Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise A, Question 1

Question:

Find the mid-point of the line joining these pairs of points:

(a) $(4, 2)$, $(6, 8)$

(b) $(0, 6)$, $(12, 2)$

(c) $(2, 2)$, $(-4, 6)$

(d) $(-6, 4)$, $(6, -4)$

(e) $(-5, 3)$, $(7, 5)$

(f) $(7, -4)$, $(-3, 6)$

(g) $(-5, -5)$, $(-11, 8)$

(h) $(6a, 4b)$, $(2a, -4b)$

(i) $(2p, -q)$, $(4p, 5q)$

(j) $(-2s, -7t)$, $(5s, t)$

(k) $(-4u, 0)$, $(3u, -2v)$

(l) $(a+b, 2a-b)$, $(3a-b, -b)$

(m) $(4\sqrt{2}, 1)$, $(2\sqrt{2}, 7)$

(n) $(-\sqrt{3}, 3\sqrt{5})$, $(5\sqrt{3}, 2\sqrt{5})$

(o) $(\sqrt{2}-\sqrt{3}, 3\sqrt{2}+4\sqrt{3})$, $(3\sqrt{2}+\sqrt{3}, -\sqrt{2}+2\sqrt{3})$

Solution:

(a) $(x_1, y_1) = (4, 2)$, $(x_2, y_2) = (6, 8)$

So $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{4+6}{2}, \frac{2+8}{2}\right) = \left(\frac{10}{2}, \frac{10}{2}\right) = (5, 5)$

(b) $(x_1, y_1) = (0, 6)$, $(x_2, y_2) = (12, 2)$

So $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{0+12}{2}, \frac{6+2}{2}\right) = \left(\frac{12}{2}, \frac{8}{2}\right) = (6, 4)$

(c) $(x_1, y_1) = (2, 2)$, $(x_2, y_2) = (-4, 6)$

So $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+(-4)}{2}, \frac{2+6}{2}\right) = \left(\frac{-2}{2}, \frac{8}{2}\right) = (-1, 4)$

(d) $(x_1, y_1) = (-6, 4)$, $(x_2, y_2) = (6, -4)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-6+6}{2}, \frac{4+(-4)}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$

(e) $(x_1, y_1) = (-5, 3)$, $(x_2, y_2) = (7, 5)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+7}{2}, \frac{3+5}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

(f) $(x_1, y_1) = (7, -4)$, $(x_2, y_2) = (-3, 6)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{7+(-3)}{2}, \frac{-4+6}{2}\right) = \left(\frac{4}{2}, \frac{2}{2}\right) = (2, 1)$

(g) $(x_1, y_1) = (-5, -5)$, $(x_2, y_2) = (-11, 8)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5+(-11)}{2}, \frac{-5+8}{2}\right) = \left(\frac{-16}{2}, \frac{3}{2}\right) = \left(-8, \frac{3}{2}\right)$

(h) $(x_1, y_1) = (6a, 4b)$, $(x_2, y_2) = (2a, -4b)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6a+2a}{2}, \frac{4b+(-4b)}{2}\right) = \left(\frac{8a}{2}, \frac{0}{2}\right) = (4a, 0)$

(i) $(x_1, y_1) = (2p, -q)$, $(x_2, y_2) = (4p, 5q)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2p+4p}{2}, \frac{-q+5q}{2}\right) = \left(\frac{6p}{2}, \frac{4q}{2}\right) = (3p, 2q)$

(j) $(x_1, y_1) = (-2s, -7t)$, $(x_2, y_2) = (5s, t)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2s+5s}{2}, \frac{-7t+t}{2}\right) = \left(\frac{3s}{2}, \frac{-6t}{2}\right) = \left(\frac{3s}{2}, -3t\right)$

(k) $(x_1, y_1) = (-4u, 0)$, $(x_2, y_2) = (3u, -2v)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4u+3u}{2}, \frac{0+(-2v)}{2}\right) = \left(\frac{-u}{2}, -v\right)$

(l) $(x_1, y_1) = (a+b, 2a-b)$, $(x_2, y_2) = (3a-b, -b)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{a+b+3a-b}{2}, \frac{2a-b+(-b)}{2}\right) = \left(\frac{4a}{2}, \frac{2a-2b}{2}\right) = (2a, a-b)$

(m) $(x_1, y_1) = (4\sqrt{2}, 1)$, $(x_2, y_2) = (2\sqrt{2}, 7)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4\sqrt{2}+2\sqrt{2}}{2}, \frac{1+7}{2}\right) = \left(\frac{6\sqrt{2}}{2}, \frac{8}{2}\right) = (3\sqrt{2}, 4)$

(n) $(x_1, y_1) = (-\sqrt{3}, 3\sqrt{5})$, $(x_2, y_2) = (5\sqrt{3}, 2\sqrt{5})$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-\sqrt{3}+5\sqrt{3}}{2}, \frac{3\sqrt{5}+2\sqrt{5}}{2}\right) = \left(\frac{4\sqrt{3}}{2}, \frac{5\sqrt{5}}{2}\right) = \left(2\sqrt{3}, \frac{5\sqrt{5}}{2}\right)$

(o) $(x_1, y_1) = (\sqrt{2} - \sqrt{3}, 3\sqrt{2} + 4\sqrt{3})$, $(x_2, y_2) = (3\sqrt{2} + \sqrt{3}, -\sqrt{2} + 2\sqrt{3})$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} + (-\sqrt{2} + 2\sqrt{3})}{2}\right)$

$= \left(\frac{\sqrt{2} - \sqrt{3} + 3\sqrt{2} + \sqrt{3}}{2}, \frac{3\sqrt{2} + 4\sqrt{3} - \sqrt{2} + 2\sqrt{3}}{2}\right)$

$= \left(\frac{4\sqrt{2}}{2}, \frac{2\sqrt{2} + 6\sqrt{3}}{2}\right)$

$= (2\sqrt{2}, \sqrt{2} + 3\sqrt{3})$

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Coordinate geometry in the (x,y) plane
Exercise A, Question 2

Question:

The line *PQ* is a diameter of a circle, where *P* and *Q* are $(-4, 6)$ and $(7, 8)$ respectively. Find the coordinates of the centre of the circle.

Solution:

$(x_1, y_1) = (-4, 6)$, $(x_2, y_2) = (7, 8)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-4 + 7}{2}, \frac{6 + 8}{2} \right) = \left(\frac{3}{2}, \frac{14}{2} \right) = \left(\frac{3}{2}, 7 \right)$

The centre is $\left(\frac{3}{2}, 7 \right)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 3

Question:

The line *RS* is a diameter of a circle, where *R* and *S* are $\left(\frac{4a}{5} , - \frac{3b}{4} \right)$ and $\left(\frac{2a}{5} , \frac{5b}{4} \right)$ respectively. Find the coordinates of the centre of the circle.

Solution:

$$\left(x_1 , y_1 \right) = \left(\frac{4a}{5} , - \frac{3b}{4} \right) , \left(x_2 , y_2 \right) = \left(\frac{2a}{5} , \frac{5b}{4} \right)$$

So $\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{\frac{4a}{5} + \frac{2a}{5}}{2} , \frac{\frac{-3b}{4} + \frac{5b}{4}}{2} \right) = \left(\frac{\frac{6a}{5}}{2} , \frac{\frac{2b}{4}}{2} \right) = \left(\frac{3a}{5} , \frac{b}{4} \right)$

The centre is $\left(\frac{3a}{5} , \frac{b}{4} \right)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 4

Question:

The line *AB* is a diameter of a circle, where *A* and *B* are $(-3, -4)$ and $(6, 10)$ respectively. Show that the centre of the circle lies on the line $y = 2x$.

Solution:

$(x_1, y_1) = (-3, -4)$, $(x_2, y_2) = (6, 10)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-3 + 6}{2}, \frac{-4 + 10}{2} \right) = \left(\frac{3}{2}, \frac{6}{2} \right) = \left(\frac{3}{2}, 3 \right)$

Substitute $x = \frac{3}{2}$ into $y = 2x$:

$y = 2 \left(\frac{3}{2} \right) = 3$ ✓

So the centre is on the line $y = 2x$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 5

Question:

The line *JK* is a diameter of a circle, where *J* and *K* are $\left(\frac{3}{4}, \frac{4}{3}\right)$ and $\left(-\frac{1}{2}, 2\right)$ respectively. Show that the centre of the circle lies on the line $y = 8x + \frac{2}{3}$.

Solution:

$$\left(x_1, y_1\right) = \left(\frac{3}{4}, \frac{4}{3}\right), \left(x_2, y_2\right) = \left(-\frac{1}{2}, 2\right)$$

So $$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{\frac{3}{4}+\left(-\frac{1}{2}\right)}{2}, \frac{\frac{4}{3}+2}{2}\right) = \left(\frac{\frac{1}{4}}{2}, \frac{\frac{10}{3}}{2}\right) = \left(\frac{1}{8}, \frac{5}{3}\right)$$

Substitute $x = \frac{1}{8}$ into $y = 8x + \frac{2}{3}$:

$$y = 8\left(\frac{1}{8}\right) + \frac{2}{3} = 1 + \frac{2}{3} = \frac{5}{3} ✓$$

So the centre is on the line $y = 8x + \frac{2}{3}$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 6

Question:

The line AB is a diameter of a circle, where A and B are $(0, -2)$ and $(6, -5)$ respectively. Show that the centre of the circle lies on the line $x - 2y - 10 = 0$.

Solution:

$(x_1, y_1) = (0, -2)$, $(x_2, y_2) = (6, -5)$

So $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 6}{2}, \frac{-2 + (-5)}{2} \right) = \left(\frac{6}{2}, \frac{-7}{2} \right) = \left(3, \frac{-7}{2} \right)$

Substitute $x = 3$ and $y = \frac{-7}{2}$ into $x - 2y - 10 = 0$:

$\left(3 \right) - 2 \left(\frac{-7}{2} \right) - 10 = 3 + 7 - 10 = 0$ ✓

So the centre is on the line $x - 2y - 10 = 0$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 7

Question:

The line *FG* is a diameter of the circle centre $(6 , 1)$. Given *F* is $(2 , - 3)$, find the coordinates of *G*.

Solution:

$(x_1 , y_1) = (a , b) , (x_2 , y_2) = (2 , - 3)$
The centre is $(6 , 1)$ so

$$\left(\frac{a + 2}{2} , \frac{b + (- 3)}{2} \right) = \left(6 , 1 \right)$$

$$\frac{a + 2}{2} = 6$$

$a + 2 = 12$
$a = 10$

$$\frac{b + (- 3)}{2} = 1$$

$$\frac{b - 3}{2} = 1$$

$b - 3 = 2$
$b = 5$

The coordinates of *G* are $(10 , 5)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 8

Question:

The line *CD* is a diameter of the circle centre $(-2a, 5a)$. Given *D* has coordinates $(3a, -7a)$, find the coordinates of *C*.

Solution:

$(x_1, y_1) = (p, q)$, $(x_2, y_2) = (3a, -7a)$

The centre is $(-2a, 5a)$ so

$$\left(\frac{p+3a}{2}, \frac{q+(-7a)}{2} \right) = \left(-2a, 5a \right)$$

$$\frac{p+3a}{2} = -2a$$

$p + 3a = -4a$
$p = -7a$

$$\frac{q+(-7a)}{2} = 5a$$

$$\frac{q-7a}{2} = 5a$$

$q - 7a = 10a$
$q = 17a$

The coordinates of *C* are $(-7a, 17a)$.

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Coordinate geometry in the (x,y) plane
Exercise A, Question 9

Question:

The points $M(3, p)$ and $N(q, 4)$ lie on the circle centre $(5, 6)$. The line MN is a diameter of the circle. Find the value of p and q.

Solution:

$(x_1, y_1) = (3, p)$, $(x_2, y_2) = (q, 4)$ so

$$\left(\frac{3+q}{2}, \frac{p+4}{2} \right) = \left(5, 6 \right)$$

$\frac{3+q}{2} = 5$

$3 + q = 10$
$q = 7$

$\frac{p+4}{2} = 6$

$p + 4 = 12$
$p = 8$

So $p = 8$, $q = 7$

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Coordinate geometry in the (x,y) plane
Exercise A, Question 10

Question:

The points $V(-4, 2a)$ and $W(3b, -4)$ lie on the circle centre $(b, 2a)$. The line VW is a diameter of the circle. Find the value of a and b.

Solution:

$(x_1, y_1) = (-4, 2a)$, $(x_2, y_2) = (3b, -4)$ so

$$\left(\frac{-4+3b}{2}, \frac{2a-4}{2} \right) = \left(b, 2a \right)$$

$$\frac{-4+3b}{2} = b$$

$-4 + 3b = 2b$
$-4 = -b$
$b = 4$

$$\frac{2a-4}{2} = 2a$$

$2a - 4 = 4a$
$-4 = 2a$
$a = -2$

So $a = -2$, $b = 4$

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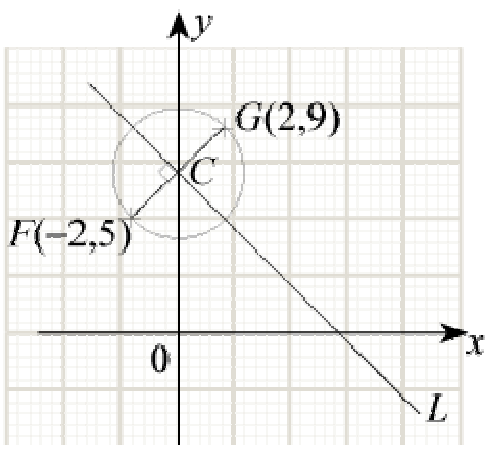
Coordinate geometry in the (x,y) plane
Exercise B, Question 1

Question:

The line *FG* is a diameter of the circle centre *C*, where *F* and *G* are $(-2, 5)$ and $(2, 9)$ respectively. The line *l* passes through *C* and is perpendicular to *FG*. Find the equation of *l*.

Solution:

(1)



(2) The gradient of *FG* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{2 - (-2)} = \frac{4}{4} = 1$$

(3) The gradient of a line perpendicular to *FG* is $\frac{-1}{(1)} = -1$.

(4) *C* is the mid-point of *FG*, so the coordinates of *C* are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 2}{2}, \frac{5 + 9}{2} \right) = \left(\frac{0}{2}, \frac{14}{2} \right) = \left(0, 7 \right)$$

(5) The equation of *l* is

$y - y_1 = m(x - x_1)$

$y - 7 = -1(x - 0)$

$y - 7 = -x$

$y = -x + 7$

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Edexcel Modular Mathematics for AS and A-Level

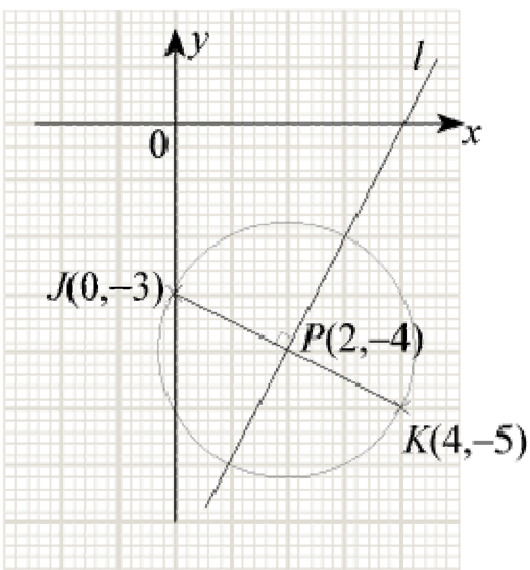
Coordinate geometry in the (x,y) plane
Exercise B, Question 2

Question:

The line JK is a diameter of the circle centre P, where J and K are $(0, -3)$ and $(4, -5)$ respectively. The line l passes through P and is perpendicular to JK. Find the equation of l. Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Solution:

(1)



(2) The gradient of JK is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-3)}{4 - 0} = \frac{-5 + 3}{4} = \frac{-2}{4} = \frac{-1}{2}$$

(3) The gradient of a line perpendicular to JK is $\frac{-1}{\left(\frac{-1}{2}\right)} = 2$

(4) P is the mid-point of JK, so the coordinates of P are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{-3 + (-5)}{2}\right) = \left(\frac{4}{2}, \frac{-8}{2}\right) = \left(2, -4\right)$$

(5) The equation of l is

$y - y_1 = m(x - x_1)$

$y - (-4) = 2(x - 2)$

$y + 4 = 2x - 4$

$0 = 2x - y - 4 - 4$

$2x - y - 8 = 0$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 3

Question:

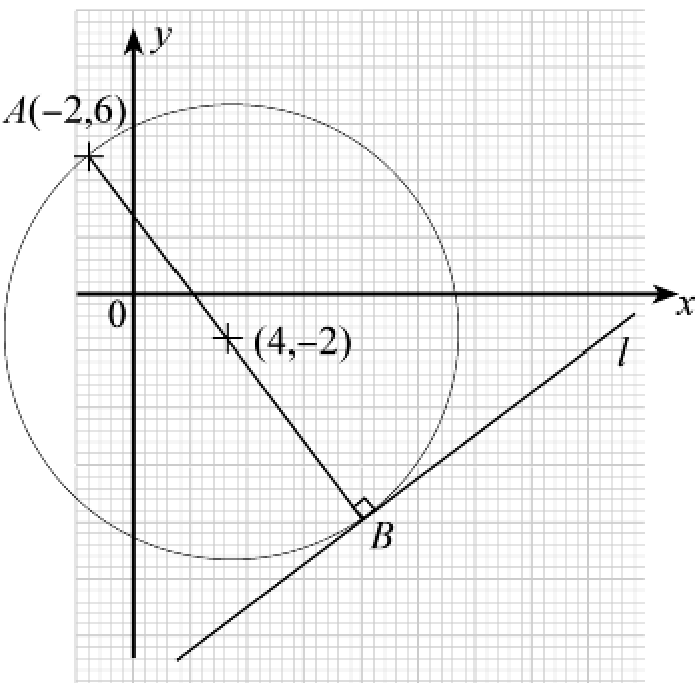
The line *AB* is a diameter of the circle centre $(4, -2)$. The line *l* passes through *B* and is perpendicular to *AB*. Given that *A* is $(-2, 6)$,

(a) find the coordinates of *B*.

(b) Hence, find the equation of *l*.

Solution:

(1)



(2) Let the coordinates of *B* be (a, b).
$(4, -2)$ is the mid-point of *AB* so

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(4, -2 \right)$$

i.e. $\left(\frac{-2 + a}{2}, \frac{6 + b}{2} \right) = \left(4, -2 \right)$

So

$\frac{-2 + a}{2} = 4$

$-2 + a = 8$
$a = 10$
and

$\frac{6 + b}{2} = -2$

$6 + b = -4$
$b = -10$

(a) The coordinates of B are $(10, -10)$.

(3) Using $(-2, 6)$ and $(4, -2)$, the gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - 6}{4 - (-2)} = \frac{-8}{6} = \frac{-4}{3}$$

(4) The gradient of a line perpendicular to AB is $\frac{-1}{\left(\frac{-4}{3}\right)} = \frac{3}{4}$

(5) The equation of l is

$$y - y_1 = m(x - x_1)$$

$$y - \left(-10\right) = \frac{3}{4}\left(x - 10\right)$$

$$y + 10 = \frac{3x}{4} - \frac{30}{4}$$

$$y = \frac{3x}{4} - \frac{30}{4} - 10$$

$$y = \frac{3x}{4} - \frac{70}{4}$$

$$y = \frac{3x}{4} - \frac{35}{2}$$

(b) The equation of l is $y = \frac{3}{4}x - \frac{35}{2}$.

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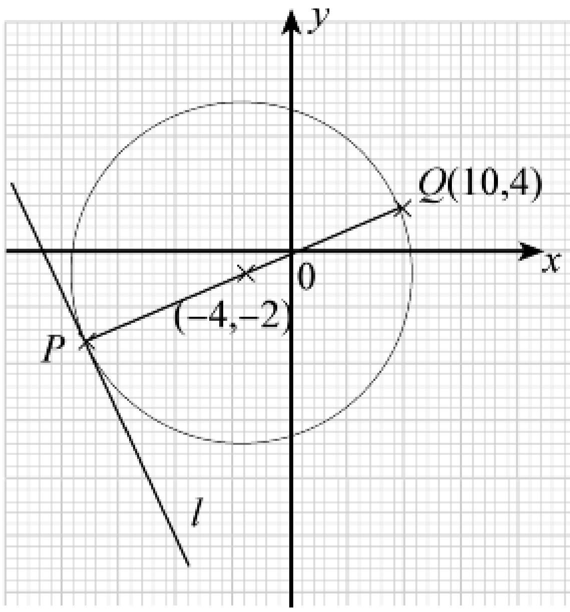
Coordinate geometry in the (x,y) plane
Exercise B, Question 4

Question:

The line *PQ* is a diameter of the circle centre $(-4 , -2)$. The line *l* passes through *P* and is perpendicular to *PQ*. Given that *Q* is $(10 , 4)$, find the equation of *l*.

Solution:

(1)



(2) Let the coordinates of *P* be (a , b) .
$(-4 , -2)$ is the mid-point of *PQ* so

$$\left(\frac{10+a}{2} , \frac{4+b}{2} \right) = \left(-4 , -2 \right)$$

$$\frac{10+a}{2} = -4$$

$10 + a = -8$
$a = -18$

$$\frac{4+b}{2} = -2$$

$4 + b = -4$
$b = -8$
The coordinates of *P* are $(-18 , -8)$.

(3) The gradient of *PQ* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{10 - (-4)} = \frac{6}{14} = \frac{3}{7}$$

(4) The gradient of a line perpendicular to *PQ* is $\frac{-1}{\left(\frac{3}{7} \right)} = \frac{-7}{3}$.

(5) The equation of *l* is

$y - y_1 = m (x - x_1)$

$y - \left(- 8 \right) = \frac{-7}{3} \left[x - \left(- 18 \right) \right]$

$y + 8 = \frac{-7}{3} \left(x + 18 \right)$

$y + 8 = \frac{-7}{3} x - 42$

$y = \frac{-7}{3} x - 50$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 5

Question:

The line *RS* is a chord of the circle centre $(5, -2)$, where *R* and *S* are $(2, 3)$ and $(10, 1)$ respectively. The line *l* is perpendicular to *RS* and bisects it. Show that *l* passes through the centre of the circle.

Solution:

(1) The gradient of *RS* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{10 - 2} = \frac{-2}{8} = \frac{-1}{4}$$

(2) The gradient of a line perpendicular to *RS* is $\frac{-1}{\left(\frac{-1}{4}\right)} = 4$.

(3) The mid-point of *RS* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 10}{2}, \frac{3 + 1}{2}\right) = \left(\frac{12}{2}, \frac{4}{2}\right) = (6, 2)$$

(4) The equation of *l* is

$y - y_1 = m(x - x_1)$

$y - 2 = 4(x - 6)$

$y - 2 = 4x - 24$

$y = 4x - 22$

(5) Substitute $x = 5$ into $y = 4x - 22$:

$y = 4(5) - 22 = 20 - 22 = -2$ ✓

So *l* passes through the centre of the circle.

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Coordinate geometry in the (x,y) plane
Exercise B, Question 6

Question:

The line MN is a chord of the circle centre $\left(1 , - \frac{1}{2} \right)$, where M and N are $(-5 , -5)$ and $(7 , 4)$ respectively. The line l is perpendicular to MN and bisects it. Find the equation of l. Write your answer in the form ax + by + c = 0, where a, b and c are integers.

Solution:

(1) The gradient of MN is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-5)}{7 - (-5)} = \frac{4 + 5}{7 + 5} = \frac{9}{12} = \frac{3}{4}$$

(2) The gradient of a line perpendicular to MN is $\frac{-1}{\left(\frac{3}{4} \right)} = \frac{-4}{3}$.

(3) The coordinates of the mid-point of MN are

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{-5 + 7}{2} , \frac{-5 + 4}{2} \right) = \left(\frac{2}{2} , \frac{-1}{2} \right) = \left(1 , \frac{-1}{2} \right)$$

(4) The equation of l is

$y - y_1 = m (x - x_1)$

$y - \left(\frac{-1}{2} \right) = \frac{-4}{3} \left(x - 1 \right)$

$y + \frac{1}{2} = \frac{-4}{3} \left(x - 1 \right)$

$y + \frac{1}{2} = \frac{-4}{3} x + \frac{4}{3}$

$(\times 6)$

$6y + 3 = -8x + 8$

$8x + 6y + 3 = 8$

$8x + 6y - 5 = 0$

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Coordinate geometry in the (x,y) plane
Exercise B, Question 7

Question:

The lines *AB* and *CD* are chords of a circle. The line $y = 2x + 8$ is the perpendicular bisector of *AB*. The line $y = -2x - 4$ is the perpendicular bisector of *CD*. Find the coordinates of the centre of the circle.

Solution:

$y = 2x + 8$
$\underline{y = -2x - 4}$
$2y = 4$
$y = 2$
Substitute $y = 2$ into $y = 2x + 8$:
$2 = 2x + 8$
$-6 = 2x$
$x = -3$
Check.
Substitute $x = -3$ and $y = 2$ into $y = -2x - 4$:
$(2) = -2(-3) - 4$
$2 = 6 - 4$
$2 = 2$ ✓
The coordinates of the centre of the circle are $(-3, 2)$.

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Coordinate geometry in the (x,y) plane
Exercise B, Question 8

Question:

The lines *EF* and *GH* are chords of a circle. The line $y = 3x - 24$ is the perpendicular bisector of *EF*. Given *G* and *F* are $(-2, 4)$ and $(4, 10)$ respectively, find the coordinates of the centre of the circle.

Solution:

(1) The gradient of *GF* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 4}{4 - (-2)} = \frac{6}{6} = 1$$

(2) The gradient of a line perpendicular to *GF* is $-\frac{1}{(1)} = -1$.

(3) The mid-point of *GF* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-2 + 4}{2}, \frac{4 + 10}{2}\right) = \left(\frac{2}{2}, \frac{14}{2}\right) = (1, 7)$$

(4) The equation of the perpendicular bisector is

$y - y_1 = m(x - x_1)$
$y - 7 = -1(x - 1)$
$y - 7 = -x + 1$
$y = -x + 8$

(5) Solving $y = -x + 8$ and $y = 3x - 24$ simultaneously:

$-x + 8 = 3x - 24$
$-4x = -32$
$x = \frac{-32}{-4}$
$x = 8$

Substitute $x = 8$ into $y = -x + 8$:

$y = -(8) + 8$
$y = -8 + 8$
$y = 0$

So the centre of the circle is $(8, 0)$.

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Coordinate geometry in the (x,y) plane
Exercise B, Question 9

Question:

The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle.

(a) Find the equation of the perpendicular bisector of
(i) *PQ*
(ii) *PR*.

(b) Hence, find the coordinates of the centre of the circle.

Solution:

(a) (i) The gradient *PQ* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 16}{11 - 3} = \frac{-4}{8} = \frac{-1}{2}$$

The gradient of a line perpendicular to *PQ* is $\frac{-1}{\left(\frac{-1}{2}\right)} = 2$.

The mid-point of *PQ* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 11}{2}, \frac{16 + 12}{2}\right) = \left(7, 14\right)$$

The equation of the perpendicular bisector of *PQ* is
$y - y_1 = m(x - x_1)$
$y - 14 = 2(x - 7)$
$y - 14 = 2x - 14$
$y = 2x$

(ii) The gradient of *PR* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 16}{-7 - 3} = \frac{-10}{-10} = 1$$

The gradient of a line perpendicular to *PR* is $-\frac{1}{(1)} = -1$.

The mid-point of *PR* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + (-7)}{2}, \frac{16 + 6}{2}\right) = \left(\frac{3 - 7}{2}, \frac{22}{2}\right) = \left(-2, 11\right)$$

The equation of the perpendicular bisector of *PR* is
$y - y_1 = m(x - x_1)$
$y - 11 = -1[x - (-2)]$
$y - 11 = -1(x + 2)$
$y - 11 = -x - 2$
$y = -x + 9$

(b) Solving $y = 2x$ and $y = -x + 9$ simultaneously:
$2x = -x + 9$
$3x = 9$
$x = 3$
Substitute $x = 3$ in $y = 2x$:
$y = 2(3)$

$y = 6$

Check.

Substitute $x = 3$ and $y = 6$ into $y = -x + 9$:

$(6) = -(3) + 9$

$6 = -3 + 9$

$6 = 6$ ✓

The coordinates of the centre are $(3, 6)$.

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Coordinate geometry in the (x,y) plane
Exercise B, Question 10

Question:

The points $A(-3, 19)$, $B(9, 11)$ and $C(-15, 1)$ lie on the circumference of a circle. Find the coordinates of the centre of the circle.

Solution:

(1) The gradient of AB is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 19}{9 - (-3)} = \frac{-8}{12} = \frac{-2}{3}$$

The gradient of a line perpendicular to AB is $\frac{-1}{\left(\frac{-2}{3}\right)} = \frac{3}{2}$.

The mid-point of AB is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 9}{2}, \frac{19 + 11}{2}\right) = \left(\frac{6}{2}, \frac{30}{2}\right) = \left(3, 15\right)$$

The equation of the perpendicular bisector of AB is

$$y - y_1 = m(x - x_1)$$

$$y - 15 = \frac{3}{2}\left(x - 3\right)$$

$$y - 15 = \frac{3}{2}x - \frac{9}{2}$$

$$y = \frac{3}{2}x + \frac{21}{2}$$

(2) The gradient of BC is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 11}{-15 - 9} = \frac{-10}{-24} = \frac{5}{12}$$

The gradient of a line perpendicular to BC is $\frac{-1}{\left(\frac{5}{12}\right)} = \frac{-12}{5}$

The mid-point of BC is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{9 + (-15)}{2}, \frac{11 + 1}{2}\right) = \left(\frac{9 - 15}{2}, \frac{11 + 1}{2}\right) = \left(\frac{-6}{2}, \frac{12}{2}\right) = \left(-3, 6\right)$$

The equation of the perpendicular bisector of BC is

$$y - y_1 = m(x - x_1)$$

$$y - 6 = \frac{-12}{5}\left[x - \left(-3\right)\right]$$

$y - 6 = \frac{-12}{5}\left(x + 3\right)$

$y - 6 = \frac{-12}{5}x - \frac{36}{5}$

$y = \frac{-12}{5}x - \frac{6}{5}$

(3) Solving $y = \frac{-12}{5}x - \frac{6}{5}$ and $y = \frac{3}{2}x + \frac{21}{2}$ simultaneously:

$\frac{3}{2}x + \frac{21}{2} = \frac{-12}{5}x - \frac{6}{5}$

$\frac{3}{2}x + \frac{12}{5}x = \frac{-6}{5} - \frac{21}{2}$

$\frac{39}{10}x = -\frac{117}{10}$

$39x = -117$
$x = -3$

Substitute $x = -3$ into $y = \frac{3}{2}x + \frac{21}{2}$:

$y = \frac{3}{2}\left(-3\right) + \frac{21}{2}$

$y = \frac{-9}{2} + \frac{21}{2}$

$y = \frac{12}{2}$

$y = 6$
Check.

Substitute $x = -3$ and $y = 6$ into $y = \frac{-12}{5}x - \frac{6}{5}$:

$\left(6\right) = \frac{-12}{5}\left(-3\right) - \frac{6}{5}$

$6 = \frac{36}{5} - \frac{6}{5}$

$6 = \frac{30}{5}$

$6 = 6$ ✓
The centre of the circle is $(-3, 6)$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise C, Question 1

Question:

Find the distance between these pairs of points:

(a) $(0, 1), (6, 9)$

(b) $(4, -6), (9, 6)$

(c) $(3, 1), (-1, 4)$

(d) $(3, 5), (4, 7)$

(e) $(2, 9), (4, 3)$

(f) $(0, -4), (5, 5)$

(g) $(-2, -7), (5, 1)$

(h) $(-4a, 0), (3a, -2a)$

(i) $(-b, 4b), (-4b, -2b)$

(j) $(2c, c), (6c, 4c)$

(k) $(-4d, d), (2d, -4d)$

(l) $(-e, -e), (-3e, -5e)$

(m) $(3\sqrt{2}, 6\sqrt{2}), (2\sqrt{2}, 4\sqrt{2})$

(n) $(-\sqrt{3}, 2\sqrt{3}), (3\sqrt{3}, 5\sqrt{3})$

(o) $(2\sqrt{3} - \sqrt{2}, \sqrt{5} + \sqrt{3}), (4\sqrt{3} - \sqrt{2}, 3\sqrt{5} + \sqrt{3})$

Solution:

(a) $(x_1, y_1) = (0, 1), (x_2, y_2) = (6, 9)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6 - 0)^2 + (9 - 1)^2}$
$= \sqrt{6^2 + 8^2}$
$= \sqrt{36 + 64}$
$= \sqrt{100}$
$= 10$

(b) $(x_1, y_1) = (4, -6), (x_2, y_2) = (9, 6)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(9 - 4)^2 + [6 - (-6)]^2}$
$= \sqrt{5^2 + 12^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

(c) $(x_1, y_1) = (3, 1)$, $(x_2, y_2) = (-1, 4)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-1-3)^2 + (4-1)^2}$
$= \sqrt{(-4)^2 + 3^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$

(d) $(x_1, y_1) = (3, 5)$, $(x_2, y_2) = (4, 7)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4-3)^2 + (7-5)^2}$
$= \sqrt{1^2 + 2^2}$
$= \sqrt{1 + 4}$
$= \sqrt{5}$

(e) $(x_1, y_1) = (2, 9)$, $(x_2, y_2) = (4, 3)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4-2)^2 + (3-9)^2}$
$= \sqrt{2^2 + (-6)^2}$
$= \sqrt{4 + 36}$
$= \sqrt{40}$
$= \sqrt{4 \times 10}$
$= \sqrt{4} \times \sqrt{10}$
$= 2\sqrt{10}$

(f) $(x_1, y_1) = (0, -4)$, $(x_2, y_2) = (5, 5)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(5-0)^2 + [5-(-4)]^2}$
$= \sqrt{5^2 + 9^2}$
$= \sqrt{25 + 81}$
$= \sqrt{106}$

(g) $(x_1, y_1) = (-2, -7)$, $(x_2, y_2) = (5, 1)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[5-(-2)]^2 + [1-(-7)]^2}$
$= \sqrt{(5+2)^2 + (1+7)^2}$
$= \sqrt{7^2 + 8^2}$
$= \sqrt{49 + 64}$
$= \sqrt{113}$

(h) $(x_1, y_1) = (-4a, 0)$, $(x_2, y_2) = (3a, -2a)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[3a-(-4a)]^2 + (-2a-0)^2}$
$= \sqrt{(3a+4a)^2 + (-2a)^2}$
$= \sqrt{(7a)^2 + (-2a)^2}$
$= \sqrt{49a^2 + 4a^2}$
$= \sqrt{53a^2}$
$= \sqrt{53}\sqrt{a^2}$
$= a\sqrt{53}$

(i) $(x_1, y_1) = (-b, 4b)$, $(x_2, y_2) = (-4b, -2b)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-4b-(-b)]^2 + (-2b-4b)^2}$
$= \sqrt{(-4b+b)^2 + (-6b)^2}$
$= \sqrt{(-3b)^2 + (-6b)^2}$
$= \sqrt{9b^2 + 36b^2}$
$= \sqrt{45b^2}$

$= \sqrt{9 \times 5 \times b^2}$
$= \sqrt{9} \sqrt{5} \sqrt{b^2}$
$= 3b \sqrt{5}$

(j) $(x_1 , y_1) = (2c , c)$, $(x_2 , y_2) = (6c , 4c)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(6c - 2c)^2 + (4c - c)^2}$
$= \sqrt{(4c)^2 + (3c)^2}$
$= \sqrt{16c^2 + 9c^2}$
$= \sqrt{25c^2}$
$= \sqrt{25} \sqrt{c^2}$
$= 5c$

(k) $(x_1 , y_1) = (- 4d , d)$, $(x_2 , y_2) = (2d , - 4d)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[2d - (- 4d)]^2 + (- 4d - d)^2}$
$= \sqrt{(2d + 4d)^2 + (- 5d)^2}$
$= \sqrt{(6d)^2 + (- 5d)^2}$
$= \sqrt{36d^2 + 25d^2}$
$= \sqrt{61d^2}$
$= \sqrt{61} \sqrt{d^2}$
$= d \sqrt{61}$

(l) $(x_1 , y_1) = (- e , - e)$, $(x_2 , y_2) = (- 3e , - 5e)$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[- 3e - (- e)]^2 + [- 5e - (- e)]^2}$
$= \sqrt{(- 3e + e)^2 + (- 5e + e)^2}$
$= \sqrt{(- 2e)^2 + (- 4e)^2}$
$= \sqrt{4e^2 + 16e^2}$
$= \sqrt{20e^2}$
$= \sqrt{4 \times 5 \times e^2}$
$= \sqrt{4} \times \sqrt{5} \times \sqrt{e^2}$
$= 2 \sqrt{5} e$

(m) $(x_1 , y_1) = (3 \sqrt{2} , 6 \sqrt{2})$, $(x_2 , y_2) = (2 \sqrt{2} , 4 \sqrt{2})$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(2 \sqrt{2} - 3 \sqrt{2})^2 + (4 \sqrt{2} - 6 \sqrt{2})^2}$
$= \sqrt{(- \sqrt{2})^2 + (- 2 \sqrt{2})^2}$
$= \sqrt{2 + 8}$
$= \sqrt{10}$

(n) $(x_1 , y_1) = (- \sqrt{3} , 2 \sqrt{3})$, $(x_2 , y_2) = (3 \sqrt{3} , 5 \sqrt{3})$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[3 \sqrt{3} - (- \sqrt{3})]^2 + (5 \sqrt{3} - 2 \sqrt{3})^2}$
$= \sqrt{(3 \sqrt{3} + \sqrt{3})^2 + (3 \sqrt{3})^2}$
$= \sqrt{(4 \sqrt{3})^2 + (3 \sqrt{3})^2}$
$= \sqrt{48 + 27}$
$= \sqrt{75}$
$= \sqrt{25 \times 3}$
$= \sqrt{25} \times \sqrt{3}$
$= 5 \sqrt{3}$

(o) $(x_1 , y_1) = (2 \sqrt{3} - \sqrt{2} , \sqrt{5} + \sqrt{3})$, $(x_2 , y_2) = (4 \sqrt{3} - \sqrt{2} , 3 \sqrt{5} + \sqrt{3})$
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[4 \sqrt{3} - \sqrt{2} - (2 \sqrt{3} - \sqrt{2})]^2 + [3 \sqrt{5} + \sqrt{3} - (\sqrt{5} + \sqrt{3})]^2}$
$= \sqrt{(4 \sqrt{3} - \sqrt{2} - 2 \sqrt{3} + \sqrt{2})^2 + (3 \sqrt{5} + \sqrt{3} - \sqrt{5} - \sqrt{3})^2}$
$= \sqrt{(2 \sqrt{3})^2 + (2 \sqrt{5})^2}$

$= \sqrt{12 + 20}$
$= \sqrt{32}$
$= \sqrt{16 \times 2}$
$= \sqrt{16} \times \sqrt{2}$
$= 4\sqrt{2}$

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Coordinate geometry in the (x,y) plane
Exercise C, Question 2

Question:

The point $(4, -3)$ lies on the circle centre $(-2, 5)$. Find the radius of the circle.

Solution:

$(x_1, y_1) = (4, -3)$, $(x_2, y_2) = (-2, 5)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-2-4)^2 + [5 - (-3)]^2}$

$= \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$

Radius of circle $= 10$.

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Coordinate geometry in the (x,y) plane

Exercise C, Question 3

Question:

The point $(14, 9)$ is the centre of the circle radius 25. Show that $(-10, 2)$ lies on the circle.

Solution:

$(x_1, y_1) = (-10, 2)$, $(x_2, y_2) = (14, 9)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[14 - (-10)]^2 + (9 - 2)^2}$

$= \sqrt{24^2 + 7^2}$

$= \sqrt{576 + 49}$

$= \sqrt{625}$

$= 25$

So $(-10, 2)$ is on the circle.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 4

Question:

The line *MN* is a diameter of a circle, where *M* and *N* are $(6, -4)$ and $(0, -2)$ respectively. Find the radius of the circle.

Solution:

$(x_1, y_1) = (6, -4), (x_2, y_2) = (0, -2)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0-6)^2 + [-2-(-4)]^2}$

$= \sqrt{(-6)^2 + (-2+4)^2}$

$= \sqrt{(-6)^2 + (2)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= \sqrt{4 \times 10}$

$= \sqrt{4} \times \sqrt{10}$

$= 2\sqrt{10}$

The diameter has length $2\sqrt{10}$.

So the radius has length $\frac{2\sqrt{10}}{2} = \sqrt{10}$.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 5

Question:

The line *QR* is a diameter of the circle centre *C*, where *Q* and *R* have coordinates $(11, 12)$ and $(-5, 0)$ respectively. The point *P* is $(13, 6)$.

(a) Find the coordinates of *C*.

(b) Show that *P* lies on the circle.

Solution:

(a) The mid-point of *QR* is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{11+(-5)}{2}, \frac{12+0}{2}\right) = \left(\frac{11-5}{2}, \frac{12}{2}\right) = \left(\frac{6}{2}, \frac{12}{2}\right) = \left(3, 6\right)$$

(b) The radius of the circle is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(11-3)^2+(12-6)^2}$

$=\sqrt{8^2+6^2}$

$=\sqrt{64+36}$

$=\sqrt{100}$

$=10$

The distance between *C* and *P* is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(13-3)^2+(6-6)^2}$

$=\sqrt{10^2+0^2}$

$=\sqrt{10^2}$

$=10$

So *P* is on the circle.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 6

Question:

The points $(-3, 19)$, $(-15, 1)$ and $(9, 1)$ are vertices of a triangle. Show that a circle centre $(-3, 6)$ can be drawn through the vertices of the triangle.

Solution:

(1) $(x_1, y_1) = (-3, 6)$, $(x_2, y_2) = (-3, 19)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[(-3) - (-3)]^2 + (19 - 6)^2}$
$= \sqrt{(-3 + 3)^2 + (13)^2}$
$= \sqrt{0^2 + 13^2}$
$= \sqrt{13^2}$
$= 13$

(2) $(x_1, y_1) = (-3, 6)$, $(x_2, y_2) = (-15, 1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-15 - (-3)]^2 + (1 - 6)^2}$
$= \sqrt{(-12)^2 + (-5)^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169}$
$= 13$

(3) $(x_1, y_1) = (-3, 6)$, $(x_2, y_2) = (9, 1)$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[9 - (-3)]^2 + (1 - 6)^2}$
$= \sqrt{(12)^2 + (-5)^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169}$
$= 13$

The distance of each vertex of the triangle to $(-3, 6)$ is 13. So a circle centre $(-3, 6)$ and radius 13 can be drawn through the vertices of the triangle.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 7

Question:

The line *ST* is a diameter of the circle c_1, where *S* and *T* are $(5, 3)$ and $(-3, 7)$ respectively.
The line *UV* is a diameter of the circle c_2 centre $(4, 4)$. The point *U* is $(1, 8)$.

(a) Find the radius of (i) c_1 (ii) c_2.

(b) Find the distance between the centres of c_1 and c_2.

Solution:

(a) (i) The centre of c_1 is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+(-3)}{2}, \frac{3+7}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1, 5)$$

The radius of c_1 is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(5-1)^2+(3-5)^2}$
$=\sqrt{4^2+(-2)^2}$
$=\sqrt{16+4}$
$=\sqrt{20}$
$=\sqrt{4\times 5}$
$=\sqrt{4}\times\sqrt{5}$
$=2\sqrt{5}$

(ii) The radius of c_2 is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(4-1)^2+(4-8)^2}$
$=\sqrt{3^2+(-4)^2}$
$=\sqrt{9+16}$
$=\sqrt{25}$
$=5$

(b) The distance between the centres is

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(1-4)^2+(5-4)^2}$
$=\sqrt{(-3)^2+(1)^2}$
$=\sqrt{9+1}$
$=\sqrt{10}$

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Coordinate geometry in the (x,y) plane
Exercise C, Question 8

Question:

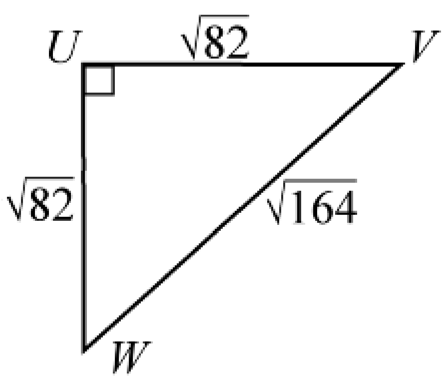
The points $U(-2, 8)$, $V(7, 7)$ and $W(-3, -1)$ lie on a circle.

(a) Show that $\triangle UVW$ has a right angle.

(b) Find the coordinates of the centre of the circle.

Solution:

(a) (1) The distance UV is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[7 - (-2)]^2 + (7 - 8)^2}$
$= \sqrt{(7 + 2)^2 + (-1)^2}$
$= \sqrt{9^2 + (-1)^2}$
$= \sqrt{81 + 1}$
$= \sqrt{82}$
(2) The distance VW is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-3 - 7)^2 + (-1 - 7)^2}$
$= \sqrt{(-10)^2 + (-8)^2}$
$= \sqrt{100 + 64}$
$= \sqrt{164}$
(3) The distance UW is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[-3 - (-2)]^2 + (-1 - 8)^2}$
$= \sqrt{(-3 + 2)^2 + (-9)^2}$
$= \sqrt{(-1)^2 + (-9)^2}$
$= \sqrt{1 + 81}$
$= \sqrt{82}$
Now $(\sqrt{82})^2 + (\sqrt{82})^2 = (\sqrt{164})^2$



i.e. $UV^2 + UW^2 = VW^2$
So, by Pythagoras' theorem, $\triangle UVW$ has a right angle at U.

(b) The angle in a semicircle is a right angle. So VW is a diameter of the circle.
The mid-point of VW is

$$\left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right) = \left(\frac{7 + (-3)}{2} , \frac{7 + (-1)}{2} \right) = \left(\frac{7 - 3}{2} , \frac{7 - 1}{2} \right) = \left(2 , 3 \right)$$

The centre of the circle is $(2 , 3)$.

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Coordinate geometry in the (x,y) plane
Exercise C, Question 9

Question:

The points $A(2,6)$, $B(5,7)$ and $C(8,-2)$ lie on a circle.

(a) Show that $\triangle ABC$ has a right angle.

(b) Find the area of the triangle.

Solution:

(a) (1) The distance AB is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(5-2)^2+(7-6)^2}$
$=\sqrt{3^2+1^2}$
$=\sqrt{9+1}$
$=\sqrt{10}$
(2) The distance BC is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(8-5)^2+(-2-7)^2}$
$=\sqrt{3^2+(-9)^2}$
$=\sqrt{9+81}$
$=\sqrt{90}$
(3) The distance AC is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(8-2)^2+(-2-6)^2}$
$=\sqrt{6^2+(-8)^2}$
$=\sqrt{36+64}$
$=\sqrt{100}$
Now $(\sqrt{10})^2+(\sqrt{90})^2=(\sqrt{100})^2$
i.e. $AB^2+BC^2=AC^2$
So, by Pythagoras' theorem, there is a right angle at B.

(b) The area of the triangle is
$\frac{1}{2}\times$ base $\times$ height $=\frac{1}{2}\times AB\times BC=\frac{1}{2}\sqrt{10}\sqrt{90}=\frac{1}{2}\sqrt{900}=\frac{1}{2}\times 30=15$

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Coordinate geometry in the (x,y) plane
Exercise C, Question 10

Question:

The points $A(-1, 9)$, $B(6, 10)$, $C(7, 3)$ and $D(0, 2)$ lie on a circle.

(a) Show that *ABCD* is a square.

(b) Find the area of *ABCD*.

(c) Find the centre of the circle.

Solution:

(a) (1) The length of *AB* is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{[6 - (-1)]^2 + (10 - 9)^2}$
$= \sqrt{7^2 + 1^2}$
$= \sqrt{49 + 1}$
$= \sqrt{50}$
(2) The length of *BC* is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(7 - 6)^2 + (3 - 10)^2}$
$= \sqrt{1^2 + (-7)^2}$
$= \sqrt{1 + 49}$
$= \sqrt{50}$
(3) The length of *CD* is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(0 - 7)^2 + (2 - 3)^2}$
$= \sqrt{(-7)^2 + (-1)^2}$
$= \sqrt{49 + 1}$
$= \sqrt{50}$
(4) The length of *DA* is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-1 - 0)^2 + (9 - 2)^2}$
$= \sqrt{(-1)^2 + 7^2}$
$= \sqrt{1 + 49}$
$= \sqrt{50}$
The sides of the quadrilateral are equal.
(5) The gradient of *AB* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 9}{6 - (-1)} = \frac{1}{7}$$

The gradient of *BC* is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 10}{7 - 6} = \frac{-7}{1} = -7$$

The product of the gradients $= -1 \left(\frac{1}{7} \times -7 = -1 \right)$.

So the line *AB* is perpendicular to *BC*.
So the quadrilateral *ABCD* is a square.

(b) The area $= \sqrt{50} \times \sqrt{50} = 50$

(c) The mid-point of *AC* is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-1 + 7}{2}, \frac{9 + 3}{2} \right) = \left(\frac{6}{2}, \frac{12}{2} \right) = \left(3, 6 \right)$$

So the centre of the circle is $(3, 6)$.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 1

Question:

Write down the equation of these circles:

(a) Centre $(3, 2)$, radius 4

(b) Centre $(-4, 5)$, radius 6

(c) Centre $(5, -6)$, radius $2\sqrt{3}$

(d) Centre $(2a, 7a)$, radius $5a$

(e) Centre $(-2\sqrt{2}, -3\sqrt{2})$, radius 1

Solution:

(a) $(x_1, y_1) = (3, 2)$, $r = 4$
So $(x-3)^2 + (y-2)^2 = 4^2$
or $(x-3)^2 + (y-2)^2 = 16$

(b) $(x_1, y_1) = (-4, 5)$, $r = 6$
So $[x-(-4)]^2 + (y-5)^2 = 6^2$
or $(x+4)^2 + (y-5)^2 = 36$

(c) $(x_1, y_1) = (5, -6)$, $r = 2\sqrt{3}$
So $(x-5)^2 + [y-(-6)]^2 = (2\sqrt{3})^2$
$(x-5)^2 + (y+6)^2 = 2^2(\sqrt{3})^2$
$(x-5)^2 + (y+6)^2 = 4 \times 3$
$(x-5)^2 + (y+6)^2 = 12$

(d) $(x_1, y_1) = (2a, 7a)$, $r = 5a$
So $(x-2a)^2 + (y-7a)^2 = (5a)^2$
or $(x-2a)^2 + (y-7a)^2 = 25a^2$

(e) $(x_1, y_1) = (-2\sqrt{2}, -3\sqrt{2})$, $r = 1$
So $[x-(-2\sqrt{2})]^2 + [y-(-3\sqrt{2})]^2 = 1^2$
or $(x+2\sqrt{2})^2 + (y+3\sqrt{2})^2 = 1$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 2

Question:

Write down the coordinates of the centre and the radius of these circles:

(a) $(x + 5)^2 + (y - 4)^2 = 9^2$

(b) $(x - 7)^2 + (y - 1)^2 = 16$

(c) $(x + 4)^2 + y^2 = 25$

(d) $(x + 4a)^2 + (y + a)^2 = 144a^2$

(e) $(x - 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27$

Solution:

(a) $(x + 5)^2 + (y - 4)^2 = 9^2$
or $[x - (-5)]^2 + (y - 4)^2 = 9^2$
The centre of the circle is $(-5 , 4)$ and the radius is 9.

(b) $(x - 7)^2 + (y - 1)^2 = 16$
or $(x - 7)^2 + (y - 1)^2 = 4^2$
The centre of the circle is $(7 , 1)$ and the radius is 4.

(c) $(x + 4)^2 + y^2 = 25$
or $[x - (-4)]^2 + (y - 0)^2 = 5^2$
The centre of the circle is $(-4 , 0)$ and the radius is 5.

(d) $(x + 4a)^2 + (y + a)^2 = 144a^2$
or $[x - (-4a)]^2 + [y - (-a)]^2 = (12a)^2$
The centre of the circle is $(-4a , -a)$ and the radius is $12a$.

(e) $(x - 3\sqrt{5})^2 + (y + \sqrt{5})^2 = 27$
or $(x - 3\sqrt{5})^2 + [y - (-\sqrt{5})]^2 = (\sqrt{27})^2$
Now $\sqrt{27} = \sqrt{9 \times 3} = \sqrt{9} \times \sqrt{3} = 3\sqrt{3}$
The centre of the circle is $(3\sqrt{5} , -\sqrt{5})$ and the radius is $3\sqrt{3}$.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 3

Question:

Find the centre and radius of these circles by first writing in the form $(x - a) ^{2} + (y - b) ^{2} = r^2$

(a) $x^2 + y^2 + 4x + 9y + 3 = 0$

(b) $x^2 + y^2 + 5x - 3y - 8 = 0$

(c) $2x^2 + 2y^2 + 8x + 15y - 1 = 0$

(d) $2x^2 + 2y^2 - 8x + 8y + 3 = 0$

Solution:

(a) $x^2 + y^2 + 4x + 9y + 3 = 0$
$x^2 + 4x + y^2 + 9y = -3$

$$(x + 2) ^{2} - 4 \left(y + \frac{9}{2} \right) ^{2} - \frac{81}{4} = -3$$

$$(x + 2) ^{2} + \left(y + \frac{9}{2} \right) ^{2} = \frac{85}{4}$$

So the centre is $(-2 , -4.5)$ and the radius is 4.61 (2 d.p.)

(b) $x^2 + y^2 + 5x - 3y - 8 = 0$
$x^2 + 5x + y^2 - 3y = 8$

$$\left(x + \frac{5}{2} \right) ^{2} - \frac{25}{4} + \left(y - \frac{3}{2} \right) ^{2} - \frac{9}{4} = 8$$

$$\left(x + \frac{5}{2} \right) ^{2} + \left(y - \frac{3}{2} \right) ^{2} = 16.5$$

So the centre is $(-2.5, 1.5)$ and the radius is 4.06 (2 d.p.)

(c) $2x^2 + 2y^2 + 8x + 15y - 1 = 0$

$$x^2 + y^2 + 4x + \frac{15}{2}y - ½ = 0$$

$$x^2 + 4x + y^2 + \frac{15}{2}y = ½$$

$$(x + 2) ^{2} - 4 + \left(y + \frac{15}{4} \right) ^{2} - \frac{225}{16} = ½$$

$$(x + 2) ^{2} + \left(y + \frac{15}{4} \right) ^{2} = 18 \frac{9}{16}$$

So the centre is $(-2 , -3.75)$ and the radius is 4.31 (2 d.p.)

(d) $2x^2 + 2y^2 - 8x + 8y + 3 = 0$

$$x^2 + y^2 - 4x + 4y + \frac{3}{2} = 0$$

$x^2 - 4x + y^2 + 4y = -\frac{3}{2}$

$(x-2)^2 - 4 + (y+2)^2 - 4 = -\frac{3}{2}$

$(x-2)^2 + (y+2)^2 = \frac{13}{2}$

So the centre is $(2, -2)$ and the radius is 2.55 (2 d.p.)

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Coordinate geometry in the (x,y) plane
Exercise D, Question 4

Question:

In each case, show that the circle passes through the given point:

(a) $(x-2)^2+(y-5)^2=13$, $(4, 8)$

(b) $(x+7)^2+(y-2)^2=65$, $(0, -2)$

(c) $x^2+y^2=25^2$, $(7, -24)$

(d) $(x-2a)^2+(y+5a)^2=20a^2$, $(6a, -3a)$

(e) $(x-3\sqrt{5})^2+(y-\sqrt{5})^2=(2\sqrt{10})^2$, $(\sqrt{5}, -\sqrt{5})$

Solution:

(a) Substitute $x=4$, $y=8$ into $(x-2)^2+(y-5)^2=13$
$(x-2)^2+(y-5)^2=(4-2)^2+(8-5)^2=2^2+3^2=4+9=13$ ✓
So the circle passes through $(4, 8)$.

(b) Substitute $x=0$, $y=-2$ into $(x+7)^2+(y-2)^2=65$
$(x+7)^2+(y-2)^2=(0+7)^2+(-2-2)^2=7^2+(-4)^2=49+16=65$ ✓
So the circle passes through $(0, -2)$.

(c) Substitute $x=7$ and $y=-24$ into $x^2+y^2=25^2$
$x^2+y^2=7^2+(-24)^2=49+576=625=25^2$ ✓
So the circle passes through $(7, -24)$.

(d) Substitute $x=6a$, $y=-3a$ into $(x-2a)^2+(y+5a)^2=20a^2$
$(x-2a)^2+(y+5a)^2=(6a-2a)^2+(-3a+5a)^2=(4a)^2+(2a)^2=16a^2+4a^2=20a^2$ ✓
So the circle passes through $(6a, -3a)$.

(e) Substitute $x=\sqrt{5}$, $y=-\sqrt{5}$ into $(x-3\sqrt{5})^2+(y-\sqrt{5})^2=(2\sqrt{10})^2$
$(x-3\sqrt{5})^2+(y-\sqrt{5})^2=(\sqrt{5}-3\sqrt{5})^2+(-\sqrt{5}-\sqrt{5})^2=(-2\sqrt{5})^2+(-2\sqrt{5})^2=4\times5+4\times5=20+20=40=(\sqrt{40})^2$
Now $\sqrt{40}=\sqrt{4\times10}=\sqrt{4}\times\sqrt{10}=2\sqrt{10}$ ✓
So the circle passes through $(\sqrt{5}, -\sqrt{5})$.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 5

Question:

The point $(4 , - 2)$ lies on the circle centre $(8 , 1)$. Find the equation of the circle.

Solution:

The radius of the circle is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(8 - 4)^2 + [1 - (- 2)]^2}$
$= \sqrt{4^2 + 3^2}$
$= \sqrt{16 + 9}$
$= \sqrt{25}$
$= 5$
The centre of the circle is $(8 , 1)$ and the radius is 5.
So $(x - 8)^2 + (y - 1)^2 = 5^2$
or $(x - 8)^2 + (y - 1)^2 = 25$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 6

Question:

The line PQ is the diameter of the circle, where P and Q are $(5, 6)$ and $(-2, 2)$ respectively. Find the equation of the circle.

Solution:

(1) The centre of the circle is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{5+(-2)}{2}, \frac{6+2}{2}\right) = \left(\frac{3}{2}, \frac{8}{2}\right) = \left(\frac{3}{2}, 4\right)$$

(2) The radius of the circle is

$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{\left(5-\frac{3}{2}\right)^2+(6-4)^2}$$

$$= \sqrt{\left(\frac{7}{2}\right)^2+(2)^2}$$

$$= \sqrt{\frac{49}{4}+4}$$

$$= \sqrt{\frac{49}{4}+\frac{16}{4}}$$

$$= \sqrt{\frac{65}{4}}$$

So the equation of the circle is

$$\left(x-\frac{3}{2}\right)^2+(y-4)^2 = \left(\sqrt{\frac{65}{4}}\right)^2$$

or $\left(x-\frac{3}{2}\right)^2+(y-4)^2 = \frac{65}{4}$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 7

Question:

The point $(1, -3)$ lies on the circle $(x-3)^2 + (y+4)^2 = r^2$. Find the value of r.

Solution:

Substitute $x = 1$, $y = -3$ into $(x-3)^2 + (y+4)^2 = r^2$
$(1-3)^2 + (-3+4)^2 = r^2$
$(-2)^2 + (1)^2 = r^2$
$4 + 1 = r^2$
$5 = r^2$
So $r = \sqrt{5}$

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Coordinate geometry in the (x,y) plane
Exercise D, Question 8

Question:

The line $y = 2x + 13$ touches the circle $x^2 + (y - 3)^2 = 20$ at $(-4, 5)$. Show that the radius at $(-4, 5)$ is perpendicular to the line.

Solution:

(1) The centre of the circle $x^2 + (y - 3)^2 = 20$ is $(0, 3)$.
(2) The gradient of the line joining $(0, 3)$ and $(-4, 5)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{-4 - 0} = \frac{2}{-4} = -\frac{1}{2}$$

(3) The gradient of $y = 2x + 13$ is 2.
(4) The product of the gradients is

$$-\frac{1}{2} \times 2 = -1$$

So the radius is perpendicular to the line.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 9

Question:

The line $x + 3y - 11 = 0$ touches the circle $(x + 1)^2 + (y + 6)^2 = 90$ at $(2, 3)$.

(a) Find the radius of the circle.

(b) Show that the radius at $(2, 3)$ is perpendicular to the line.

Solution:

(a) The radius of the circle $(x + 1)^2 + (y + 6)^2 = 90$ is $\sqrt{90}$.
$\sqrt{90} = \sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}$

(b) (1) The centre of the circle $(x + 1)^2 + (y + 6)^2 = 90$ is $(-1, -6)$.
(2) The gradient of the line joining $(-1, -6)$ and $(2, 3)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - (-6)}{2 - (-1)} = \frac{3 + 6}{2 + 1} = \frac{9}{3} = 3$$

(3) Rearrange $x + 3y - 11 = 0$ into the form $y = mx + c$
$x + 3y - 11 = 0$
$3y - 11 = -x$
$3y = -x + 11$
$y = -\frac{1}{3}x + \frac{11}{3}$

So the gradient of $x + 3y - 11 = 0$ is $-\frac{1}{3}$.

(4) The product of the gradients is

$3 \times -\frac{1}{3} = -1$

So the radius is perpendicular to the line.

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Coordinate geometry in the (x,y) plane
Exercise D, Question 10

Question:

The point $P(1, -2)$ lies on the circle centre $(4, 6)$.

(a) Find the equation of the circle.

(b) Find the equation of the tangent to the circle at P.

Solution:

(a) (1) The radius of the circle is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-1)^2 + [6-(-2)]^2} = \sqrt{3^2 + 8^2} = \sqrt{9+64} = \sqrt{73}$
(2) The equation of the circle is
$(x-4)^2 + (y-6)^2 = (\sqrt{73})^2$
or $(x-4)^2 + (y-6)^2 = 73$

(b) (1) The gradient of the line joining $(1, -2)$ and $(4, 6)$ is
$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-2)}{4-1} = \frac{6+2}{3} = \frac{8}{3}$

(2) The gradient of the tangent is $\frac{-1}{\left(\frac{8}{3}\right)} = -\frac{3}{8}$.

(3) The equation of the tangent to the circle at $(1, -2)$ is
$y - y_1 = m(x - x_1)$
$y - \left(-2\right) = -\frac{3}{8}\left(x-1\right)$
$y + 2 = -\frac{3}{8}\left(x-1\right)$
$8y + 16 = -3(x-1)$
$8y + 16 = -3x + 3$
$3x + 8y + 16 = 3$
$3x + 8y + 13 = 0$

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Coordinate geometry in the (x,y) plane
Exercise E, Question 1

Question:

Find where the circle $(x-1)^2+(y-3)^2=45$ meets the x-axis.

Solution:

Substitute $y=0$ into $(x-1)^2+(y-3)^2=45$
$(x-1)^2+(-3)^2=45$
$(x-1)^2+9=45$
$(x-1)^2=36$
$x-1=\pm\sqrt{36}$
$x-1=\pm 6$
So $x-1=6 \Rightarrow x=7$
and $x-1=-6 \Rightarrow x=-5$
The circle meets the x-axis at $(7,0)$ and $(-5,0)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 2

Question:

Find where the circle $(x-2)^2+(y+3)^2=29$ meets the *y*-axis.

Solution:

Substitute $x=0$ into $(x-2)^2+(y+3)^2=29$
$(-2)^2+(y+3)^2=29$
$4+(y+3)^2=29$
$(y+3)^2=25$
$y+3=\pm\sqrt{25}$
$y+3=\pm 5$
So $y+3=5 \Rightarrow y=2$
and $y+3=-5 \Rightarrow y=-8$
The circle meets the *y*-axis at $(0,2)$ and $(0,-8)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 3

Question:

The circle $(x - 3)^2 + (y + 3)^2 = 34$ meets the x-axis at $(a , 0)$ and the y-axis at $(0 , b)$. Find the possible values of a and b.

Solution:

(1) Substitute $x = a$, $y = 0$ into $(x - 3)^2 + (y + 3)^2 = 34$
$(a - 3)^2 + (3)^2 = 34$
$(a - 3)^2 + 9 = 34$
$(a - 3)^2 = 25$
$a - 3 = \pm \sqrt{25}$
$a - 3 = \pm 5$
So $a - 3 = 5 \Rightarrow a = 8$
and $a - 3 = -5 \Rightarrow a = -2$
The circle meets the x-axis at $(8 , 0)$ and $(-2 , 0)$.
(2) Substitute $x = 0$, $y = b$ into $(x - 3)^2 + (y + 3)^2 = 34$
$(-3)^2 + (b + 3)^2 = 34$
$9 + (b + 3)^2 = 34$
$(b + 3)^2 = 25$
$b + 3 = \pm \sqrt{25}$
$b + 3 = \pm 5$
So $b + 3 = 5 \Rightarrow b = 2$
and $b + 3 = -5 \Rightarrow b = -8$
The circle meets the y-axis at $(0 , 2)$ and $(0 , -8)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 4

Question:

The line $y = x + 4$ meets the circle $(x - 3)^2 + (y - 5)^2 = 34$ at A and B. Find the coordinates of A and B.

Solution:

Substitute $y = x + 4$ into $(x - 3)^2 + (y - 5)^2 = 34$
$(x - 3)^2 + [(x + 4) - 5]^2 = 34$
$(x - 3)^2 + (x + 4 - 5)^2 = 34$
$(x - 3)^2 + (x - 1)^2 = 34$
$x^2 - 6x + 9 + x^2 - 2x + 1 = 34$
$2x^2 - 8x + 10 = 34$
$2x^2 - 8x - 24 = 0$
$x^2 - 4x - 12 = 0$
$(x - 6) (x + 2) = 0$
So $x = 6$ and $x = - 2$

Substitute $x = 6$ into $y = x + 4$
$y = 6 + 4$
$y = 10$
Substitute $x = - 2$ into $y = x + 4$
$y = - 2 + 4$
$y = 2$

The coordinates of A and B are $(6 , 10)$ and $(- 2 , 2)$.

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Coordinate geometry in the (x,y) plane
Exercise E, Question 5

Question:

Find where the line $x + y + 5 = 0$ meets the circle $(x + 3)^2 + (y + 5)^2 = 65$.

Solution:

Rearranging $x + y + 5 = 0$
$y + 5 = - x$
$y = - x - 5$
Substitute $y = - x - 5$ into $(x + 3)^2 + (y + 5)^2 = 65$
$(x + 3)^2 + [(- x - 5) + 5]^2 = 65$
$(x + 3)^2 + (- x - 5 + 5)^2 = 65$
$(x + 3)^2 + (- x)^2 = 65$
$x^2 + 6x + 9 + x^2 = 65$
$2x^2 + 6x + 9 = 65$
$2x^2 + 6x - 56 = 0$
$x^2 + 3x - 28 = 0$
$(x + 7) (x - 4) = 0$
So $x = - 7$ and $x = 4$

Substitute $x = - 7$ into $y = - x - 5$
$y = - (- 7) - 5$
$y = 7 - 5$
$y = 2$
Substitute $x = 4$ into $y = - x - 5$
$y = - (4) - 5$
$y = - 4 - 5$
$y = - 9$

So the line meets the circle at $(- 7 , 2)$ and $(4 , - 9)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane

Exercise E, Question 6

Question:

Show that the line $y = x - 10$ does not meet the circle $(x - 2)^2 + y^2 = 25$.

Solution:

Substitute $y = x - 10$ into $(x - 2)^2 + y^2 = 25$
$(x - 2)^2 + (x - 10)^2 = 25$
$x^2 - 4x + 4 + x^2 - 20x + 100 = 25$
$2x^2 - 24x + 104 = 25$
$2x^2 - 24x + 79 = 0$
Now $b^2 - 4ac = (- 24)^2 - 4 (2) (79) = 576 - 632 = - 56$
As $b^2 - 4ac < 0$ then $2x^2 - 24x + 79 = 0$ has no real roots.
So the line does not meet the circle.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise E, Question 7

Question:

Show that the line $x + y = 11$ is a tangent to the circle $x^2 + (y - 3)^2 = 32$.

Solution:

Rearranging $x + y = 11$
$y = 11 - x$
Substitute $y = 11 - x$ into $x^2 + (y - 3)^2 = 32$
$x^2 + [(11 - x) - 3]^2 = 32$
$x^2 + (11 - x - 3)^2 = 32$
$x^2 + (8 - x)^2 = 32$
$x^2 + 64 - 16x + x^2 = 32$
$2x^2 - 16x + 64 = 32$
$2x^2 - 16x + 32 = 0$
$x^2 - 8x + 16 = 0$
$(x - 4)(x - 4) = 0$
The line meets the circle at $x = 4$ (only).
So the line is a tangent.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise E, Question 8

Question:

Show that the line $3x - 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$.

Solution:

Rearrange $3x - 4y + 25 = 0$
$3x + 25 = 4y$
$4y = 3x + 25$
$y = \frac{3}{4}x + \frac{25}{4}$

Substitute $y = \frac{3}{4}x + \frac{25}{4}$ into $x^2 + y^2 = 25$

$x^2 + \left(\frac{3}{4}x + \frac{25}{4} \right)^2 = 25$

$x^2 + \frac{9}{16}x^2 + \frac{150}{16}x + \frac{625}{16} = 25$

$\frac{25}{16}x^2 + \frac{150}{16}x + \frac{225}{16} = 0$

$25x^2 + 150x + 225 = 0$
$x^2 + 6x + 9 = 0$
$(x + 3)(x + 3) = 0$
The line meets the circle at $x = -3$ (only).
So the line is a tangent.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise E, Question 9

Question:

The line $y = 2x - 2$ meets the circle $(x - 2)^2 + (y - 2)^2 = 20$ at A and B.

(a) Find the coordinates of A and B.

(b) Show that AB is a diameter of the circle.

Solution:

(a) Substitute $y = 2x - 2$ into $(x - 2)^2 + (y - 2)^2 = 20$
$(x - 2)^2 + [(2x - 2) - 2]^2 = 20$
$(x - 2)^2 + (2x - 4)^2 = 20$
$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$
$5x^2 - 20x + 20 = 20$
$5x^2 - 20x = 0$
$5x (x - 4) = 0$
So $x = 0$ and $x = 4$

Substitute $x = 0$ into $y = 2x - 2$
$y = 2 (0) - 2$
$y = 0 - 2$
$y = - 2$
Substitute $x = 4$ into $y = 2x - 2$
$y = 2 (4) - 2$
$y = 8 - 2$
$y = 6$

So the coordinates of A and B are $(0 , - 2)$ and $(4 , 6)$.

(b) (1) The length of AB is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4 - 0)^2 + [6 - (- 2)]^2}$
$= \sqrt{4^2 + (6 + 2)^2}$
$= \sqrt{4^2 + 8^2}$
$= \sqrt{16 + 64}$
$= \sqrt{80}$
$= \sqrt{4 \times 20}$
$= \sqrt{4} \times \sqrt{20}$
$= 2 \sqrt{20}$
The radius of the circle $(x - 2)^2 + (y - 2)^2 = 20$ is $\sqrt{20}$.
So the length of the chord AB is twice the length of the radius.
AB is a diameter of the circle.
(2) Substitute $x = 2$, $y = 2$ into $y = 2x - 2$
$2 = 2 (2) - 2 = 4 - 2 = 2$ ✓
So the line $y = 2x - 2$ joining A and B passes through the centre $(2 , 2)$ of the circle.
So AB is a diameter of the circle.

Solutionbank C2
Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise E, Question 10

Question:

The line $x + y = a$ meets the circle $(x - p)^2 + (y - 6)^2 = 20$ at $(3, 10)$, where a and p are constants.

(a) Work out the value of a.

(b) Work out the two possible values of p.

Solution:

(a) Substitute $x = 3$, $y = 10$ into $x + y = a$
$(3) + (10) = a$
So $a = 13$

(b) Substitute $x = 3$, $y = 10$ into $(x - p)^2 + (y - 6)^2 = 20$
$(3 - p)^2 + (10 - 6)^2 = 20$
$(3 - p)^2 + 4^2 = 20$
$(3 - p)^2 + 16 = 20$
$(3 - p)^2 = 4$
$(3 - p) = \sqrt{4}$
$3 - p = \pm 2$
So $3 - p = 2 \Rightarrow p = 1$
and $3 - p = -2 \Rightarrow p = 5$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise F, Question 1

Question:

The line $y = 2x - 8$ meets the coordinate axes at A and B. The line AB is a diameter of the circle. Find the equation of the circle.

Solution:

Substitute $x = 0$ into $y = 2x - 8$
$y = 2(0) - 8$
$y = -8$
Substitute $y = 0$ into $y = 2x - 8$
$0 = 2x - 8$
$2x = 8$
$x = 4$
The line meets the coordinate axes at $(0, -8)$ and $(4, 0)$
The coordinates of the centre of the circle is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 4}{2}, \frac{-8 + 0}{2} \right) = \left(\frac{4}{2}, -\frac{8}{2} \right) = \left(2, -4 \right)$$

The length of the diameter is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(4 - 0)^2 + [0 - (-8)]^2}$
$= \sqrt{4^2 + 8^2}$
$= \sqrt{16 + 64}$
$= \sqrt{80}$
$= \sqrt{16 \times 5}$
$= \sqrt{16} \times \sqrt{5}$
$= 4\sqrt{5}$
So the length of the radius is $\frac{4\sqrt{5}}{2} = 2\sqrt{5}$.

The centre of the circle is $(2, -4)$ and the radius is $2\sqrt{5}$.
So the equation is
$(x - x_1)^2 + (y - y_1)^2 = r^2$
$(x - 2)^2 + [y - (-4)]^2 = (2\sqrt{5})^2$
$(x - 2)^2 + (y + 4)^2 = 20$

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Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane
Exercise F, Question 2

Question:

The circle centre $(8 , 10)$ meets the x-axis at $(4 , 0)$ and $(a , 0)$.

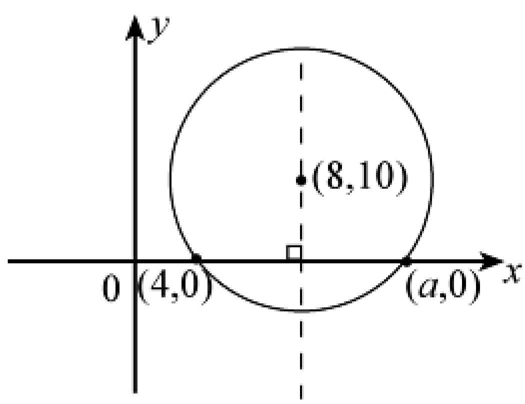
(a) Find the radius of the circle.

(b) Find the value of a.

Solution:

(a) The radius is
$\sqrt{(x_2 - x_1) ^2 + (y_2 - y_1) ^2}$
$= \sqrt{(8 - 4) ^2 + (10 - 0) ^2}$
$= \sqrt{4^2 + 10^2}$
$= \sqrt{16 + 100}$
$= \sqrt{116}$
$= 2 \sqrt{29}$

(b)



The centre is on the perpendicular bisector of $(4 , 0)$ and $(a , 0)$. So
$\frac{4 + a}{2} = 8$

$4 + a = 16$
$a = 12$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 3

Question:

The circle $(x - 5)^2 + y^2 = 36$ meets the *x*-axis at *P* and *Q*. Find the coordinates of *P* and *Q*.

Solution:

Substitute $y = 0$ into $(x - 5)^2 + y^2 = 36$
$(x - 5)^2 = 36$
$x - 5 = \sqrt{36}$
$x - 5 = \pm 6$
So $x - 5 = 6 \Rightarrow x = 11$
and $x - 5 = -6 \Rightarrow x = -1$
The coordinates of *P* and *Q* are $(-1, 0)$ and $(11, 0)$.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 4

Question:

The circle $(x+4)^2+(y-7)^2=121$ meets the y-axis at $(0, m)$ and $(0, n)$. Find the value of m and n.

Solution:

Substitute $x=0$ into $(x+4)^2+(y-7)^2=121$
$4^2+(y-7)^2=121$
$16+(y-7)^2=121$
$(y-7)^2=105$
$y-7=\pm\sqrt{105}$
So $y=7\pm\sqrt{105}$
The values of m and n are $7+\sqrt{105}$ and $7-\sqrt{105}$.

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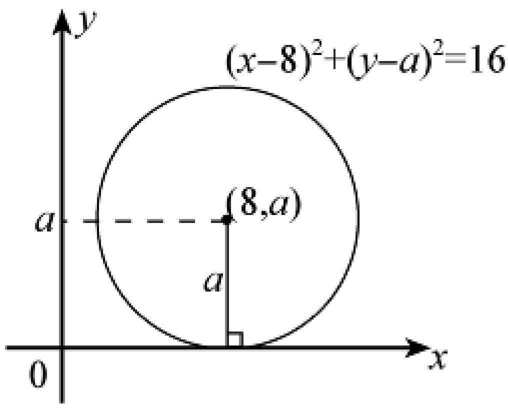
Coordinate geometry in the (x,y) plane
Exercise F, Question 5

Question:

The line $y = 0$ is a tangent to the circle $(x - 8)^2 + (y - a)^2 = 16$. Find the value of a.

Solution:



The radius of the circle is $\sqrt{16} = 4$.
So $a = 4$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 6

Question:

The point $A(-3, -7)$ lies on the circle centre $(5, 1)$.
Find the equation of the tangent to the circle at A.

Solution:

The gradient of the line joining $(-3, -7)$ and $(5, 1)$ is

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-7)}{5 - (-3)} = \frac{1 + 7}{5 + 3} = \frac{8}{8} = 1$$

So the gradient of the tangent is $-\frac{1}{(1)} = -1$.

The equation of the tangent is
$y - y_1 = m(x - x_1)$
$y - (-7) = -1[x - (-3)]$
$y + 7 = -1(x + 3)$
$y + 7 = -x - 3$
$y = -x - 10$ or $x + y + 10 = 0$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 7

Question:

The circle $(x+3)^2+(y+8)^2=100$ meets the positive coordinate axes at $A(a,0)$ and $B(0,b)$.

(a) Find the value of a and b.

(b) Find the equation of the line AB.

Solution:

(a) Substitute $y=0$ into $(x+3)^2+(y+8)^2=100$
$(x+3)^2+8^2=100$
$(x+3)^2+64=100$
$(x+3)^2=36$
$x+3=\pm\sqrt{36}$
$x+3=\pm 6$
So $x+3=6 \Rightarrow x=3$
and $x+3=-6 \Rightarrow x=-9$
As $a>0$, $a=3$.

Substitute $x=0$ into $(x+3)^2+(y+8)^2=100$
$3^2+(y+8)^2=100$
$9+(y+8)^2=100$
$(y+8)^2=91$
$y+8=\pm\sqrt{91}$
So $y+8=\sqrt{91} \Rightarrow y=\sqrt{91}-8$
and $y+8=-\sqrt{91} \Rightarrow y=-\sqrt{91}-8$
As $b>0$, $b=\sqrt{91}-8$.

(b) The equation of the line joining $(3,0)$ and $(0,\sqrt{91}-8)$ is

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y-0}{(\sqrt{91}-8)-0}=\frac{x-3}{0-3}$$

$$\frac{y}{\sqrt{91}-8}=\frac{x-3}{-3}$$

$$y=\left(\sqrt{91}-8\right)\times\left(\frac{x-3}{-3}\right)$$

$$y=\left(\frac{\sqrt{91}-8}{-3}\right)\left(x-3\right)$$

$$y=\left(\frac{8-\sqrt{91}}{3}\right)\left(x-3\right)$$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 8

Question:

The circle $(x+2)^2 + (y-5)^2 = 169$ meets the positive coordinate axes at $C(c, 0)$ and $D(0, d)$.

(a) Find the value of c and d.

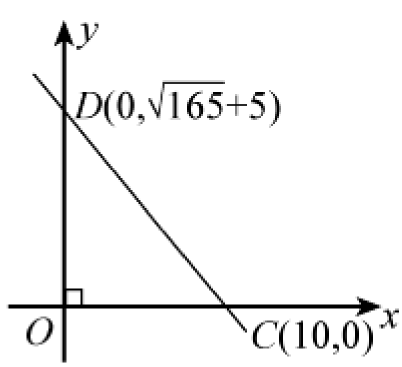
(b) Find the area of $\triangle OCD$, where O is the origin.

Solution:

(a) Substitute $y = 0$ into $(x+2)^2 + (y-5)^2 = 169$
$(x+2)^2 + (-5)^2 = 169$
$(x+2)^2 + 25 = 169$
$(x+2)^2 = 144$
$x + 2 = \pm\sqrt{144}$
$x + 2 = \pm 12$
So $x + 2 = 12 \Rightarrow x = 10$
and $x + 2 = -12 \Rightarrow x = -14$
As $c > 0$, $c = 10$.

Substitute $x = 0$ into $(x+2)^2 + (y-5)^2 = 169$
$2^2 + (y-5)^2 = 169$
$4 + (y-5)^2 = 169$
$(y-5)^2 = 165$
$y - 5 = \pm\sqrt{165}$
So $y - 5 = \sqrt{165} \Rightarrow y = \sqrt{165} + 5$
and $y - 5 = -\sqrt{165} \Rightarrow y = -\sqrt{165} + 5$
As $d > 0$, $d = \sqrt{165} + 5$.

(b)



The area of $\triangle OCD$ is
$\frac{1}{2} \times 10 \times \left(\sqrt{165} + 5\right) = 5\left(\sqrt{165} + 5\right)$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 9

Question:

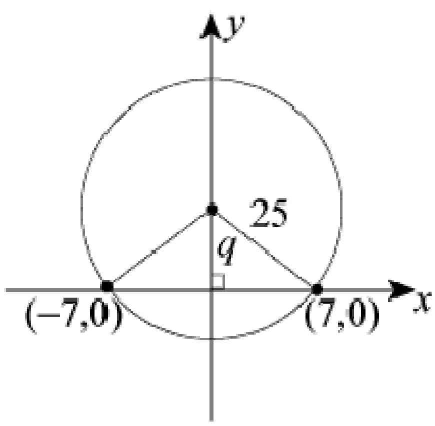
The circle, centre (p, q) radius 25, meets the x-axis at $(-7, 0)$ and $(7, 0)$, where $q > 0$.

(a) Find the value of p and q.

(b) Find the coordinates of the points where the circle meets the y-axis.

Solution:

(a) By symmetry $p = 0$.



Using Pythagoras' theorem
$q^2 + 7^2 = 25^2$
$q^2 + 49 = 625$
$q^2 = 576$
$q = \pm\sqrt{576}$
$q = \pm 24$
As $q > 0$, $q = 24$.

(b) The circle meets the y-axis at $q \pm r$; i.e.
at $24 + 25 = 49$
and $24 - 25 = -1$
So the coordinates are $(0, 49)$ and $(0, -1)$.

Solutionbank C2

Edexcel Modular Mathematics for AS and A-Level

Coordinate geometry in the (x,y) plane

Exercise F, Question 10

Question:

Show that $(0, 0)$ lies inside the circle $(x-5)^2 + (y+2)^2 = 30$.

Solution:

The distance between $(0, 0)$ and $(5, -2)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5-0)^2 + (-2-0)^2}$

$= \sqrt{5^2 + (-2)^2}$

$= \sqrt{25 + 4}$

$= \sqrt{29}$

The radius of the circle is $\sqrt{30}$.

As $\sqrt{29} < \sqrt{30}$ $(0, 0)$ lies inside the circle.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 11

Question:

The points $A(-4,0)$, $B(4,8)$ and $C(6,0)$ lie on a circle. The lines *AB* and *BC* are chords of the circle. Find the coordinates of the centre of the circle.

Solution:

(1) The gradient of *AB* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{8-0}{4-(-4)}=\frac{8}{4+4}=\frac{8}{8}=1$$

(2) The gradient of a line perpendicular to *AB* is $\frac{-1}{(1)}=-1$.

(3) The mid-point of *AB* is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-4+4}{2},\frac{0+8}{2}\right)=\left(\frac{0}{2},\frac{8}{2}\right)=\left(0,4\right)$$

(4) The equation of the perpendicular bisector of *AB* is

$y-y_1=m(x-x_1)$

$y-4=-1(x-0)$

$y-4=-x$

$y=-x+4$

(5) The gradient of *BC* is

$$\frac{y_2-y_1}{x_2-x_1}=\frac{0-8}{6-4}=\frac{-8}{2}=-4$$

(6) The gradient of a line perpendicular to *BC* is $-\frac{1}{(-4)}=\frac{1}{4}$.

(7) The mid-point of *BC* is

$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{4+6}{2},\frac{8+0}{2}\right)=\left(\frac{10}{2},\frac{8}{2}\right)=\left(5,4\right)$$

(8) The equation of the perpendicular bisector of *BC* is

$y-y_1=m(x-x_1)$

$y-4=\frac{1}{4}\left(x-5\right)$

$y-4=\frac{1}{4}x-\frac{5}{4}$

$y=\frac{1}{4}x+\frac{11}{4}$

(9) Solving $y=-x+4$ and $y=\frac{1}{4}x+\frac{11}{4}$ simultaneously

$\frac{1}{4}x+\frac{11}{4}=-x+4$

$\frac{5}{4}x + \frac{11}{4} = 4$

$\frac{5}{4}x = \frac{5}{4}$

$x = 1$
Substitute $x = 1$ into $y = -x + 4$
$y = -1 + 4$
$y = 3$
So coordinates of the centre of the circle are $(1, 3)$.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 12

Question:

The points $R(-4, 3)$, $S(7, 4)$ and $T(8, -7)$ lie on a circle.

(a) Show that $\triangle RST$ has a right angle.

(b) Find the equation of the circle.

Solution:

(a) (1) The distance between R and S is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[7 - (-4)]^2 + (4 - 3)^2}$

$= \sqrt{(7 + 4)^2 + 1^2}$

$= \sqrt{11^2 + 1^2}$

$= \sqrt{121 + 1}$

$= \sqrt{122}$

(2) The distance between S and T is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 7)^2 + (-7 - 4)^2}$

$= \sqrt{1^2 + (-11)^2}$

$= \sqrt{1 + 121}$

$= \sqrt{122}$

(3) The distance between R and T is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{[8 - (-4)]^2 + (-7 - 3)^2}$

$= \sqrt{(8 + 4)^2 + (-10)^2}$

$= \sqrt{12^2 + (-10)^2}$

$= \sqrt{144 + 100}$

$= \sqrt{244}$

By Pythagoras' theorem

$(\sqrt{122})^2 + (\sqrt{122})^2 = (\sqrt{244})^2$

So $\triangle RST$ has a right angle (at S).

(b) (1) The radius of the circle is

$\frac{1}{2} \times \text{diameter} = \frac{1}{2}\sqrt{244} = \frac{1}{2}\sqrt{4 \times 61} = \frac{1}{2}\sqrt{4} \times \sqrt{61} = \frac{1}{2} \times 2\sqrt{61} = \sqrt{61}$

(2) The centre of the circle is the mid-point of RT:

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

$= \left(\frac{-4 + 8}{2}, \frac{3 + (-7)}{2} \right) = \left(\frac{4}{2}, -\frac{4}{2} \right) = \left(2, -2 \right)$

So the equation of the circle is

$(x - 2)^2 + (y + 2)^2 = (\sqrt{61})^2$

or $(x - 2)^2 + (y + 2)^2 = 61$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 13

Question:

The points $A(-7,7)$, $B(1,9)$, $C(3,1)$ and $D(-7,1)$ lie on a circle. The lines *AB* and *CD* are chords of the circle.

(a) Find the equation of the perpendicular bisector of (i) *AB* (ii) *CD*.

(b) Find the coordinates of the centre of the circle.

Solution:

(a) (i) (1) The gradient of the line joining *A* and *B* is
$$\frac{y_2-y_1}{x_2-x_1}=\frac{9-7}{1-(-7)}=\frac{2}{1+7}=\frac{2}{8}=\frac{1}{4}$$

(2) The gradient of a line perpendicular to *AB* is $-\frac{1}{m}=\frac{-1}{\left(\frac{1}{4}\right)}=-4$

(3) The mid-point of *AB* is
$$\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{-7+1}{2},\frac{7+9}{2}\right)=\left(\frac{-6}{2},\frac{16}{2}\right)=\left(-3,8\right)$$

(4) The equation of the perpendicular bisector of *AB* is
$y-y_1=m(x-x_1)$
$y-8=-4[x-(-3)]$
$y-8=-4(x+3)$
$y-8=-4x-12$
$y=-4x-4$

(ii) (1) The gradient of the line joining *C* and *D* is
$$\frac{y_2-y_1}{x_2-x_1}=\frac{1-1}{-7-3}=\frac{0}{-10}=0$$

So the line is horizontal.

(2) The mid-point of *CD* is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{3+(-7)}{2},\frac{1+1}{2}\right)=\left(\frac{-4}{2},\frac{2}{2}\right)=\left(-2,1\right)$

(3) The equation of the perpendicular bisector of *CD* is $x=-2$
i.e. the vertical line through $(-2,1)$

(b) Solving $y=-4x-4$ and $x=-2$ simultaneously,
substitute $x=-2$ into $y=-4x-4$
$y=-4(-2)-4=8-4=4$
So the centre of the circle is $(-2,4)$.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 14

Question:

The centres of the circles $(x-8)^2+(y-8)^2=117$ and $(x+1)^2+(y-3)^2=106$ are *P* and *Q* respectively.

(a) Show that *P* lies on $(x+1)^2+(y-3)^2=106$.

(b) Find the length of *PQ*.

Solution:

(a) The centre of $(x-8)^2+(y-8)^2=117$ is $(8, 8)$.
Substitute $(8, 8)$ into $(x+1)^2+(y-3)^2=106$
$(8+1)^2+(8-3)^2=9^2+5^2=81+25=106$ ✓
So $(8, 8)$ lies on the circle $(x+1)^2+(y-3)^2=106$.

(b) As *Q* is the centre of the circle $(x+1)^2+(y-3)^2=106$ and *P* lies on this circle, the length *PQ* must equal the radius.
So PQ = $\sqrt{106}$

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Coordinate geometry in the (x,y) plane

Exercise F, Question 15

Question:

The line $y = -3x + 12$ meets the coordinate axes at A and B.

(a) Find the coordinates of A and B.

(b) Find the coordinates of the mid-point of AB.

(c) Find the equation of the circle that passes through A, B and O, where O is the origin.

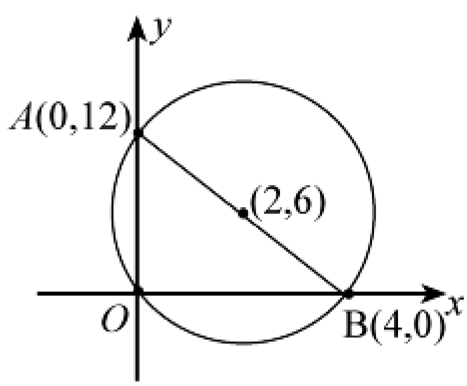
Solution:

(a) $y = -3x + 12$
(1) Substitute $x = 0$ into $y = -3x + 12$
$y = -3(0) + 12 = 12$
So A is $(0, 12)$.
(2) Substitute $y = 0$ into $y = -3x + 12$
$0 = -3x + 12$
$3x = 12$
$x = 4$
So B is $(4, 0)$.

(b) The mid-point of AB is

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 4}{2}, \frac{12 + 0}{2} \right) = \left(2, 6 \right)$$

(c)



$\angle AOB = 90°$, so AB is a diameter of the circle.
The centre of the circle is the mid-point of AB, i.e. $(2, 6)$.
The length of the diameter AB is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(4 - 0)^2 + (0 - 12)^2}$$
$$= \sqrt{4^2 + (-12)^2}$$
$$= \sqrt{16 + 144}$$
$$= \sqrt{160}$$

So the radius of the circle is $\frac{\sqrt{160}}{2}$.

The equation of the circle is

$$(x-2)^2+(y-6)^2=\left(\frac{\sqrt{160}}{2}\right)^2$$

$$(x-2)^2+(y-6)^2=\frac{160}{4}$$

$$(x-2)^2+(y-6)^2=40$$

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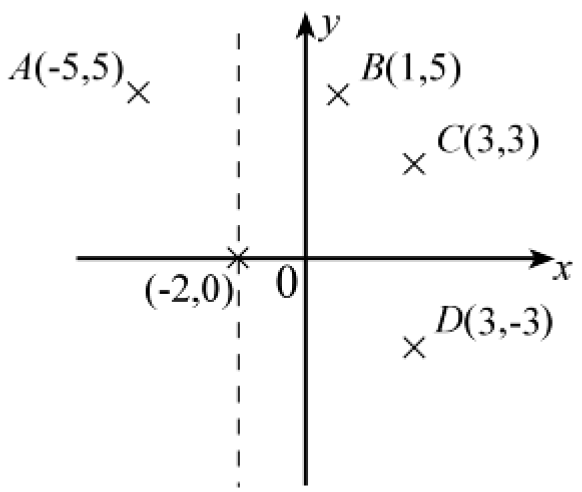
Coordinate geometry in the (x,y) plane
Exercise F, Question 16

Question:

The points $A(-5,5)$, $B(1,5)$, $C(3,3)$ and $D(3,-3)$ lie on a circle. Find the equation of the circle.

Solution:

(1)



(2) The mid-point of *AB* is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-5+1}{2}, \frac{5+5}{2}\right) = \left(\frac{-4}{2}, \frac{10}{2}\right) = \left(-2, 5\right)$$

So the equation of the perpendicular bisector of *AB* is $x = -2$.

(3) The mid-point of *CD* is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3+3}{2}, \frac{3+(-3)}{2}\right) = \left(\frac{6}{2}, \frac{3-3}{2}\right) = \left(3, \frac{0}{2}\right) = \left(3, 0\right)$$

So the equation of the perpendicular bisector of *CD* is $y = 0$.

(4) The perpendicular bisectors intersect at $(-2, 0)$.

(5) The radius is the distance between $(-2, 0)$ and $(-5, 5)$

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{[-5-(-2)]^2+(5-0)^2}$

$= \sqrt{(-5+2)^2+(5)^2}$

$= \sqrt{(-3)^2+(5)^2}$

$= \sqrt{9+25}$

$= \sqrt{34}$

(6) So the equation of the circle centre $(-2, 0)$ and radius $\sqrt{34}$ is

$[x-(-2)]^2+(y-0)^2 = (\sqrt{34})^2$

$(x+2)^2+y^2 = 34$

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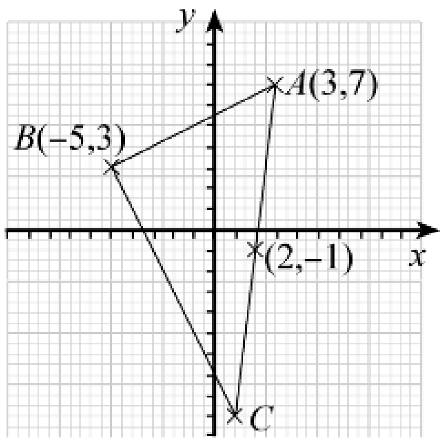
Coordinate geometry in the (x,y) plane
Exercise F, Question 17

Question:

The line AB is a chord of a circle centre $(2, -1)$, where A and B are $(3, 7)$ and $(-5, 3)$ respectively. AC is a diameter of the circle. Find the area of $\triangle ABC$.

Solution:

(1)



(2) Let the coordinates of C be (p, q).
$(2, -1)$ is the mid-point of $(3, 7)$ and (p, q)

So $\frac{3+p}{2} = 2$ and $\frac{7+q}{2} = -1$

$\frac{3+p}{2} = 2$

$3 + p = 4$
$p = 1$

$\frac{7+q}{2} = -1$

$7 + q = -2$
$q = -9$
So the coordinates of C are $(1, -9)$.

(3) The length of AB is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-5-3)^2 + (3-7)^2}$
$= \sqrt{(-8)^2 + (-4)^2}$
$= \sqrt{64 + 16}$
$= \sqrt{80}$

The length of BC is
$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(-5-1)^2 + [3-(-9)]^2}$
$= \sqrt{(-6)^2 + (3+9)^2}$
$= \sqrt{(-6)^2 + (12)^2}$
$= \sqrt{36 + 144}$
$= \sqrt{180}$

(4) The area of $\triangle ABC$ is

$\frac{1}{2}\sqrt{180}\sqrt{80} = \frac{1}{2}\sqrt{14400} = \frac{1}{2}\sqrt{144 \times 100} = \frac{1}{2}\sqrt{144} \times \sqrt{100} = \frac{1}{2} \times 12 \times 10 = 60$

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Coordinate geometry in the (x,y) plane
Exercise F, Question 18

Question:

The points $A(-1, 0)$, $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $C\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ are the vertices of a triangle.

(a) Show that the circle $x^2 + y^2 = 1$ passes through the vertices of the triangle.

(b) Show that $\triangle ABC$ is equilateral.

Solution:

(a) (1) Substitute $(-1, 0)$ into $x^2 + y^2 = 1$
$(-1)^2 + (0)^2 = 1 + 0 = 1$ ✓
So $(-1, 0)$ is on the circle.

(2) Substitute $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ into $x^2 + y^2 = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$ ✓

So $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is on the circle.

(3) Substitute $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ into $x^2 + y^2 = 1$

$\left(\frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$ ✓

So $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is on the circle.

(b) (1) The distance between $(-1, 0)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[\frac{1}{2} - \left(-1\right)\right]^2 + \left(\frac{\sqrt{3}}{2} - 0\right)^2}$

$= \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{3}{4}}$

$= \sqrt{\frac{12}{4}}$

$= \sqrt{3}$

(2) The distance between $(-1, 0)$ and $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left[\frac{1}{2} - \left(-1\right)\right]^2 + \left(\frac{-\sqrt{3}}{2} - 0\right)^2}$

$= \sqrt{\left(\frac{1}{2} + 1\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$

$= \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2}\right)^2}$

$= \sqrt{\frac{9}{4} + \frac{3}{4}}$

$= \sqrt{\frac{12}{4}}$

$= \sqrt{3}$

(3) The distance between $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(\frac{-\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)^2}$

$= \sqrt{0^2 + (-\sqrt{3})^2}$

$= \sqrt{0 + 3}$

$= \sqrt{3}$

So *AB*, *BC* and *AC* all equal $\sqrt{3}$.

$\triangle ABC$ is equilateral.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 19

Question:

The points $P(2,2)$, $Q(2+\sqrt{3},5)$ and $R(2-\sqrt{3},5)$ lie on the circle $(x-2)^2+(y-4)^2=r^2$.

(a) Find the value of r.

(b) Show that $\triangle PQR$ is equilateral.

Solution:

(a) Substitute $(2,2)$ into $(x-2)^2+(y-4)^2=r^2$
$(2-2)^2+(2-4)^2=r^2$
$0^2+(-2)^2=r^2$
$r^2=4$
$r=2$

(b) (1) The distance between $(2,2)$ and $(2+\sqrt{3},5)$ is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(2+\sqrt{3}-2)^2+(5-2)^2}$
$=\sqrt{(\sqrt{3})^2+3^2}$
$=\sqrt{3+9}$
$=\sqrt{12}$
(2) The distance between $(2,2)$ and $(2-\sqrt{3},5)$ is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{(2-\sqrt{3}-2)^2+(5-2)^2}$
$=\sqrt{(-\sqrt{3})^2+(3)^2}$
$=\sqrt{3+9}$
$=\sqrt{12}$
(3) The distance between $(2+\sqrt{3},5)$ and $(2-\sqrt{3},5)$ is
$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$=\sqrt{[(2-\sqrt{3})-(2+\sqrt{3})]^2+(5-5)^2}$
$=\sqrt{(2-\sqrt{3}-2-\sqrt{3})^2+0^2}$
$=\sqrt{(-2\sqrt{3})^2}$
$=\sqrt{(-2)^2\times(\sqrt{3})^2}$
$=\sqrt{4\times3}$
$=\sqrt{12}$
So PQ, QR and PR all equal $\sqrt{12}$.
$\triangle PQR$ is equilateral.

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Coordinate geometry in the (x,y) plane
Exercise F, Question 20

Question:

The points $A(-3, -2)$, $B(-6, 0)$ and $C(p, q)$ lie on a circle centre $\left(-\frac{5}{2}, 2\right)$. The line BC is a diameter of the circle.

(a) Find the value of p and q.

(b) Find the gradient of (i) AB (ii) AC.

(c) Show that AB is perpendicular to AC.

Solution:

(a) The mid-point of $(-6, 0)$ and (p, q) is $\left(-\frac{5}{2}, 2\right)$.

So $\left(\frac{-6+p}{2}, \frac{0+q}{2}\right) = \left(-\frac{5}{2}, 2\right)$

$\frac{-6+p}{2} = -\frac{5}{2}$

$-6 + p = -5$

$p = -5 + 6$

$p = 1$

$\frac{0+q}{2} = 2$

$\frac{q}{2} = 2$

$q = 4$

(b) (i) The gradient of the line joining $(-3, -2)$ and $(-6, 0)$ is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-2)}{-6-(-3)} = \frac{2}{-6+3} = \frac{2}{-3} = -\frac{2}{3}$

(ii) The gradient of the line joining $(-3, -2)$ and $(1, 4)$ is

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{1-(-3)} = \frac{4+2}{1+3} = \frac{6}{4} = \frac{3}{2}$

(c) Two lines are perpendicular if $m_1 \times m_2 = -1$.

Now $-\frac{2}{3} \times \frac{3}{2} = -1$ ✓

So AB is perpendicular to AC.