$

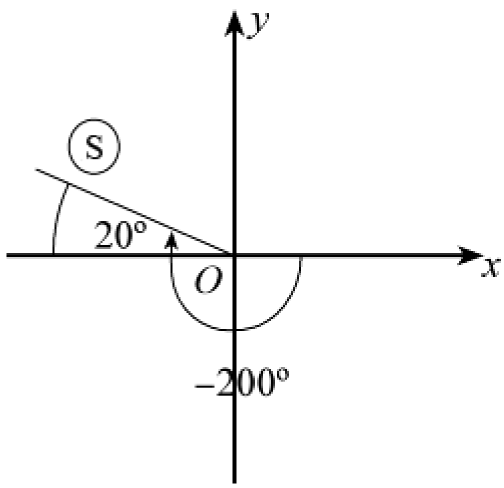
(b)



80° is the acute angle.

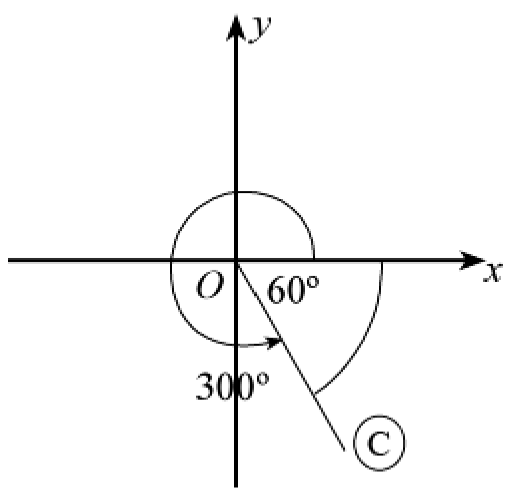
In fourth quadrant sin is − ve.
So sin $(-80)° = -\sin 80°$

(c)



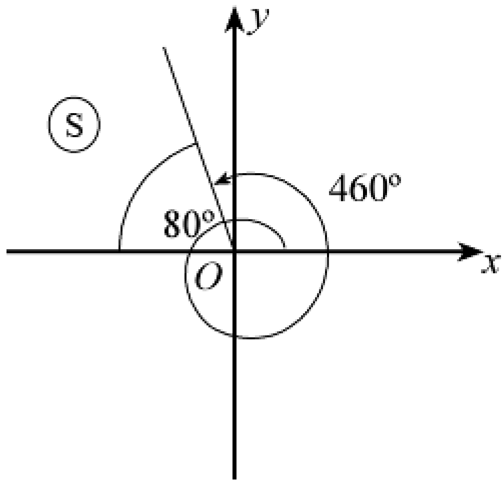
20° is the acute angle.
In second quadrant sin is +ve.
So sin $(-200)° = +\sin 20°$

(d)



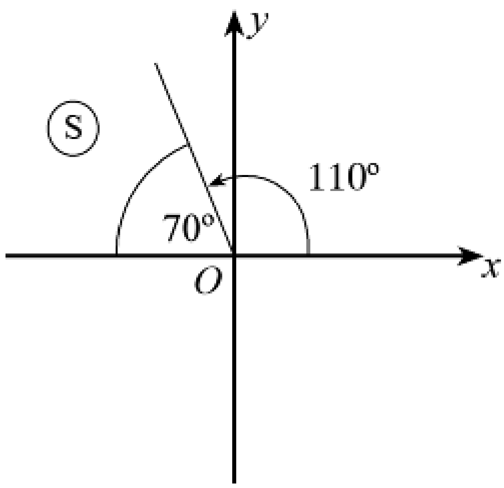
60° is the acute angle.
In fourth quadrant sin is − ve.
So sin $300° = -\sin 60°$

(e)



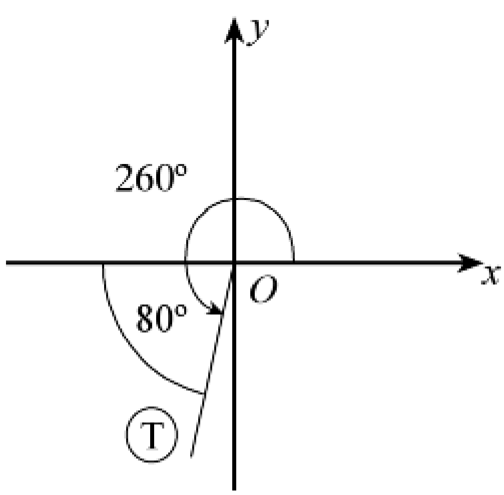
80° is the acute angle.
In second quadrant sin is +ve.
So $\sin\ 460° = +\sin\ 80°$

(f)



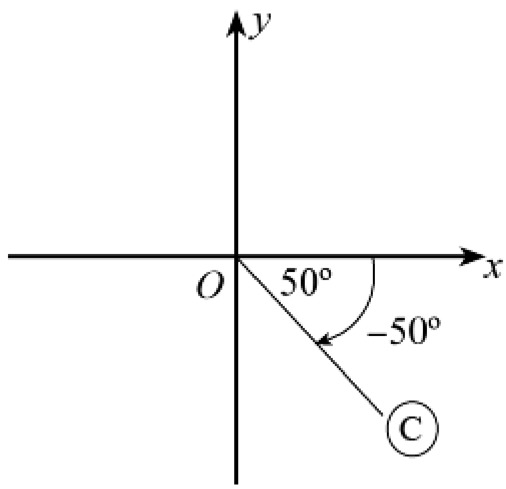
70° is the acute angle.
In second quadrant cos is − ve.
So $\cos\ 110° = -\cos\ 70°$

(g)



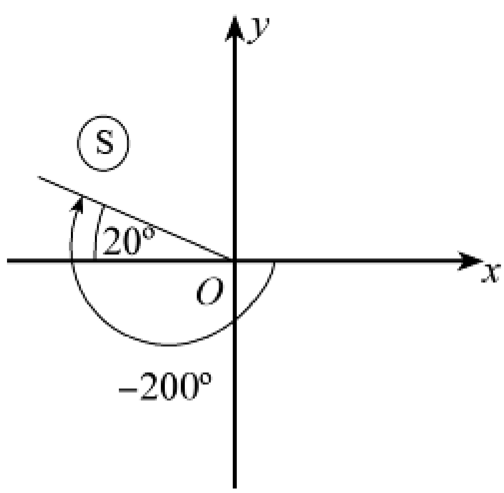
80° is the acute angle.
In third quadrant cos is − ve.
So $\cos 260° = -\cos 80°$

(h)



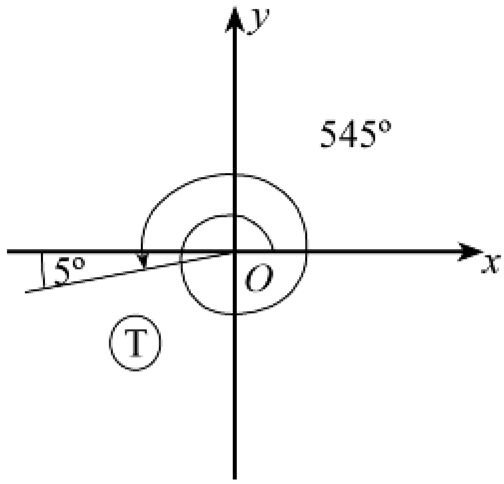
50° is the acute angle.
In fourth quadrant cos is +ve.
So $\cos (-50)° = +\cos 50°$

(i)



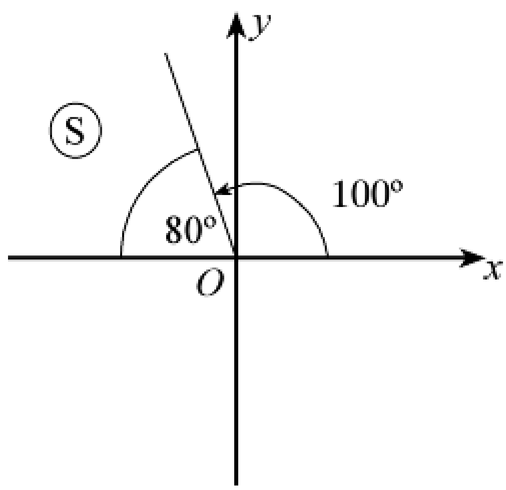
20° is the acute angle.
In second quadrant cos is − ve.
So $\cos (-200)° = -\cos 20°$

(j)



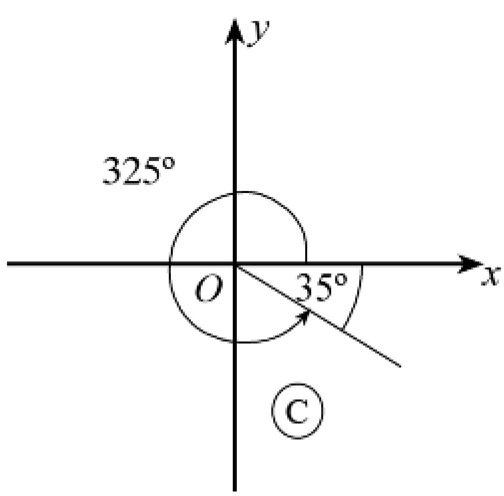
5° is the acute angle.
In third quadrant cos is − ve.
So $\cos 545° = -\cos 5°$

(k)



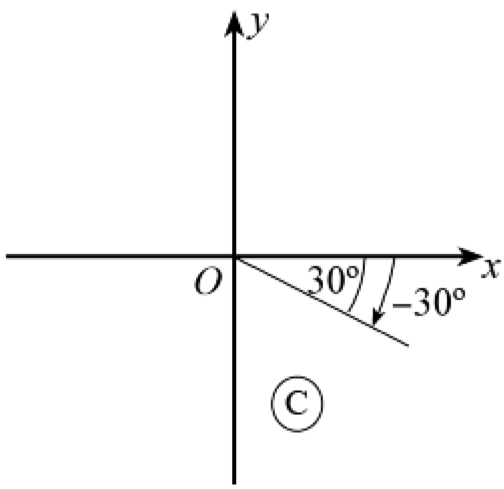
80° is the acute angle.
In second quadrant tan is − ve.
So $\tan 100° = -\tan 80°$

(l)



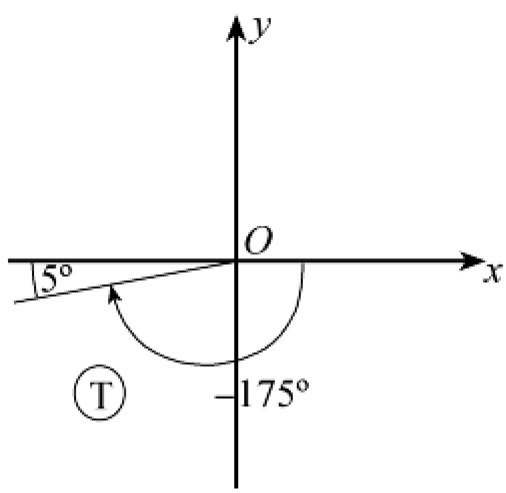
35° is the acute angle.
In fourth quadrant tan is − ve.
So $\tan 325° = -\tan 35°$

(m)



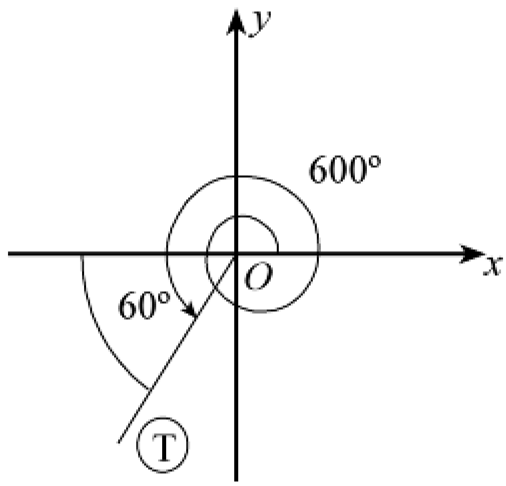
30° is the acute angle.
In fourth quadrant tan is − ve.
So $\tan(-30)° = -\tan 30°$

(n)



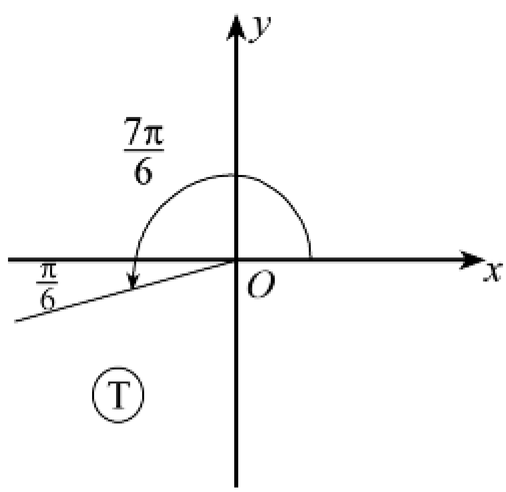
5° is the acute angle.
In third quadrant tan is +ve.
So $\tan(-175)° = +\tan 5°$

(o)



60° is the acute angle.
In third quadrant tan is +ve.
So $\tan 600° = + \tan 60°$

(p)

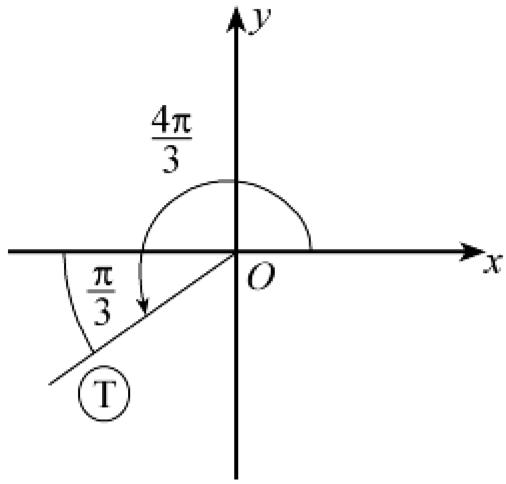


$\frac{\pi}{6}$ is the acute angle.

In third quadrant sin is − ve.

So $\sin \frac{7\pi}{6} = - \sin \frac{\pi}{6}$

(q)

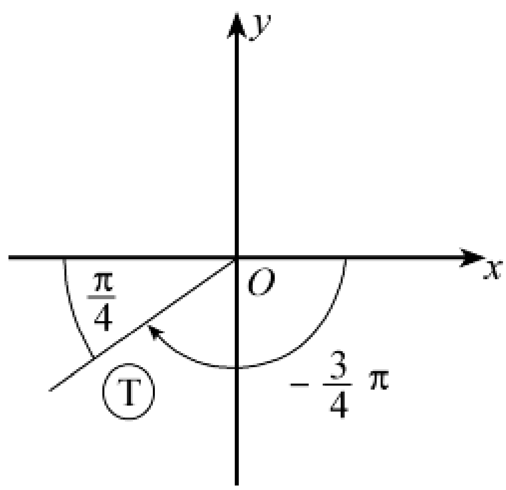


$\frac{\pi}{3}$ is the acute angle.

In third quadrant cos is − ve.

So $\cos \frac{4\pi}{3} = -\cos \frac{\pi}{3}$

(r)

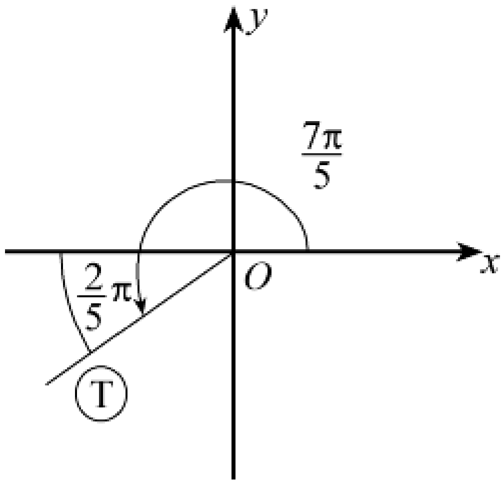


$\frac{\pi}{4}$ is the acute angle.

In third quadrant cos is − ve.

So $\cos \left(-\frac{3}{4}\pi \right) = -\cos \frac{\pi}{4}$

(s)

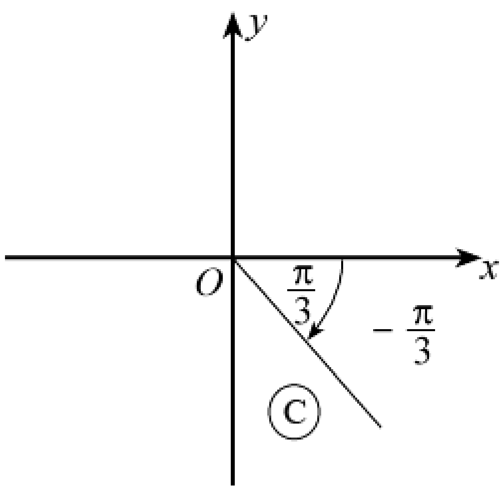


$\frac{2\pi}{5}$ is the acute angle.

In third quadrant tan is +ve.

So tan $\frac{7\pi}{5}$ = + tan $\frac{2\pi}{5}$

(t)

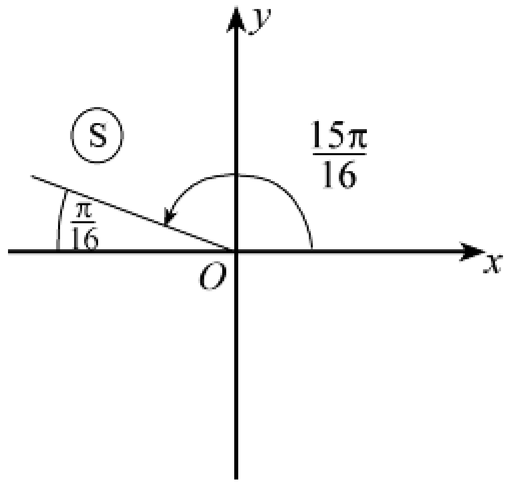


$\frac{\pi}{3}$ is the acute angle.

In fourth quadrant tan is − ve.

So tan $\left(-\frac{\pi}{3}\right) = -\tan\frac{\pi}{3}$

(u)

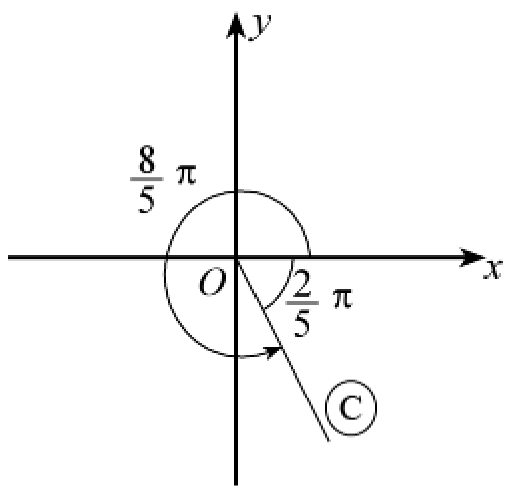


$\frac{\pi}{16}$ is the acute angle.

In second quadrant sin is +ve.

So $\sin\ \frac{15\pi}{16} = +\sin\ \frac{\pi}{16}$

(v)

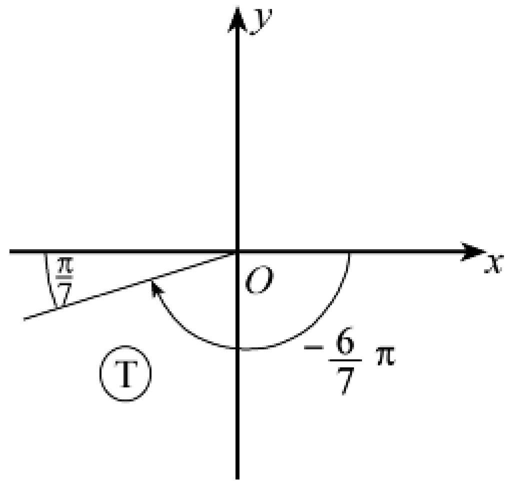


$\frac{2}{5}\pi$ is the acute angle.

In fourth quadrant cos is +ve.

So $\cos\ \frac{8}{5}\pi = +\cos\ \frac{2}{5}\pi$

(w)

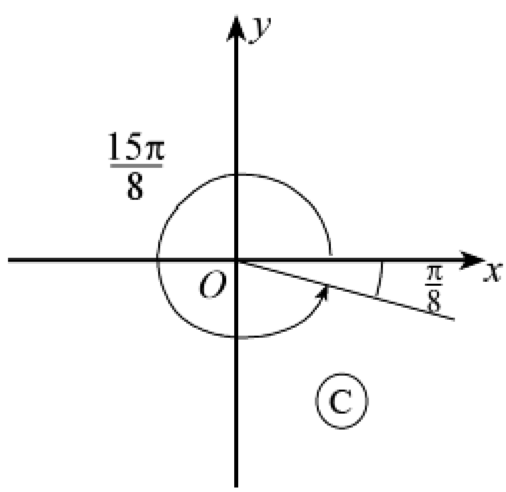


$\frac{\pi}{7}$ is the acute angle.

In third quadrant sin is − ve.

So $\sin\left(-\frac{6\pi}{7}\right) = -\sin\frac{\pi}{7}$

(x)



$\frac{\pi}{8}$ is the acute angle.

In fourth quadrant tan is − ve.

So $\tan\frac{15\pi}{8} = -\tan\frac{\pi}{8}$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise C, Question 2

Question:

(*Note:* Do not use a calculator.)

Given that θ is an acute angle measured in degrees, express in terms of $\sin\theta$:

(a) $\sin(-\theta)$

(b) $\sin(180° + \theta)$

(c) $\sin(360° - \theta)$

(d) $\sin-(180° + \theta)$

(e) $\sin(-180° + \theta)$

(f) $\sin(-360° + \theta)$

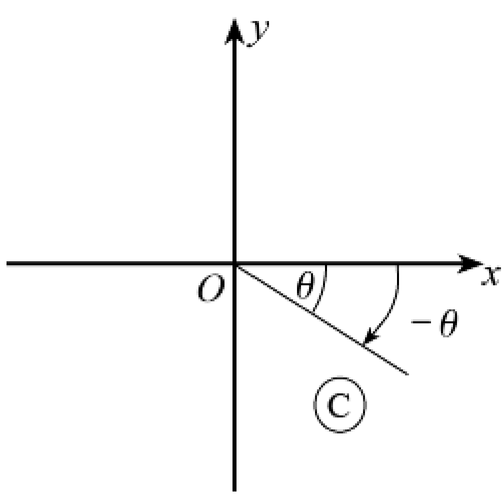
(g) $\sin(540° + \theta)$

(h) $\sin(720° - \theta)$

(i) $\sin(\theta + 720°)$

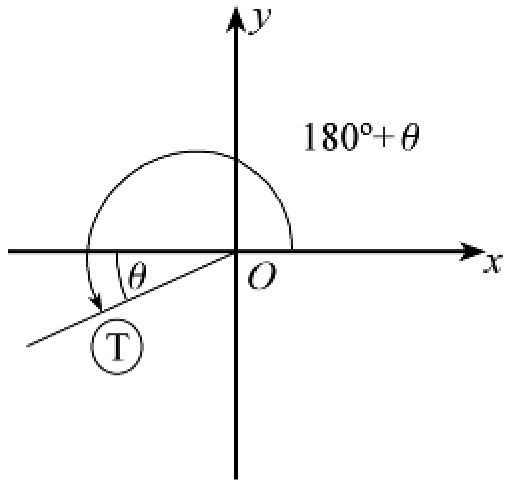
Solution:

(a)



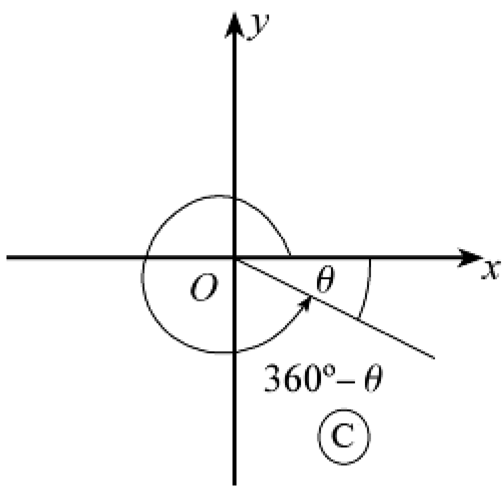
sin is − ve in this quadrant.
So $\sin(-\theta) = -\sin\theta$

(b)



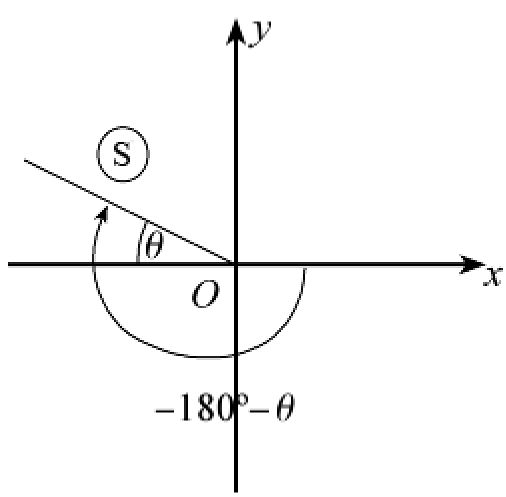
sin is − ve in this quadrant.
So $\sin(180° + \theta) = -\sin\theta$

(c)



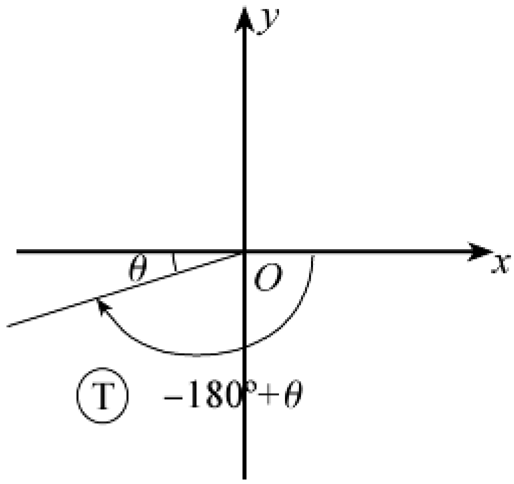
sin is − ve in this quadrant.
So $\sin(360° - \theta) = -\sin\theta$

(d)



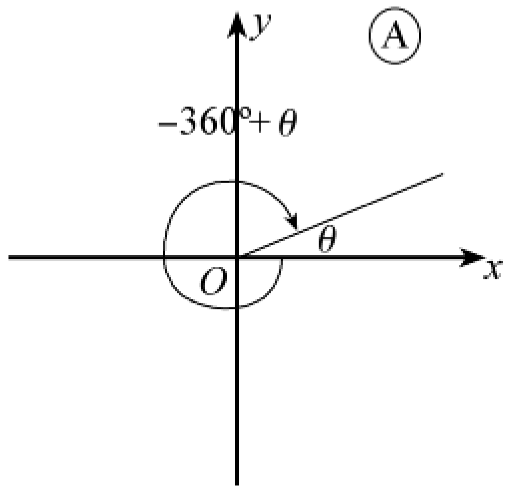
sin is +ve in this quadrant.
So $\sin -(180° + \theta) = +\sin\theta$

(e)



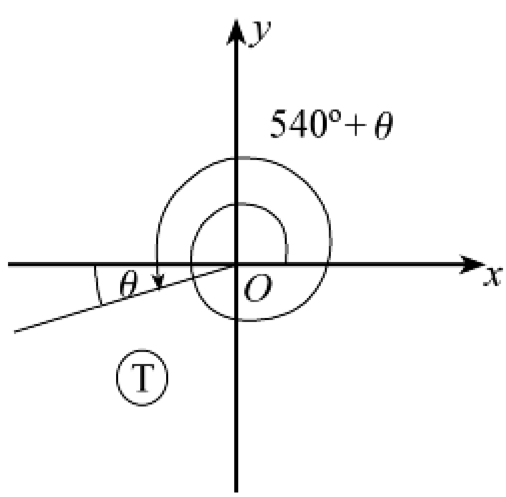
sin is $-$ ve in this quadrant.
So $\sin(-180° + \theta) = -\sin\theta$

(f)



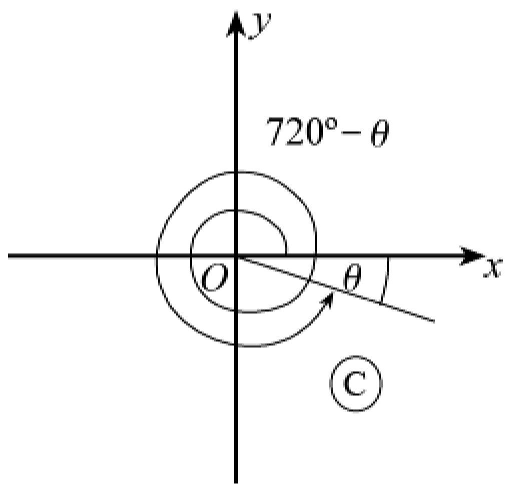
sin is +ve in this quadrant.
So $\sin(-360° + \theta) = +\sin\theta$

(g)



sin is $-$ ve in this quadrant.
So $\sin(540° + \theta) = -\sin\theta$

(h)



sin is − ve in this quadrant.
So $\sin(720° - \theta) = -\sin\theta$

(i) $\theta + 720°$ is in the first quadrant with θ to the horizontal.
So $\sin(\theta + 720°) = +\sin\theta$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise C, Question 3

Question:

(*Note:* Do not use a calculator.)

Given that θ is an acute angle measured in degrees, express in terms of cos θ or tan θ:

(a) $\cos(180° - \theta)$

(b) $\cos(180° + \theta)$

(c) $\cos(-\theta)$

(d) $\cos - (180° - \theta)$

(e) $\cos(\theta - 360°)$

(f) $\cos(\theta - 540°)$

(g) $\tan(-\theta)$

(h) $\tan(180° - \theta)$

(i) $\tan(180° + \theta)$

(j) $\tan(-180° + \theta)$

(k) $\tan(540° - \theta)$

(l) $\tan(\theta - 360°)$

Solution:

(a) $180° - \theta$ is in the second quadrant where cos is $-$ ve, and the angle to the horizontal is θ, so
$\cos(180° - \theta) = -\cos\theta$

(b) $180° + \theta$ is in the third quadrant, at θ to the horizontal, so
$\cos(180° + \theta) = -\cos\theta$

(c) $-\theta$ is in the fourth quadrant, at θ to the horizontal, so
$\cos(-\theta) = +\cos\theta$

(d) $-180° + \theta$ is in the third quadrant, at θ to the horizontal, so
$\cos(-180° + \theta) = -\cos\theta$

(e) $\theta - 360°$ is in the first quadrant, at θ to the horizontal, so
$\cos(\theta - 360°) = +\cos\theta$

(f) $\theta - 540°$ is in the third quadrant, at θ to the horizontal, so
$\cos(\theta - 540°) = -\cos\theta$

(g) $\tan(-\theta) = -\tan\theta$ as $-\theta$ is in the fourth quadrant.

(h) $\tan(180° - \theta) = -\tan\theta$ as $(180° - \theta)$ is in the second quadrant.

(i) $\tan(180° + \theta) = +\tan\theta$ as $(180° + \theta)$ is in the third quadrant.

(j) $\tan(-180° + \theta) = +\tan\theta$ as $(-180° + \theta)$ is in the third quadrant.

(k) $\tan(540° - \theta) = -\tan\theta$ as $(540° - \theta)$ is in the second quadrant.

(l) $\tan(\theta - 360°) = +\tan\theta$ as $(\theta - 360°)$ is in the first quadrant.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise C, Question 4

Question:

(*Note:* Do not use a calculator.)

A function f is an even function if $f(-\theta) = f(\theta)$.

A function f is an odd function if $f(-\theta) = -f(\theta)$

Using your results from questions 2(a), 3(c) and 3(g), state whether $\sin\theta$, $\cos\theta$ and $\tan\theta$ are odd or even functions.

Solution:

As $\sin(-\theta) = -\sin\theta$ (question **2a**)
$\sin\theta$ is an odd function.

As $\cos(-\theta) = +\cos\theta$ (question **3c**)
$\cos\theta$ is an even function.

As $\tan(-\theta) = -\tan\theta$ (question **3g**)
$\tan\theta$ is an odd function.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise D, Question 1

Question:

Express the following as trigonometric ratios of either 30°, 45° or 60°, and hence find their exact values.

(a) $\sin 135°$

(b) $\sin(-60°)$

(c) $\sin 330°$

(d) $\sin 420°$

(e) $\sin(-300°)$

(f) $\cos 120°$

(g) $\cos 300°$

(h) $\cos 225°$

(i) $\cos(-210°)$

(j) $\cos 495°$

(k) $\tan 135°$

(l) $\tan(-225°)$

(m) $\tan 210°$

(n) $\tan 300°$

(o) $\tan(-120°)$

Solution:

(a) $\sin 135° = +\sin 45°$ (135° is in the second quadrant at 45° to the horizontal)
So $\sin 135° = \frac{\sqrt{2}}{2}$

(b) $\sin(-60)° = -\sin 60°$ ($-60°$ is in the fourth quadrant at 60° to the horizontal)
So $\sin\left(-60\right)° = -\frac{\sqrt{3}}{2}$

(c) $\sin 330° = -\sin 30°$ (330° is in the fourth quadrant at 30° to the horizontal)
So $\sin 330° = -\frac{1}{2}$

(d) $\sin 420° = +\sin 60°$ (on second revolution)

So $\sin 420° = \frac{\sqrt{3}}{2}$

(e) $\sin(-300)° = +\sin 60°$ ($-300°$ is in the first quadrant at 60° to the horizontal)

So $\sin\left(-300\right)° = \frac{\sqrt{3}}{2}$

(f) $\cos 120° = -\cos 60°$ (120° is in the second quadrant at 60° to the horizontal)

So $\cos 120° = -\frac{1}{2}$

(g) $\cos 300° = +\cos 60°$ (300° is in the fourth quadrant at 60° to the horizontal)

So $\cos 300° = \frac{1}{2}$

(h) $\cos 225° = -\cos 45°$ (225° is in the third quadrant at 45° to the horizontal)

So $\cos 225° = -\frac{\sqrt{2}}{2}$

(i) $\cos(-210°) = -\cos 30°$ ($-210°$ is in the second quadrant at 30° to the horizontal)

So $\cos\left(-210°\right) = -\frac{\sqrt{3}}{2}$

(j) $\cos 495° = -\cos 45°$ (495° is in the second quadrant at 45° to the horizontal)

So $\cos 495° = -\frac{\sqrt{2}}{2}$

(k) $\tan 135° = -\tan 45°$ (135° is in the second quadrant at 45° to the horizontal)
So $\tan 135° = -1$

(l) $\tan(-225°) = -\tan 45°$ ($-225°$ is in the second quadrant at 45° to the horizontal)
So $\tan(-225°) = -1$

(m) $\tan 210° = +\tan 30°$ (210° is in the third quadrant at 30° to the horizontal)

So $\tan 210° = \frac{\sqrt{3}}{3}$

(n) $\tan 300° = -\tan 60°$ (300° is in the fourth quadrant at 60° to the horizontal)
So $\tan 300° = -\sqrt{3}$

(o) $\tan(-120°) = +\tan 60°$ ($-120°$ is in the third quadrant at 60° to the horizontal)
So $\tan(-120°) = \sqrt{3}$

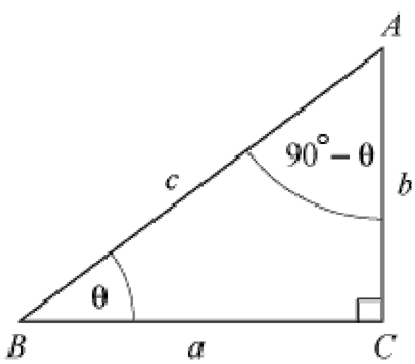
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise D, Question 2

Question:

In Section 8.3 you saw that $\sin 30° = \cos 60°$, $\cos 30° = \sin 60°$, and $\tan 60° = \frac{1}{\tan 30°}$. These are particular examples of the general results: $\sin(90° - \theta) = \cos\theta$, and $\cos(90° - \theta) = \sin\theta$, and $\tan(90° - \theta) = \frac{1}{\tan\theta}$, where the angle θ is measured in degrees. Use a right-angled triangle *ABC* to verify these results for the case when θ is acute.

Solution:



With $\angle B = \theta$, $\angle A = (90° - \theta)$

$\sin\theta = \frac{b}{c}$, $\cos\left(90° - \theta\right) = \frac{b}{c}$

So $\cos(90° - \theta) = \sin\theta$

$\cos\theta = \frac{a}{c}$, $\sin\left(90° - \theta\right) = \frac{a}{c}$

So $\sin(90° - \theta) = \cos\theta$

$\tan\theta = \frac{b}{a}$, $\tan\left(90° - \theta\right) = \frac{a}{b} = \frac{1}{\left(\frac{b}{a}\right)} = \frac{1}{\tan\theta}$

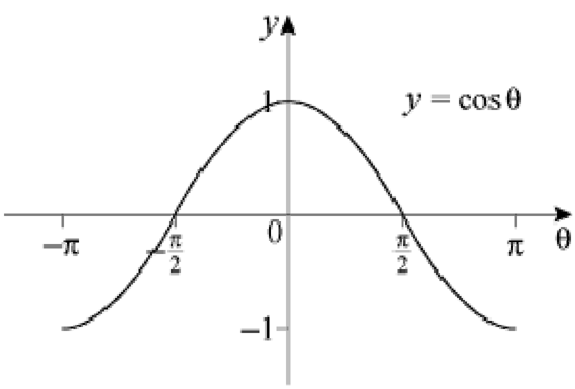
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise E, Question 1

Question:

Sketch the graph of $y = \cos\ \theta$ in the interval $-\pi \leq \theta \leq \pi$.

Solution:



.

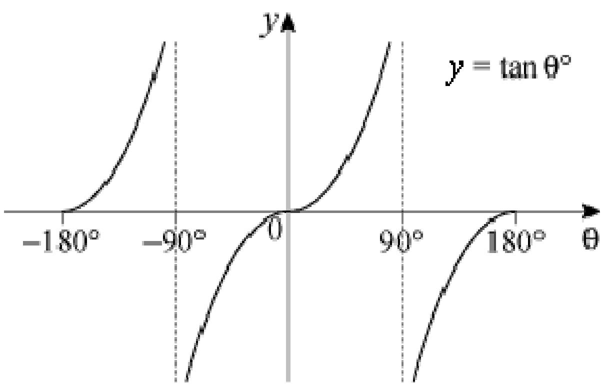
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise E, Question 2

Question:

Sketch the graph of $y = \tan\ \theta°$ in the interval $-180 \leq \theta \leq 180$.

Solution:



.

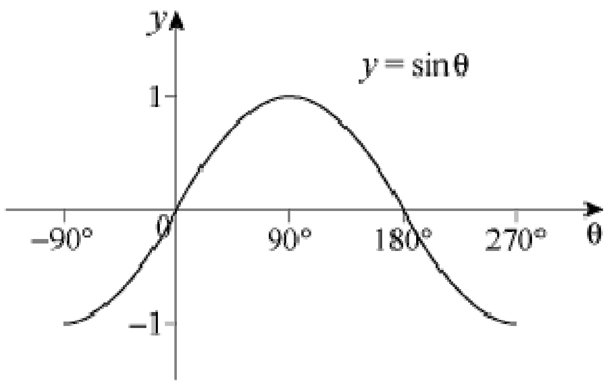
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise E, Question 3

Question:

Sketch the graph of $y = \sin \theta°$ in the interval $-90 \leq \theta \leq 270$.

Solution:



.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise F, Question 1

Question:

Write down (i) the maximum value, and (ii) the minimum value, of the following expressions, and in each case give the smallest positive (or zero) value of x for which it occurs.

(a) $\cos x°$

(b) $4 \sin x°$

(c) $\cos (- x) °$

(d) $3 + \sin x°$

(e) $- \sin x°$

(f) $\sin 3x°$

Solution:

(a) (i) Maximum value of $\cos x° = 1$, occurs when $x = 0$.
(ii) Minimum value is -1, occurs when $x = 180$.

(b) (i) Maximum value of $\sin x° = 1$, so maximum value of $4 \sin x° = 4$, occurs when $x = 90$.
(ii) Minimum value of $4 \sin x°$ is -4, occurs when $x = 270$.

(c) The graph of $\cos (- x) °$ is a reflection of the graph of $\cos x°$ in the y-axis.
This is the same curve; $\cos (- x) ° = \cos x°$.
(i) Maximum value of $\cos (- x) ° = 1$, occurs when $x = 0$.
(ii) Minimum value of $\cos (- x) ° = - 1$, occurs when $x = 180$.

(d) The graph of $3 + \sin x°$ is the graph of $\sin x°$ translated by +3 vertically.
(i) Maximum = 4, when $x = 90$.
(ii) Minimum = 2, when $x = 270$.

(e) The graph of $- \sin x°$ is the reflection of the graph of $\sin x°$ in the x-axis.
(i) Maximum = 1, when $x = 270$.
(ii) Minimum = -1, when $x = 90$.

(f) The graph of $\sin 3x°$ is the graph of $\sin x°$ stretched by $\frac{1}{3}$ in the x direction.

(i) Maximum = 1, when $x = 30$.
(ii) Minimum = -1, when $x = 90$.

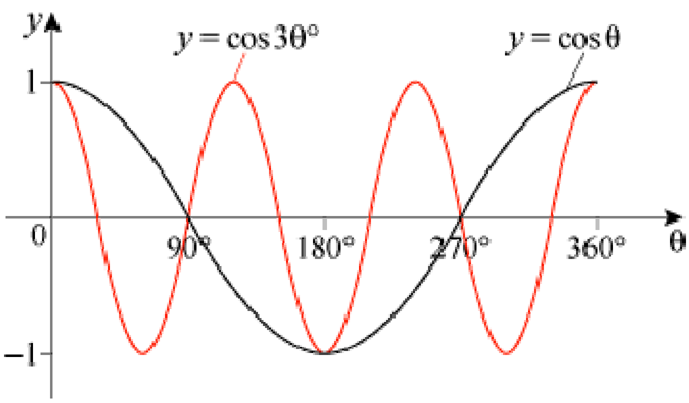
Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise F, Question 2

Question:

Sketch, on the same set of axes, in the interval $0 \leq \theta \leq 360°$, the graphs of $\cos\theta$ and $\cos 3\theta$.

Solution:



.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise F, Question 3

Question:

Sketch, on separate axes, the graphs of the following, in the interval $0 \leq \theta \leq 360°$. Give the coordinates of points of intersection with the axes, and of maximum and minimum points where appropriate.

(a) $y = -\cos \theta$

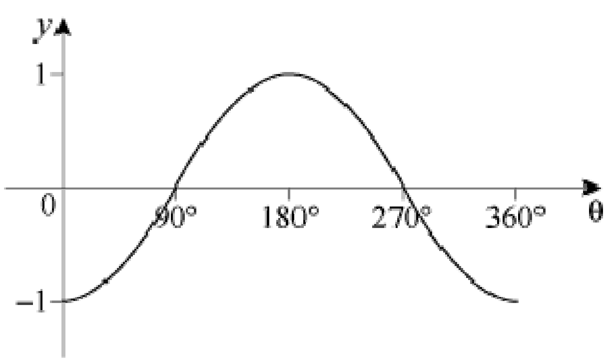
(b) $y = \frac{1}{3} \sin \theta$

(c) $y = \sin \frac{1}{3}\theta$

(d) $y = \tan (\theta - 45°)$

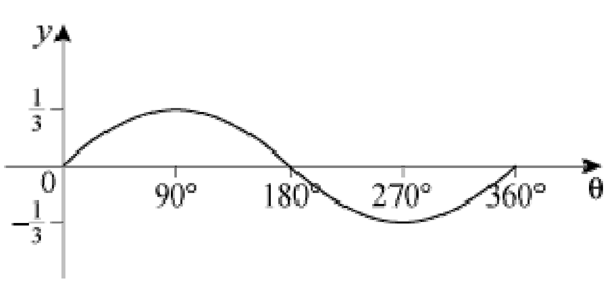
Solution:

(a) The graph of $y = -\cos \theta$ is the graph of $y = \cos \theta$ reflected in the θ-axis.



Meets θ-axis at (90°, 0), (270°, 0)
Meets y-axis at (0°, −1)
Maximum at (180°, 1)
Minima at (0°, −1) and (360°, −1)

(b) The graph of $y = \frac{1}{3} \sin \theta$ is the graph of $y = \sin \theta$ stretched by scale factor $\frac{1}{3}$ in y direction.

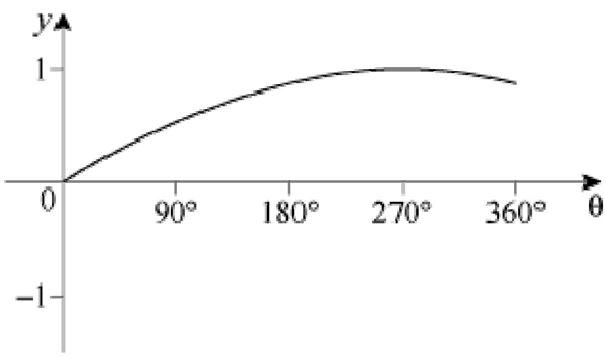


Meets θ-axis at (0°, 0), (180°, 0), (360°, 0)
Meets y-axis at (0°, 0)

Maximum at $\left(90°, \frac{1}{3}\right)$

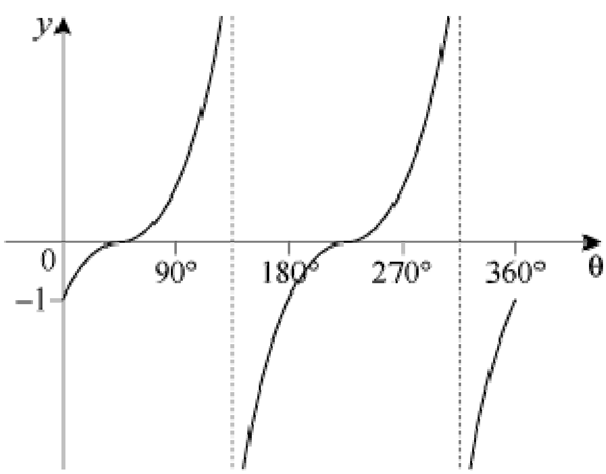
Minimum at $\left(270°, -\frac{1}{3}\right)$

(c) The graph of $y = \sin \frac{1}{3}\theta$ is the graph of $y = \sin \theta$ stretched by scale factor 3 in θ direction.



Only meets axes at origin
Maximum at (270°, 1)

(d) The graph of $y = \tan(\theta - 45°)$ is the graph of $\tan \theta$ translated by 45° to the right.



Meets θ-axis at (45°, 0), (225°, 0)
Meets y-axis at (0°, − 1)
(Asymptotes at $\theta = 135°$ and $\theta = 315°$)

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise F, Question 4

Question:

Sketch, on separate axes, the graphs of the following, in the interval $-180 \leq \theta \leq 180$. Give the coordinates of points of intersection with the axes, and of maximum and minimum points where appropriate.

(a) $y = -2 \sin \theta°$

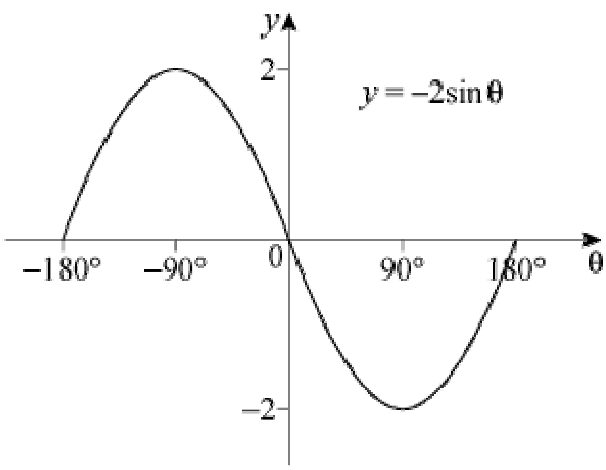
(b) $y = \tan (\theta + 180)°$

(c) $y = \cos 4\theta°$

(d) $y = \sin (-\theta)°$

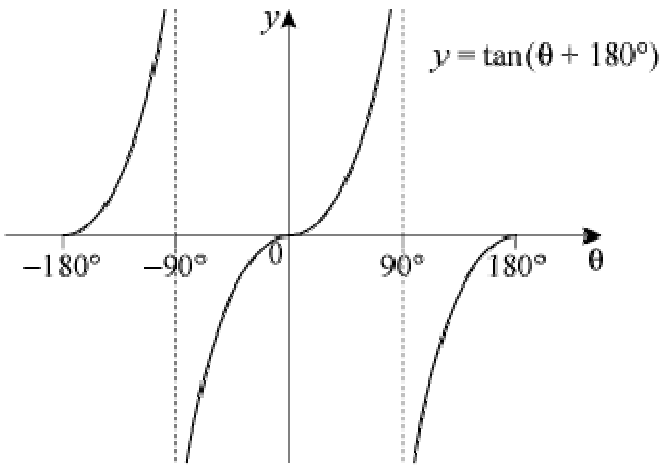
Solution:

(a) This is the graph of $y = \sin \theta°$ stretched by scale factor -2 in the y direction
(i.e. reflected in the θ-axis and scaled by 2 in the y direction).



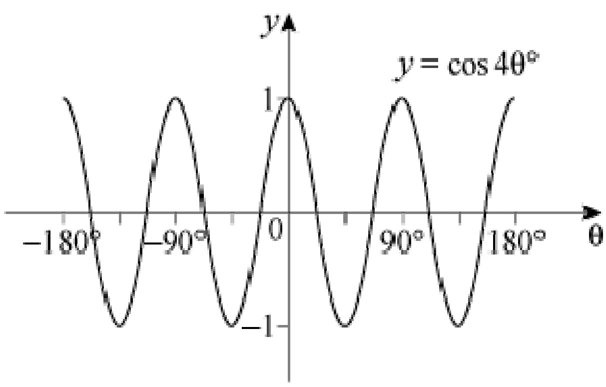
Meets θ-axis at $(-180°, 0)$, $(0°, 0)$, $(180°, 0)$
Maximum at $(-90°, 2)$
Minimum at $(90°, -2)$

(b) This is the graph of $y = \tan \theta°$ translated by 180° to the left.



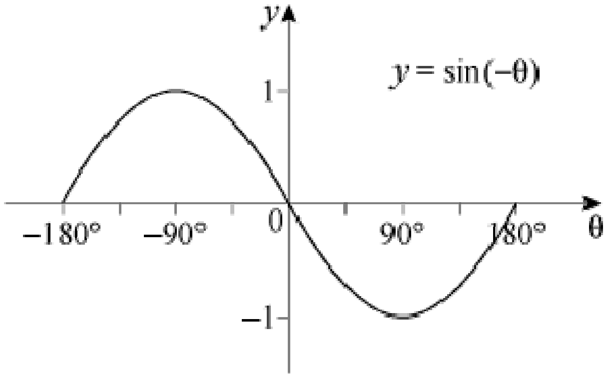
As tan θ ° has a period of 180°
tan (θ + 180) ° = tan θ °
Meets θ-axis at (− 180°, 0), (0°, 0), (180°, 0)

(c) This is the graph of y = cos θ ° stretched by scale factor $\frac{1}{4}$ horizontally.



Meets θ-axis at $\left(-157\frac{1}{2}°, 0 \right)$, $\left(-112\frac{1}{2}°, 0 \right)$, $\left(-67\frac{1}{2}°, 0 \right)$, $\left(-22\frac{1}{2}°, 0 \right)$, $\left(22\frac{1}{2}°, 0 \right)$, $\left(67\frac{1}{2}°, 0 \right)$, $\left(112\frac{1}{2}°, 0 \right)$, $\left(157\frac{1}{2}°, 0 \right)$
Meets y-axis at (0°, 1)
Maxima at (− 180°, 1) , (− 90°, 1), (0°, 1), (90°, 1), (180°, 1)
Minima at (− 135°, − 1), (− 45°, − 1), (45°, − 1), (135°, − 1)

(d) This is the graph of y = sin θ ° reflected in the y-axis. (This is the same as y = − sin θ ° .)



Meets θ-axis at $(-180°, 0)$, $(0°, 0)$, $(180°, 0)$
Maximum at $(-90°, 1)$
Minimum at $(90°, -1)$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise F, Question 5

Question:

In this question θ is measured in radians. Sketch, on separate axes, the graphs of the following in the interval $-2\pi \leq \theta \leq 2\pi$. In each case give the periodicity of the function.

(a) $y = \sin \frac{1}{2}\theta$

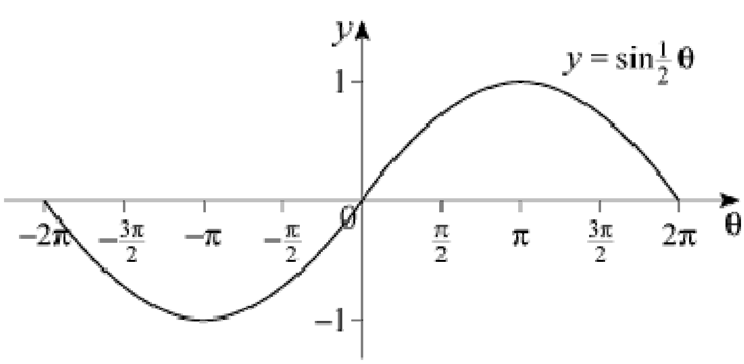
(b) $y = -\frac{1}{2}\cos \theta$

(c) $y = \tan\left(\theta - \frac{\pi}{2}\right)$

(d) $y = \tan 2\theta$

Solution:

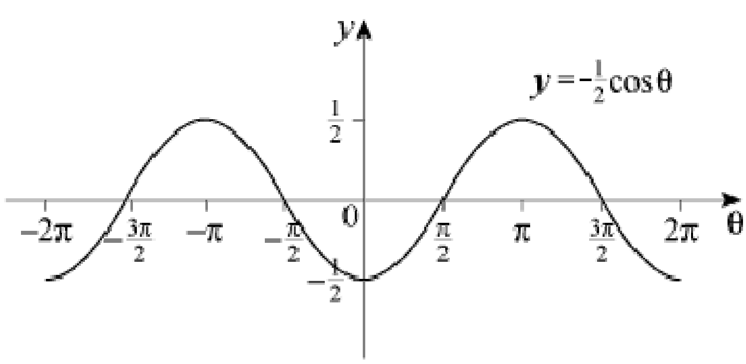
(a) This is the graph of $y = \sin \theta$ stretched by scale factor 2 horizontally.
Period $= 4\pi$



(b) This is the graph of $y = \cos \theta$ stretched by scale factor $-\frac{1}{2}$ vertically.

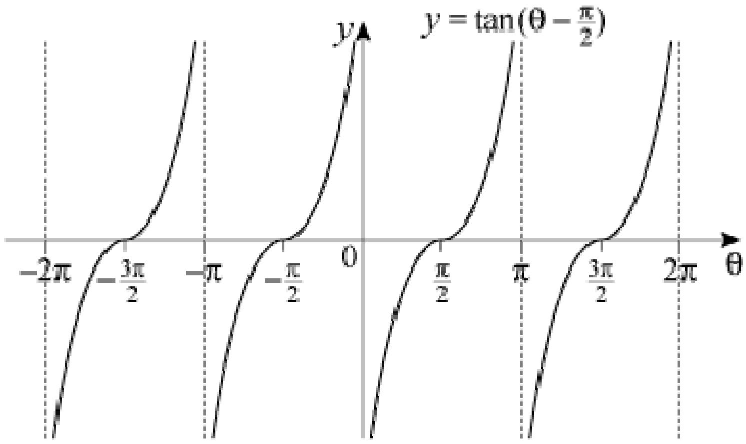
(Equivalent to reflection, in θ-axis and stretching vertically by $+\frac{1}{2}$.)

Period $= 2\pi$



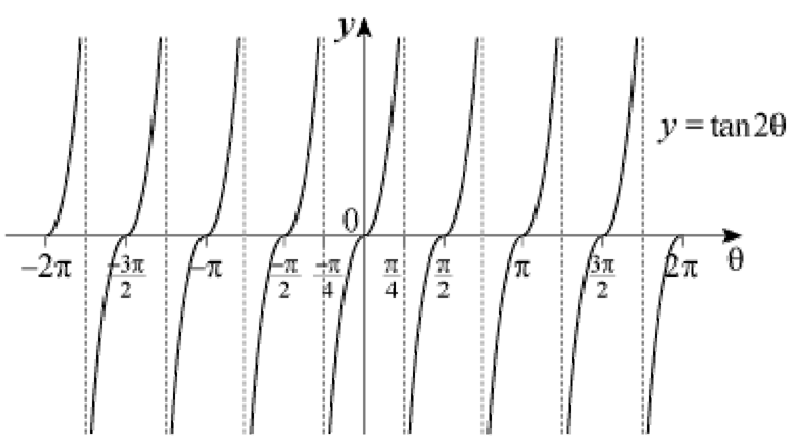
(c) This is the graph of $y = \tan\ \theta$ translated by $\frac{\pi}{2}$ to the right.

Period = π



(d) This is the graph of $y = \tan\ \theta$ stretched by scale factor $\frac{1}{2}$ horizontally.

Period $= \frac{\pi}{2}$



Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise F, Question 6

Question:

(a) By considering the graphs of the functions, or otherwise, verify that:
(i) $\cos\theta = \cos(-\theta)$

(ii) $\sin\theta = -\sin(-\theta)$

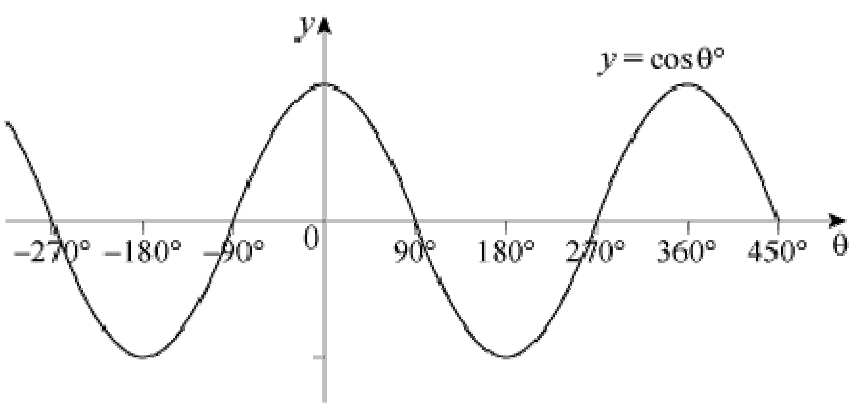
(iii) $\sin(\theta - 90°) = -\cos\theta$

(b) Use the results in (a) (ii) and (iii) to show that $\sin(90° - \theta) = \cos\theta$.

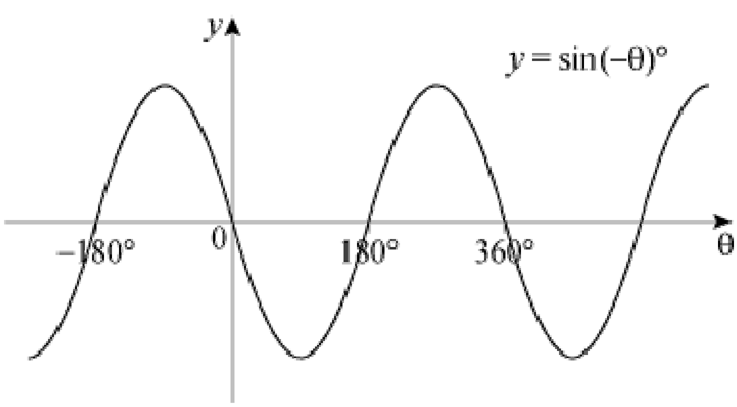
(c) In Example 11 you saw that $\cos(\theta - 90°) = \sin\theta$. Use this result with part (a) (i) to show that $\cos(90° - \theta) = \sin\theta$.

Solution:

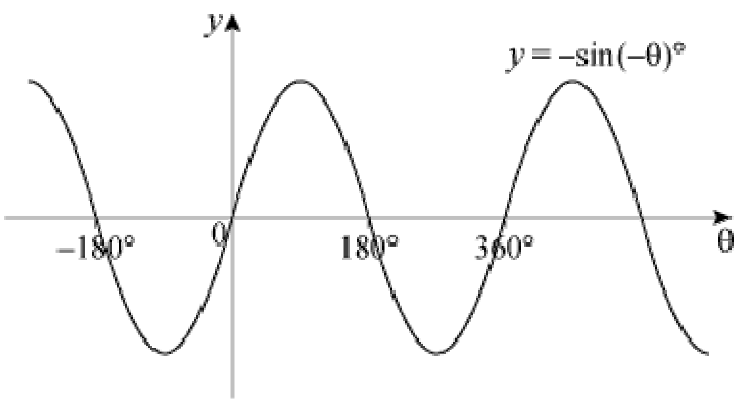
(a) (i) $y = \cos(-\theta)$ is a reflection of $y = \cos\theta$ in the y-axis, which is the same curve, so $\cos\theta = \cos(-\theta)$.



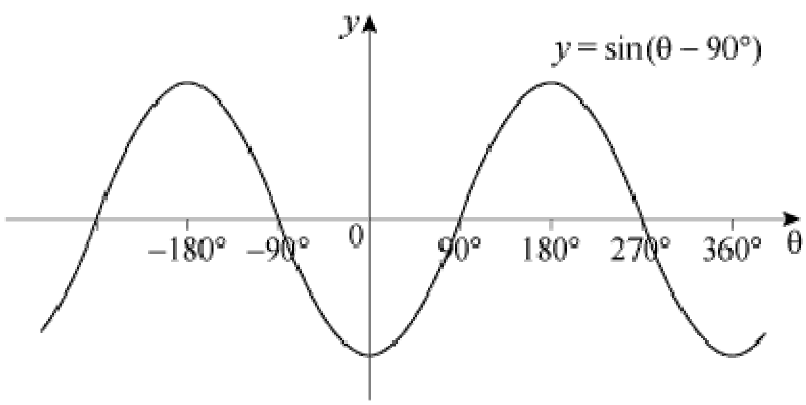
(ii) $y = \sin(-\theta)$ is a reflection of $y = \sin\theta$ in the y-axis



$y = -\sin(-\theta)$ is a reflection of $y = \sin(-\theta)$ in the θ-axis, which is the graph of $y = \sin\theta$, so $-\sin(-\theta) = \sin\theta$.



(iii) $y = \sin(\theta - 90°)$ is the graph of $y = \sin\theta$ translated by 90° to the right, which is the graph of $y = -\cos\theta$, so $\sin(\theta - 90°) = -\cos\theta$.



(b) Using (a) (ii), $\sin(90° - \theta) = -\sin[-(90° - \theta)] = -\sin(\theta - 90°)$
Using (a) (iii), $-\sin(\theta - 90°) = -(-\cos\theta) = \cos\theta$
So $\sin(90° - \theta) = \cos\theta$.

(c) Using (a)(i), $\cos(90° - \theta) = \cos(\theta - 90°) = \sin\theta$, using Example 11.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 1

Question:

Write each of the following as a trigonometric ratio of an acute angle:

(a) $\cos 237°$

(b) $\sin 312°$

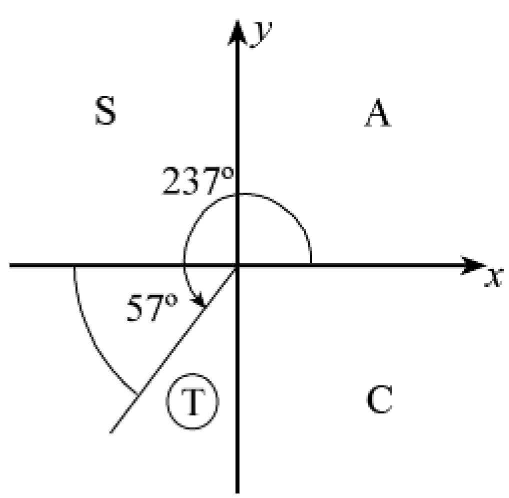
(c) $\tan 190°$

(d) $\sin 2.3^c$

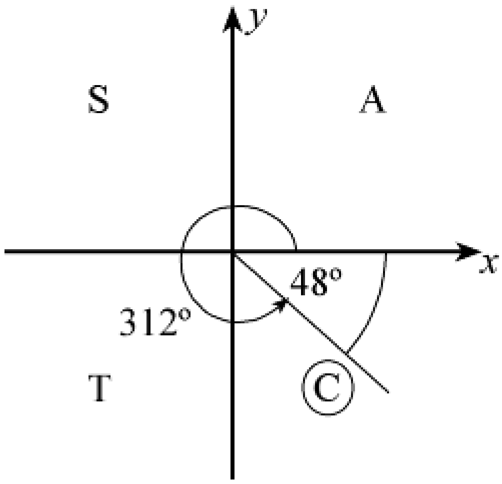
(e) $\cos \left(-\frac{\pi}{15} \right)$

Solution:

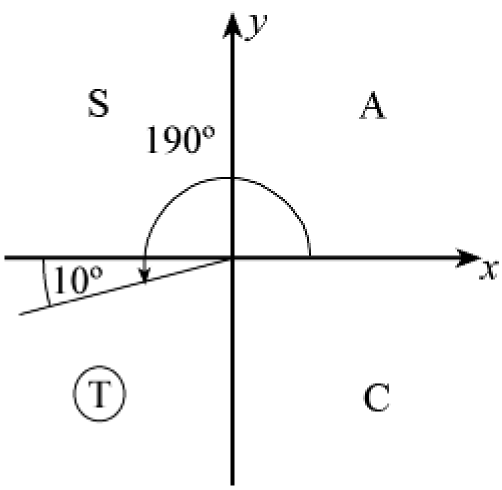
(a) 237° is in the third quadrant so $\cos 237°$ is $-$ ve.
The angle made with the horizontal is 57°.
So $\cos 237° = -\cos 57°$



(b) 312° is in the fourth quadrant so $\sin 312°$ is $-$ ve.
The angle to the horizontal is 48°.
So $\sin 312° = -\sin 48°$



(c) 190° is in the third quadrant so tan 190° is +ve.
The angle to the horizontal is 10°.
So $\tan 190° = + \tan 10°$



(d) 2.3 radians (131.78 … °) is in the second quadrant so $\sin 2.3^c$ is +ve.
The angle to the horizontal is $(\pi - 2.3)$ radians $= 0.84$ radians (2 s.f.).
So $\sin 2.3^c = + \sin 0.84^c$

(e) $-\left(\frac{\pi}{15}\right)$ is in the fourth quadrant so $\cos\left(-\frac{\pi}{15}\right)$ is +ve.

The angle to the horizontal is $\frac{\pi}{15}$.

So $\cos\left(-\frac{\pi}{15}\right) = + \cos\left(\frac{\pi}{15}\right)$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 2

Question:

Without using your calculator, work out the values of:

(a) $\cos 270°$

(b) $\sin 225°$

(c) $\cos 180°$

(d) $\tan 240°$

(e) $\tan 135°$

(f) $\cos 690°$

(g) $\sin \frac{5\pi}{3}$

(h) $\cos \left(-\frac{2\pi}{3} \right)$

(i) $\tan 2\pi$

(j) $\sin \left(-\frac{7\pi}{6} \right)$

Solution:

(a) $\sin 270° = -1$ (see graph of $y = \sin \theta$)

(b) $\sin 225° = \sin \left(180 + 45 \right)° = -\sin 45° = -\frac{\sqrt{2}}{2}$

(c) $\cos 180° = -1$ (see graph of $y = \cos \theta$)

(d) $\tan 240° = \tan (180 + 60)° = +\tan 60°$ (third quadrant)
So $\tan 240° = +\sqrt{3}$

(e) $\tan 135° = -\tan 45°$ (second quadrant)
So $\tan 135° = -1$

(f) $\cos 690° = \cos (360 + 330)° = \cos 330° = +\cos 30°$ (fourth quadrant)
So $\cos 690° = +\frac{\sqrt{3}}{2}$

(g) $\sin \frac{5\pi}{3} = -\sin \frac{\pi}{3}$ (fourth quadrant)

So $\sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

(h) $\cos \left(-\frac{2\pi}{3} \right) = -\cos \frac{\pi}{3}$ (third quadrant)

So $\cos \left(-\frac{2\pi}{3} \right) = -\frac{1}{2}$

(i) $\tan 2\pi = 0$ (see graph of $y = \tan \theta$)

(j) $\sin \left(-\frac{7\pi}{6} \right) = +\sin \left(\frac{\pi}{6} \right)$ (second quadrant)

So $\sin \left(-\frac{7\pi}{6} \right) = +\frac{1}{2}$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 3

Question:

Describe geometrically the transformations which map:

(a) The graph of $y = \tan x°$ onto the graph of $\tan \frac{1}{2}x°$.

(b) The graph of $y = \tan \frac{1}{2}x°$ onto the graph of $3 + \tan \frac{1}{2}x°$.

(c) The graph of $y = \cos x°$ onto the graph of $-\cos x°$.

(d) The graph of $y = \sin (x - 10)°$ onto the graph of $\sin (x + 10)°$.

Solution:

(a) A stretch of scale factor 2 in the x direction.

(b) A translation of $+3$ in the y direction.

(c) A reflection in the x-axis

(d) A translation of $+20$ in the negative x direction (i.e. 20 to the left).

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 4

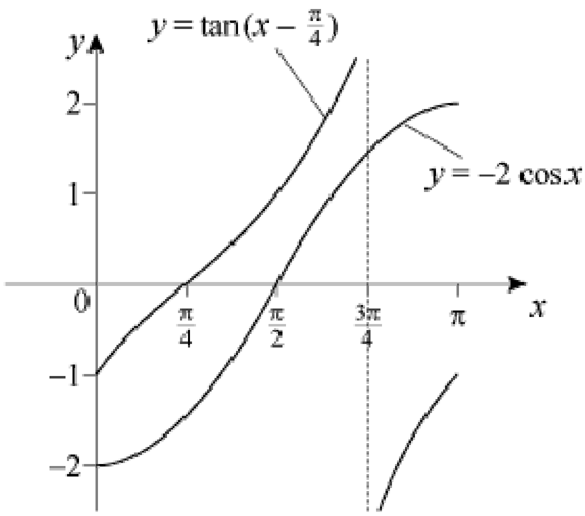
Question:

(a) Sketch on the same set of axes, in the interval $0 \leq x \leq \pi$, the graphs of $y = \tan\left(x - \frac{1}{4}\pi\right)$ and $y = -2\cos x$, showing the coordinates of points of intersection with the axes.

(b) Deduce the number of solutions of the equation $\tan\left(x - \frac{1}{4}\pi\right) + 2\cos x = 0$, in the interval $0 \leq x \leq \pi$.

Solution:

4 (a)



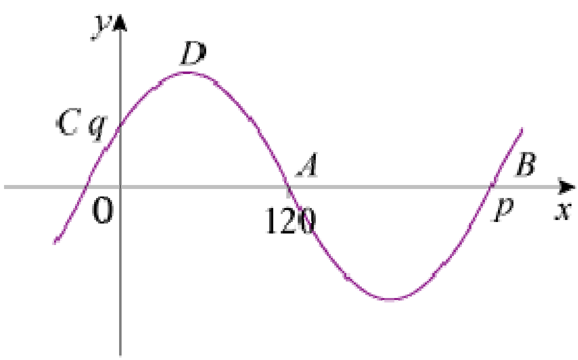
(b) There are no solutions of $\tan\left(x - \frac{\pi}{4}\right) + 2\cos x = 0$ in the interval $0 \leq x \leq \pi$, since $y = \tan\left(x - \frac{\pi}{4}\right)$ and $y = -2\cos x$ do not intersect in the interval.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 5

Question:

The diagram shows part of the graph of $y = f(x)$. It crosses the x-axis at $A(120, 0)$ and $B(p, 0)$. It crosses the y-axis at $C(0, q)$ and has a maximum value at D, as shown.



Given that $f(x) = \sin(x + k)°$, where $k > 0$, write down:

(a) the value of p

(b) the coordinates of D

(c) the smallest value of k

(d) the value of q

Solution:

(a) As it is the graph of $y = \sin x°$ translated, the gap between A and B is 180, so $p = 300$.

(b) The difference in the x-coordinates of D and A is 90, so the x-coordinate of D is 30.
The maximum value of y is 1, so $D = (30, 1)$.

(c) For the graph of $y = \sin x°$, the first positive intersection with the x-axis would occur at 180. The point A is at 120 and so the curve has been translated by 60 to the left.
$k = 60$

(d) The equation of the curve is $y = \sin(x + 60)°$.
When $x = 0$, $y = \sin 60° = \frac{\sqrt{3}}{2}$, so $q = \frac{\sqrt{3}}{2}$.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 6

Question:

Consider the function $f(x) = \sin px$, $p \in \mathbb{R}$, $0 \leq x \leq 2\pi$.

The closest point to the origin that the graph of f(x) crosses the x-axis has x-coordinate $\frac{\pi}{5}$.

(a) Sketch the graph of f(x).

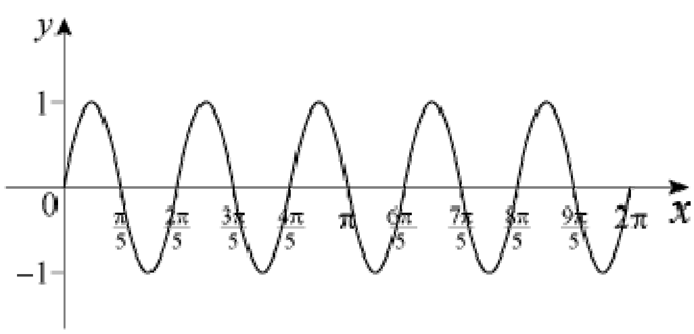
(b) Write down the period of f(x).

(c) Find the value of p.

Solution:

(a) The graph is that of $y = \sin x$ stretched in the x direction.

Each 'half-wave' has interval $\frac{\pi}{5}$.



(b) The period is a 'wavelength', i.e. $\frac{2\pi}{5}$.

(c) The stretch factor is $\frac{1}{p}$.

As 2π has been reduced to $\frac{2\pi}{5}$, 2π has been multiplied by $\frac{1}{5}$ which is $\frac{1}{p}$ $\Rightarrow$ $p = 5$.

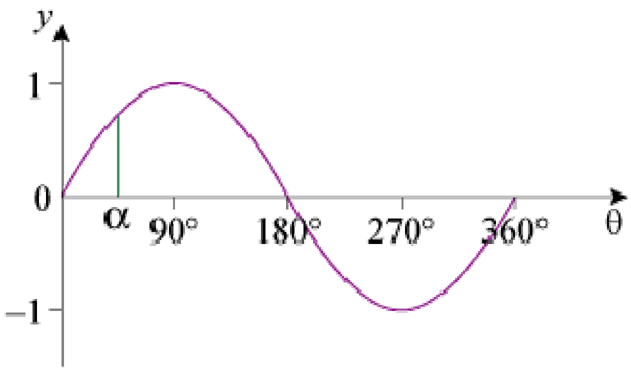
The curve is $y = \sin 5x$, there are 5 'waves' in 0 to 2π.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 7

Question:

The graph below shows $y = \sin\theta$, $0 \leq \theta \leq 360°$, with one value of θ $(\theta = \alpha°)$ marked on the axis.

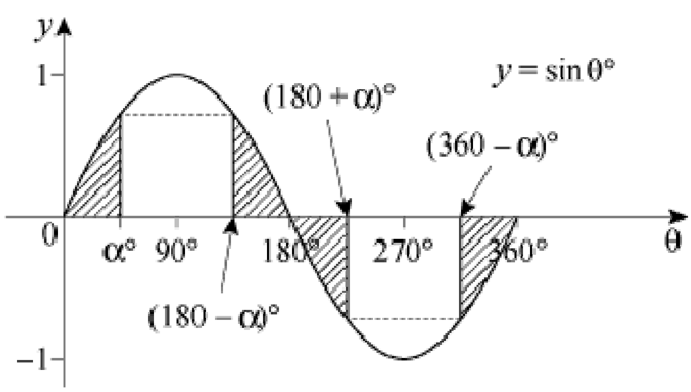


(a) Copy the graph and mark on the θ-axis the positions of $(180 - \alpha)°$, $(180 + \alpha)°$, and $(360 - \alpha)°$.

(b) Establish the result $\sin\alpha° = \sin(180 - \alpha)° = -\sin(180 + \alpha)° = -\sin(360 - \alpha)°$.

Solution:

(a) The four shaded regions are congruent.



(b) $\sin\alpha°$ and $\sin(180 - \alpha)°$ have the same y value (call it k).
So $\sin\alpha° = \sin(180 - \alpha)°$
$\sin(180 + \alpha)°$ and $\sin(360 - \alpha)°$ have the same y value, which will be $-k$.
So $\sin\alpha° = \sin(180 - \alpha)° = -\sin(180 + \alpha)° = -\sin(360 - \alpha)°$

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Graphics of trigonometric functions
Exercise G, Question 8

Question:

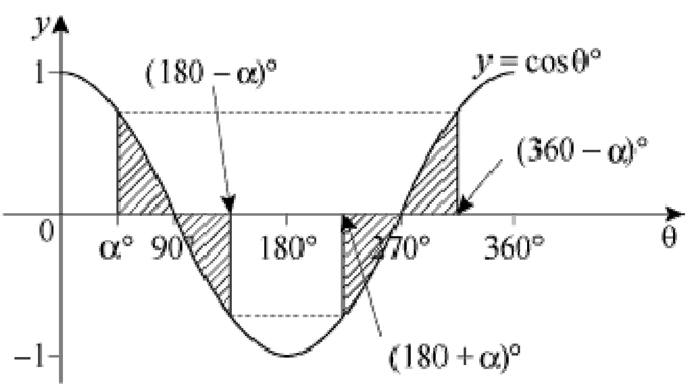
(a) Sketch on separate axes the graphs of $y = \cos \theta$ $(0 \leq \theta \leq 360°)$ and $y = \tan \theta$ $(0 \leq \theta \leq 360°)$, and on each θ-axis mark the point $(\alpha°, 0)$ as in question 7.

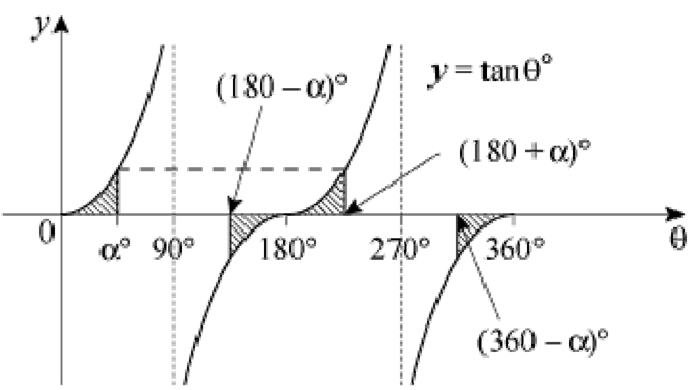
(b) Verify that:
(i) $\cos \alpha° = -\cos (180 - \alpha)° = -\cos (180 + \alpha)° = \cos (360 - \alpha)°$.

(ii) $\tan \alpha° = -\tan (180 - \alpha)° = -\tan (180 + \alpha)° = -\tan (360 - \alpha)°$.

Solution:

(a)





(b) (i) From the graph of $y = \cos \theta°$, which shows four congruent shaded regions, if the y value at $\alpha°$ is k, then y at $(180 - \alpha)°$ is $-k$, y at $(180 + \alpha)°$ is $-k$ and y at $(360 - \alpha)°$ is $+k$.
So $\cos \alpha° = -\cos (180 - \alpha)° = -\cos (180 + \alpha)° = \cos (360 - \alpha)°$

(ii) From the graph of $y = \tan \theta°$, if the y value at $\alpha°$ is k, then at $(180 - \alpha)°$ it is $-k$, at $(180 + \alpha)°$ it is $+k$ and at $(360 - \alpha)°$ it is $-k$.
So $\tan \alpha° = -\tan (180 - \alpha)° = +\tan (180 + \alpha)° = -\tan (360 - \alpha)°$