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Mark Scheme (Results)

Summer 2016

Pearson Edexcel GCE
Decision Mathematics 2

6690/01

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $*$ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.

- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1. (a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited just once	B2, 1, 0 **(2)**
(b)	A – D – C – F – B – E – G – A 12+16+19+25+14+41+22 = 149 A – D – C – F – G – E – B – A 12+16+19+25+41+14+31 = 158	M1 A1 A1 **(3)**
(c)	RMST weight = 86 (miles) 86 + 12 + 15 = 113 (miles)	B1 M1 A1 **(3)**
(d)	113 ≤ optimal distance ≤ 149	B2, 1, 0 **(2)** **10 marks**

Notes for Question 1

a1B1: Understands the difference is connected to the number of times each vertex may be visited (but maybe incorrectly attributed). Must be an attempt at a difference (so must refer to both the classical and practical problems explicitly). Technical language (vertex/node) must be correct. Need not imply each/every/all (oe) vertices for this first mark
a2B1: Correctly identifies which is classical and which is practical and correctly states the difference. Must imply that each/every/all (oe) vertices are visited, so for example, 'the practical problem visits a vertex at least once while the classical visits a vertex only once' is B1B0 (note that B0B1 is not possible in (a))

b1M1: Either one correct route, must return to A, or one correct length stated (do not isw in part (b) if correct lengths seen but are then doubled)
b1A1: One correct route, must return to A and corresponding length correct
b2A1: Both routes correct and their corresponding lengths correct

c1B1: CAO for RMST weight (either 86 or 20 + 16 + 14 + 19 + 17) – maybe implied by later working
c1M1: Adding 12 + 15 (the two least weighted arcs) to their RMST length (the length of their RMST must be in the interval $66 \leq \text{RMST} \leq 106$) – this mark maybe implied by the correct value for the lower bound
c1A1: CAO - if 113 seen without working then award all 3 marks in (c)

d1B1: Their numbers correctly used, accept any inequalities or any indication of interval from their 113 to their 149 (so 113 – 149 can score this mark). This mark is dependent on two totals seen in (b), however, neither of the two totals need to be correct. Please note that UB > LB for this mark
d2B1: CAO (no follow through on their values) including correct inequalities or equivalent set notation (but condone $113 < \text{optimal distance} \leq 149$)

Question Number	Scheme	Marks
2. (a)	Saturated arcs: SB, SC, AE, DT, FT	M1 A1 **(2)**
(b)	59	B1 **(1)**
(c)	$C_1 = 72$, $C_2 = 86$	B1 B1 **(2)**
(d)	SABCFET	B1 **(1)**
(e)	The cut through DT, AE and CF (or DT, AE, BC and SC) has a value 62 Value of the flow is 62, so by max flow – min cut theorem, flow is maximal	M1 A1 **(2)** **8 marks**

Notes for Question 2

a1M1: All correct – accept one omission **and**/**or** one extra arc
a1A1: CAO

b1B1: CAO (59)

c1B1: CAO (72)
c2B1: CAO (86)

d1B1: CAO (accept, SA, AB, BC, CF, FE, ET)

e1M1: The arcs of the correct cut stated **or** a correct cut drawn on their diagram in the answer book – please check carefully for this
e1A1: Must have stated the (maximum) flow as 62 and a conclusion based on the max flow min cut theorem - they must use all four words, ‘max’, ‘flow’, ‘min’, ‘cut’ in their conclusion

Question Number	Scheme	Marks
3. (a)	Since maximising, subtract all elements from some value ≥ 72	
	e.g $\begin{bmatrix} 11 & 22 & 25 & 49 \\ 1 & 10 & 52 & 11 \\ 2 & 23 & 24 & 23 \\ 0 & 4 & 5 & 5 \end{bmatrix}$	M1
	Reduce rows $\begin{bmatrix} 0 & 11 & 14 & 38 \\ 0 & 9 & 51 & 10 \\ 0 & 21 & 22 & 21 \\ 0 & 4 & 5 & 5 \end{bmatrix}$ and then columns $\begin{bmatrix} 0 & 7 & 9 & 33 \\ 0 & 5 & 46 & 5 \\ 0 & 17 & 17 & 16 \\ 0 & 0 & 0 & 0 \end{bmatrix}$	M1 A1
	$\begin{bmatrix} 0 & 2 & 4 & 28 \\ 0 & 0 & 41 & 0 \\ 0 & 12 & 12 & 11 \\ 5 & 0 & 0 & 0 \end{bmatrix}$ followed by $\begin{bmatrix} 0 & 0 & 2 & 26 \\ 2 & 0 & 41 & 0 \\ 0 & 10 & 10 & 9 \\ 7 & 0 & 0 & 0 \end{bmatrix}$	M1 A1ft M1 A1
	Optimal allocation is F = 1, A = 2, Z = 3, E = 4	A1 **(8)**
(b)	(Total score is =) 248	B1 **(1)** **9 marks**

Notes for Question 3

a1M1: Subtracting from some value which must be ≥ 72 **or** all values made negative **and** then adding a value which must be ≥ 72. Condone no more than two errors
a2M1: Reducing rows **and then** columns – candidates may combine the two stages of converting from a maximum to a minimum problem and row reduction which is acceptable
a1A1: CAO
a3M1: Double covered +e; one uncovered –e ; and one single covered unchanged. 2 lines to 3 lines needed
a2A1ft: Follow through on their previous table – no errors
a4M1: One double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table)
a3A1: CSO on final table (so must have scored all previous marks)
a4A1: CAO – this mark is dependent on all M marks being awarded

b1B1: CAO

SC: Minimising

After row reduction $\begin{bmatrix} 38 & 27 & 24 & 0 \\ 51 & 42 & 0 & 41 \\ 22 & 1 & 0 & 1 \\ 5 & 1 & 0 & 0 \end{bmatrix}$ and then after column reduction $\begin{bmatrix} 33 & 26 & 24 & 0 \\ 46 & 41 & 0 & 41 \\ 17 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

scores M0 M1A1 M0A0 M0A0 A0 B0 (so 2 marks max)

<table>
<tr><td>Question Number</td><td>Scheme</td><td>Marks</td></tr>
<tr><td>4. (a)</td><td>e.g.variable x was increased first, since it has become a basic variable</td><td>B1 (1)</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td>b.v.</td><td>x</td><td>y</td><td>z</td><td>r</td><td>s</td><td>t</td><td>Value</td><td>row ops</td></tr>
<tr><td>z</td><td>0</td><td>2.5</td><td>1</td><td>0.5</td><td>-1.5</td><td>0</td><td>5</td><td>$R_1 \div 2$</td></tr>
<tr><td>x</td><td>1</td><td>-5.5</td><td>0</td><td>-1.5</td><td>5.5</td><td>0</td><td>3</td><td>$R_2 - 3R_1$</td></tr>
<tr><td>t</td><td>0</td><td>3.5</td><td>0</td><td>0.5</td><td>2.5</td><td>1</td><td>8</td><td>$R_3 + R_1$</td></tr>
<tr><td>P</td><td>0</td><td>13</td><td>0</td><td>2</td><td>-5</td><td>0</td><td>27</td><td>$R_4 + 4R_1$</td></tr>
</table>
</td><td>M1 A1
M1
A1ft A1 (5)</td></tr>
<tr><td>(c)</td><td>$P + 13y + 2r - 5s = 27$</td><td>B1ft (1)</td></tr>
<tr><td>(d)</td><td>$P = 27 - 13y - 2r + 5s$, so we can increase the profit by increasing s, hence not optimal</td><td>B2,1,0 (2)
9 marks</td></tr>
</table>

Notes for Question 4

a1B1: e.g. identifies *x*, refers to basic variable (oe)

If pivoting on a negative value or on a value from the *x* or *y* column then no marks in (b), (c) or (d)

b1M1: Correct pivot located (2 in column *z*), attempt to divide row
b1A1: Pivot row correct **including change of b.v.** (so the *r* must be replaced with a *z*)
b2M1: **All** values in one of the non-pivot rows correct **or** one of the non zero-and-one columns (*y*, *r*, *s* or value) correct following through their choice of pivot from column *z*
b2A1ft: Row operations used correctly at least twice, i.e. **two** of the non zero-and-one columns (*y*, *r*, *s* or value) correct following through their choice of pivot from column *z*
b3A1: CAO – no follow through – all values and row operations correctly stated – allow if row operations given in terms of old row 1 – **ignore b.v. column for this mark**

c1B1ft: Dependent on the second M mark earned in (b) – must be an equation containing *P* (please note that $P = 13y + 2r - 5s + 27$ is incorrect)

d1B1: Must have gained both M marks in (b) **and** must refer to **increasing** *y*, *r* or *s*. Do not accept 'negatives in profit row' with no further explanation given
d2B1: CAO – dependent on the correct profit equation in (c). Specifically identifies *s* as the next variable that could be increased and states 'not optimal' (oe)

Pivoting on the 3 in the *z* column (can score a maximum of B1 M0A0M1A1A0 B1 B0B0 – so 4/9)

b.v.	*x*	*y*	*z*	*r*	*s*	*t*	Value
r	-2/3	11/3	0	1	-11/3	0	-2
z	1/3	2/3	1	0	1/3	0	6
t	1/3	5/3	0	0	13/3	1	9
P	4/3	17/3	0	0	7/3	0	31

Question Number	Scheme	Marks
5. (a)	(total) supply > (total) demand	B1 **(1)**
(b)		B1 **(1)**

	A	B	C	Dummy	Supply
1	18	23	20	0	15
2	22	17	25	0	36
3	24	21	19	0	28
4	21	22	17	0	20
Demand	40	20	25	14	

(c) B1 **(1)**

	A	B	C	D	Supply
1	15				15
2	25	11			36
3		9	19		28
4			6	14	20
Demand	40	20	25	14	

(d) M1 A1

	A	B	C	D
1	15			
2	$19-\theta$	$17+\theta$		
3		$3-\theta$	25	θ
4	$6+\theta$			$14-\theta$

giving

	A	B	C	D
1	15			
2	16	20		
3			25	3
4	9			11

Exiting cell is B3

M1 A1

Shadow costs		18	13	16	-3
		A	B	C	D
0	1	X	10	4	3
4	2	X	X	5	-1
3	3	3	5	X	X
3	4	X	6	-2	X

Entering cell is C4

M1 A1

	A	B	C	D
1				
2				
3			$25-\theta$	$3+\theta$
4			θ	$11-\theta$

giving

	A	B	C	D
1	15			
2	16	20		
3			14	14
4	9		11	

Exiting Cell is D4

M1 A1 **(6)**

Question Number	Scheme	Marks
(e)	<table><tr><td>Shadow costs</td><td></td><td>18</td><td>13</td><td>14</td><td>-5</td></tr><tr><td></td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr><tr><td>0</td><td>1</td><td>X</td><td>10</td><td>6</td><td>5</td></tr><tr><td>4</td><td>2</td><td>X</td><td>X</td><td>7</td><td>1</td></tr><tr><td>5</td><td>3</td><td>1</td><td>3</td><td>X</td><td>X</td></tr><tr><td>3</td><td>4</td><td>X</td><td>6</td><td>X</td><td>2</td></tr></table> Optimal since no negative improvement indices	M1 A1 A1 **(3)** **12 marks**

Notes for Question 5

a1B1: CAO (or to make demand = supply or because demand $\neq$ supply (oe))

b1B1: CAO

c1B1: CAO

d1M1: A valid route, only one empty square, D3 used, θ's balance
d1A1: Correct route, up to an improved solution (seven numbers no zeros)
d2M1: Finding 8 shadow costs and 9 improvement indices
d2A1: Shadow costs [Alt: A(0), B(-5), C(-2), D(-21), 1(18), 2(22), 3(21), 4(21)] and II CAO
d3M1: A valid route, their most negative II chosen, only one empty square used, θ's balance
d3A1: CSO (for part d) – so all previous marks in this part must have been awarded - **including exiting and entering cells stated correctly**

e1M1: Finding 8 shadow costs and all 9 improvement indices or at least 1 negative II found
e1A1: CAO for the shadow costs [Alt: A(0), B(-5), C(-4), D(-23), 1(18), 2 (22), 3(23), 4(21)] and the 9 positive IIs
e2A1: CSO (for part e) + reason + optimal

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>6. (a)</td><td>Row mins {-3, 0, -4} Column max {5, 5, 4}
Row maximin (0) $\neq$ column minimax (4) (so not stable)</td><td>M1
A1 (2)</td></tr>
<tr><td>(b)</td><td>E.g. add 5 to each element
Let p_1, p_2, p_3 be the probability of (A) playing 1, 2 and 3 respectively
(where $p_1, p_2, p_3 \geq 0$)
Let V = value of the game (to player A)

Maximise (P =) V

Subject to:
$V - 10p_1 - 7p_2 - p_3 + r = 0$
$V - 2p_1 - 10p_2 - 4p_3 + s = 0$
$V - 6p_1 - 5p_2 - 9p_3 + t = 0$
$p_1 + p_2 + p_3 (+u) = 1$
$(r, s, t, u \geq 0)$</td><td>B1
B1
B1
B1

M1 A1 A1 (7)</td></tr>
<tr><td>(c)</td><td>e.g. (adding 5 to each element)

<table>
<tr><th>b.v.</th><th>V</th><th>p_1</th><th>p_2</th><th>p_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>Value</th></tr>
<tr><td>r</td><td>1</td><td>-10</td><td>-7</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>s</td><td>1</td><td>-2</td><td>-10</td><td>-4</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>t</td><td>1</td><td>-6</td><td>-5</td><td>-9</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>u</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td>P</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>
</td><td>B1 M1 A1 (3)

12 marks</td></tr>
</table>

Notes for Question 6

a1M1: Finding row minimums and column maximums – condone one error
a1A1: CAO must state that $0 \neq 4$ (oe) – if $0 \neq 4$ stated with no working then award M1A0 only

b1B1: Making all terms non-negative (any addition ≥ 4 is acceptable)
b2B1: Defining probability variables
b3B1: Defining V
b4B1: 'maximise' + function/expression
b1M1: At least three (of the four) equations **or** inequalities in V, p_1, p_2, p_3 (with all p_i terms in the first three constraint equations having correct signs for the coefficients) – condone no slack variables for this mark
b1A1: CAO - the three constraints involving V and p_i expressed as equations with slack variables
b2A1: Probability sum **equation** correct (allow presence of a slack variable in this equation)

c1B1: All row and column labels correct for Simplex tableau
c1M1: Any two (numerical in nature) rows correct following from their constraints **or** a 'correct' answer (no follow through) with either one column or one row or one of both (so both a row and column) missing
c1A1: CAO – candidates may not label columns or rows in the order as given above – please check these carefully. Furthermore, candidates may add any value ≥ 4 which will change the nine bolded values above (so if +4 has been used this will increase each of the bolded values by +1). If all these bolded values are different then check the candidate's original constraints in (b) to see if consistent with equations seen earlier

Question Number	Scheme	Marks

7. (a)

Stage	State	Action	Dest	Value
May	3	1	0	75 + 150 = 225*
(4)	2	2	0	50 + 150 = 200*
	1	3	0	25 + 150 = 175*
	0	4	0	150 + 400 = 550*
April	3	3	0	75 + 150+ + 550 = 775*
(6)		4	1	75 + 150 + 400 + 175 = 800
		5	2	75 + 150 + 400 + 200 = 825
	2	4	0	50 + 150 + 400 + 550 = 1150
		5	1	50 + 150 + 400 + 175 = 775*
	1	5	0	25 + 150 + 400 + 550 = 1125*
March	3	3	1	75 + 150 + + 1125 = 1350*
(5)		4	2	75 + 150 + 400 + 775 = 1400
		5	3	75 + 150 + 400 + 775 = 1400
	2	4	1	50 + 150 + 400 + 1125 = 1725
		5	2	50 + 150 + 400 + 775 = 1375*
	1	5	1	25 + 150 + 400 + 1125 = 1700*
Feb	3	0	1	75 + 1700 = 1775
(2)		1	2	75 + 150 + 1375 = 1600
		2	3	75 + 150 + 1350 = 1575*
	2	1	1	50 + 150 + 1700 = 1900
		2	2	50 + 150 + 1375 = 1575
		3	3	50 + 150 + 1350 = 1550*
	1	2	1	25 + 150 + 1700 = 1875
		3	2	25 + 150 + 1375 = 1550*
		4	3	25 + 150 + 400 + 1350 = 1925
	0	3	1	150 + 1700 = 1850*
		4	2	150 + 400 + 1375 = 1925
		5	3	150 + 400 + 1350 = 1900
Jan	0	2	0	150 + 1850 = 2000
(2)		3	1	150 + 1550 = 1700*
		4	2	150 + 400 + 1550 = 2100
		5	3	150 + 400 + 1575 = 2125

M1 A1 A1 (April)

M1 A1ft A1 (March)

M1 A1ft A1 (February)

M1 A1 (January)

Month	January	February	March	April	May
Number made	3	3	5	5	3

B1

Minimum cost: (£) 1700

B1 **(13)**

(b)

$700 \times 19 - (6050 + \text{their } 1700) = (£)\ 5550$

M1 A1 **(2)**

15 marks

Question Number	Scheme	Marks

Notes for Question 7

All M marks – must bring optimal result from previous stage into calculations so for the first stage (April) if none of 225, 200, 175 or 550 (the optimal results from May) are used then M0. Ignore extra rows. Condone and credit rows that have been crossed out if they can still be read. Must have right 'ingredients' (storage costs, overhead costs, additional worker cost) at least once per stage (as an example for the six rows in April we must see at least one of these rows having a calculation that has either three or four values). Must have values in two of the three colums (State, Action, Dest). If no working seen then the number stated in the Value column must be correct to imply the correct method has been used

a1M1: First stage (April) completed. At least 6 rows, 'something' in each cell (but see M mark guidance above)
a1A1: Any two states correct (condone extra rows)
a2A1: CAO for first stage. No extra rows

a2M1: Second stage (March) completed. At least 6 rows, something in each cell (see M mark guidance above)
a3A1ft: Any two states correct – ft their * values/their smallest value from previous stage (condone extra rows)
a4A1: CAO for second stage. No extra rows

a3M1: Third stage (February) completed. At least 12 rows, something in each cell (see M mark guidance above)
a5A1ft: Any two states correct – ft their * values/their smallest value from previous stage (condone extra rows)
a6A1: CAO for third stage. No extra rows

a4M1: Fourth stage completed. At least 4 rows, something in each cell (see M mark guidance above)
a7A1: CAO (no ft) for fourth stage. No extra rows

a1B1:CAO – **but must have scored all previous M marks**
a2B1:CAO – condone lack of units - **but must have scored all previous M marks**

b1M1: $700\times19-(5\times450+19\times200+\text{their }1700)$ **or** $7250-\text{their }1700$. **Must have scored at least two M marks in (a)**
b1A1: CAO (condone lack of units) – correct answer with no working can score both marks in this part (**but is still dependent on at least two M marks awarded in (a)**)

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